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Constraint on time-reversal tests in fully chaotic nuclear systems

E. D. Davis
Fisika Departement, Universiteit van Stellenbosch, 7599 Stellenbosch, Republic of South Africa
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In tests exploiting the unique sensitivity of a fully chaotic system like the compound nucleus, the possibility exists that unknown resonance parameters and interaction matrix elements take on values which preclude the observation of a violation of time-reversal invariance independent of the strength of the time-reversal noninvariant interaction. A nontrivial constraint on time-reversal tests is implied, namely, that the observable has to be sampled a minimum number of times $M_{\min}$ where $M_{\min} \geq 3$. Several experiments purporting to be tests of time reversal do not satisfy this condition.

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There is now quite a diverse class of reaction tests of time reversal exploiting the characteristics of the chaotic quantum many-body system, the compound nucleus (CN). This class includes traditional detailed balance (TDB) measurements in the strongly overlapping resonance regime, most recently performed by Blanke *et al.* [1], the capture-asymmetry shift (CAS) measurement performed by Barabanov *et al.* [2], and the fivefold correlation (FC) and threefold correlation (TC) measurements proposed in [3]. Considerable effort is currently being devoted to the realization of these FC and TC measurements [4], while, motivated by [5], a quantitative survey of prospects for TDB measurements in the isolated resonance regime has appeared recently [6].

All these tests probe for the presence of an antisymmetric contribution $S^{(a)}$ to the S matrix arising from the coupling of CN resonances (μ and ν) by a time-reversal noninvariant interaction V' (with the standard choice of phase convention, matrix elements of V' are imaginary; throughout, $V'_{\mu\nu}$ denotes this imaginary part). The tests are of interest in that, as illustrated by the example of analogous on-resonance parity-violation measurements [7], they can benefit from considerable enhancements in sensitivity. Generically, there is (in the terminology of [8]) resonance enhancement, referring to the fact that $(S^{(a)})_{\text{on-resonance}}/(S^{(a)})_{\text{off-resonance}} \sim D/\Gamma$, where D is the average spacing between resonances and Γ their average width (in the isolated resonance regime of heavy nuclei $D/\Gamma \sim 10^2$–10^3), and dynamic enhancement, referring to the fact that matrix elements $V'_{\mu\nu}$ appear in a combination $[V'_{\mu\nu}/(E_\mu - E_\nu)$, where E_μ and E_ν denote resonance energies] which scales as $\sqrt{N}$, where N is the number of principal single-particle components of the states involved (typically, $N \sim 10^6$ for the CN states of interest).

Despite their sensitivity, it is unlikely that these reaction tests of time reversal will do more than constrain the magnitude of V'. In themselves, bounds on V' can be instructive as the example of the limit on the electric dipole moment of the neutron illustrates. However, for CN reaction tests, the extraction of a bound is not straightforward. In the planning and implementation of CN reaction tests of time reversal, it has been customary to assume that it is enough to perform *one* measurement at a suitably chosen energy. In this Brief Report, we argue on the basis of statistical considerations that an experiment involving only a single measurement cannot usefully constrain the stength of time-reversal noninvariant interactions.

The difficulties associated with the extraction of a bound have their origin in the fact that the analysis of a time-reversal test requires a stochastic treatment of the underlying observable. To indicate the character of this theoretical treatment, let us consider (for the sake of definiteness) the case of FC measurements. In a FC experiment, a null measurement at resonance μ constrains a linear combination of the form [8]

$$\begin{aligned}\Delta_\mu &= \frac{1}{\Gamma_\mu} \sum_{\substack{\lambda \\ \lambda\neq\mu}} \left(\gamma^\mu_{nj}\gamma^\lambda_{nj'} - \gamma^\mu_{nj'}\gamma^\lambda_{nj}\right) \frac{V'_{\lambda\mu}}{(E_\lambda - E_\mu)} \\ &\equiv \sum_{\substack{\lambda \\ \lambda\neq\mu}} A^{(\mu)}_\lambda V'_{\lambda\mu}, \end{aligned} \tag{1}$$

where the sum extends over adjacent resonances of the same spin and parity (γ^λ_{nj} denotes the partial width amplitude for emission of a neutron of spin j). The sum in Eq. (1) is, in general, over very many resonances. Thus, in common with other CN reaction test obervables, Δ_μ involves a large number of independent matrix elements $V'_{\lambda\mu}$ which cannot be determined individually from experiment. Nor, because of the complexity of CN states, can they be reliably calculated from a nuclear model.

The stochastic approach [8] circumvents this problem by allowing for all possible values of the matrix elements $V'_{\lambda\mu}$. Fortunately, because of the fully chaotic character of a CN system, the nature of CN matrix element distributions is simple. On general grounds [9], it can be assumed that the distribution of values of CN matrix elements $V'_{\lambda\mu}$ for different pairs of resonances λ and μ is a zero-centered Gaussian. A single parameter then suffices to characterize the distribution of matrix elements $V'_{\lambda\mu}$, namely, its variance v^2, and, in the corresponding analysis of time-reversal test data, attention is focused on extraction of information on this parameter.

Within the stochastic treatment, the value of Δ_μ is viewed as being drawn from a zero-centered Gaussian of variance

$$v_\mu^2 \;=\; \sum_{\substack{\lambda \\ \lambda\neq\mu}} (A_\lambda^{(\mu)})^2\, v^2 \;\equiv\; A_\mu^2\, v^2. \qquad (2)$$

In the event that all the resonance parameters are known (i.e., all the coefficients $A_\lambda^{(\mu)}$ are known), introduction of the auxiliary observable $\delta_\mu = \Delta_\mu/A_\mu$ is indicated [10]. Like the matrix elements $V'_{\lambda\mu}$, δ_μ has a Gaussian distribution of zero mean and variance v^2, a fact which can be used to obtain v^2. Should there be any unknown resonance parameters, then the stochastic treatment can be generalized accordingly [8]. The distribution of the auxiliary observable which replaces δ_μ remains symmetric but is, in general, distinctly non-Gaussian, being somewhat broader.

Let us now turn to the issues involved in the extraction of a bound on v^2. The sensitivity of individual measurements is of primary importance in determining the level at which a reaction test can distinguish if v^2 is nonzero or not. However, within the stochastic analysis of a single null measurement, the possibility that the values of the matrix elements $V'_{\lambda\mu}$ (and any resonance parameters which may be unknown) have caused the observable (e.g., Δ_μ) to take on a value which precludes detection at the level of sensitivity of the experiment cannot be excluded *a priori.* This possibility arises no matter what the magnitude of v is. In short, *it is not appropriate to assume that a single null measurement automatically places a useful bound on* v^2. On a more mundane level, we note that, within standard sampling theory, it is not possible to infer an estimate for the variance of a random variable without knowledge of at least *two* independent values and this restriction holds regardless of the actual magnitude of the variance.

It is apparent that we need a criterion which identifies whether or not a CN reaction test experiment does, from a statistical point of view, usefully constrain time-reversal noninvariance — i.e., sets a bound on v^2 at the nominal level of sensitivity of the experiment at a reasonable confidence level. In arriving at a criterion, we find it helpful to turn this issue around and pose the question, under what circumstances would an experiment *not* be suitable? We believe (and the arguments we present below seem to confirm) that these statistical concerns boil down to the following restriction: There is a certain minimum number of independent measurements, $M_{\min}$, which need to be performed. The example cited above of estimating the variance of a random variable within standard sampling theory indicates that this condition should prove nontrivial; at the very least, $M_{\min} \geq 2$.

To arrive at a more quantitative result, we need to decide upon a specific method for the statistical evaluation of the reaction test's data. Unfortunately, no entirely satisfactory prescription is currently available (although a Bayesian-type approach would be adequate if ambiguities concerning the choice of *prior* distribution can be resolved). Nevertheless, we have found that a simple-minded approach based on the use of "standard errors" [11] is instructive (the defect of this approach is that it does not guarantee that estimates for v^2 are positive). Supposing a data set $\{X_i\}$ comprising M independent measurements ($i = 1,\ldots,M$) of a time-reversal test observable X, we use

$$m_2 = \frac{1}{M}\sum_i X_i^2 \qquad (3)$$

to estimate v^2 and

$$m_4 = \frac{1}{M}\sum_i X_i^4 \qquad (4)$$

to gauge the uncertainty inherent in this estimate. (It suffices to consider only even moments because, in all cases of interest, odd moments vanish.)

To proceed along these lines, some input on the form of the probability distribution $p_{\rm expt}(\{X_i\})$ for a data set $\{X_i\}$ is required. The nontrivial component is the theoretical distribution function $p_{\rm th}$ for the value X of a reaction test observable in the absence of any experimental uncertainties. It accommodates, along the lines developed in [8], the possible variations in the matrix elements $V'_{\mu\nu}$ (as well as, if need be, the unknown resonance parameters) and is parametrized in terms of the unknown v^2. In fact, it is the conditional probability $p_{\rm th}(X|v^2)$ and, without loss of generality, may be taken to have variance v^2 [8]. To obtain $p_{\rm expt}(\{X_i\})$, it remains to convolute $p_{\rm th}$ with distributions describing the experimental uncertainties. In line with standard practice, we take the experimental error associated with the ith measurement to be drawn from a zero-centered Gaussian of variance σ_i^2. Thus, the desired experimental probability distribution

$$p_{\rm expt}(\{X_i\}) \;=\; \int d[Y_i] \prod_{i=1}^{M} \frac{1}{\sqrt{2\pi\sigma_i^2}} \exp\!\left[-\frac{(X_i-Y_i)^2}{2\sigma_i^2}\right] \times\, p_{\rm th}(Y_i|v^2), \qquad (5)$$

where $d[Y_i]$ denotes integration over all the Y_i's. Odd moments of $p_{\rm th}(X|v^2)$ and, hence, $p_{\rm expt}(\{X_i\})$ all vanish.

Let us now take up the use of m_2 and m_4. Trivially, the average value $\langle m_2\rangle$ of m_2 is

$$\langle m_2\rangle \;\equiv\; \int d[X_i]\, p_{\rm expt}(\{X_i\})\, m_2 \;=\; v^2 + \frac{1}{M}\sum_i \sigma_i^2. \quad (6)$$

Hence, an estimator (in the jargon of statistics) for v^2 is $\theta = m_2 - \theta_{\rm err}$, where $\theta_{\rm err}$ is the arithmetic average of the estimators $\theta_{\rm err}(i)$ for the uncertainties σ_i^2. The latter are taken to be known independently from an assessment of counting statistics and systematic errors. We assume that θ evaluates to a positive number (otherwise the present sampling method fails).

The standard error in our estimate θ for v^2 is its variance $\mathrm{var}(\theta)$. Under the assumption that m_2 and θ_{err} are uncorrelated (reasonable if we assume errors are dominated by counting statistics), the variance in θ is $\mathrm{var}(\theta) = \mathrm{var}(m_2) + \mathrm{var}(\theta_{\mathrm{err}})$. A lower bound to the variance in θ (which will suffice below) is thus given by

$$\mathrm{var}(m_2) \equiv \langle m_4 \rangle - \langle m_2 \rangle^2 = \frac{1}{M}\left[\frac{2}{M}\sum_i (v^2+\sigma_i^2)^2 + \kappa v^4\right], \qquad (7)$$

where we have introduced the kurtosis κ of $p_{\mathrm{th}}(X|v^2)$ which allows us to express the fourth moment of this distribution in terms of its second through $\langle X^4\rangle = (\kappa+3)\langle X^2\rangle^2$ [here, $\langle X^k\rangle$ denotes the kth moment of $p_{\mathrm{th}}(X|v^2)$]. We note that the kurtosis is a measure of the broadness of the distribution (relative to that of a Gaussian of variance v^2) which is independent of the magnitude of v^2.

Armed with these results for θ and $\mathrm{var}(\theta)$, we now consider the circumstances under which the inequality

$$\theta^2 \gg \mathrm{var}(\theta) \qquad (8)$$

is satisfied. Adopting the above lower bound for $\mathrm{var}(\theta)$ [with v^2 replaced by its estimator θ and each σ_i^2 by its estimator $\theta_{\mathrm{err}}(i)$], Eq. (8) implies

$$(M-2-\kappa)\left(\frac{\theta}{\theta_{\mathrm{err}}}\right)^2 \gg 4\left(\frac{\theta}{\theta_{\mathrm{err}}}\right) + 2(1+r), \qquad (9)$$

where

$$r = \frac{1}{M}\sum_i [\theta_{\mathrm{err}}(i) - \theta_{\mathrm{err}}]^2/\theta_{\mathrm{err}}^2 \,. \qquad (10)$$

Observe that, since we assume $\theta > 0$ and r is always positive, the right-hand side of Eq. (9) is positive. Thus, from the fact that the left-hand side of Eq. (9) must be positive if the inequality is to be satisfied, we read off the following condition: The sample size M must be such that

$$M \geq M_{\mathrm{min}} \equiv \mathrm{int}(\kappa) + 3, \qquad (11)$$

where $\mathrm{int}(\kappa)$ denotes the largest integer less than or equal to κ. Representative values of M_{min} for CN reaction tests of time reversal are quoted in Table I (complete spin assignments are assumed). In all cases of interest, $\kappa \geq 0$ so that $M_{\mathrm{min}} \geq 3$.

What is the significance of the result in Eq. (11)? We claim that it serves to delineate experiments which are suitable for setting bounds on time-reversal noninvariance from those which are not. We note that, if the inequality in Eq. (11) is not satisfied, then the inequality in Eq. (8) cannot hold — i.e., θ cannot be distinguished from zero. This conclusion applies independent of the magnitude of v^2 or its relation to the nominal sensitivity of the experiment (as measured by θ_{err}). Thus, unless Eq. (11) is satisfied, an experiment will be unable to distinguish the estimator θ for v^2 from zero even in the event that v^2 exceeds the nominal level of sensitivity of the experiment. Clearly, interpretation of the data of an experiment which does not fufill Eq. (11) as null measurements must be suspect.

The expression obtained for M_{min} is plausible. The occurrence of the number 3 in the expression reflects the fact that, in the absence of any prior information, at least three values of a random variable are required to gauge the reliability of an estimate of the variance. Likewise, the dependence on κ arises because the broader (narrower) $p_{\mathrm{th}}(x|v^2)$ is, the more (fewer) measurements one would expect to have to perform to pin down its variance v^2. For these reasons, we believe that the M_{min} criterion should survive in a statistical analysis which guarantees that θ is positive.

The implications of the M_{min} criterion in Eq. (11) for CN reaction tests of time reversal are fairly sobering. In the planning of the epithermal TC measurement [4] and the implementation of the CAS experiment reported in [2], it has been assumed that it is enough to perform *one* measurement at a suitably chosen energy. Even if complete spectroscopic information on resonance parameters were available in the CAS experiment (which is not the case), meaning that p_{th} could be chosen so that $\kappa = 0$, at least two more measurements would be required to satisfy the M_{min} criterion. We conclude that the CAS experiment of [2] cannot legitimately be used to constrain time-reversal noninvariance. More worrying perhaps is the implication that the current strategy for the projected epithermal TC measurement of sitting on a single p-wave resonance in ^{139}La (at 0.63 eV) is misguided. Further p-wave resonances of the same spin must be found in ^{139}La or this choice of target must be discarded.

Not all time-reversal tests fall foul of the M_{min} criterion. The proposal for epithermal FC measurements discussed in Ref. [12] can easily accommodate the M_{min} criterion. The same is true of the TDB measurements envisaged by Drake *et al.* [6]. In this latter proposal, there is some freedom in the choice of target. Five are considered in Ref. [6]: ^{23}Na, ^{27}Al, ^{31}P, ^{35}Cl, and ^{39}K. In view of the M_{min} criterion, the first two would have to be discarded on the basis of the currently available data (in neither have more than two resonance pairs of the same spin and parity been identified). Of the remainder, ^{35}Cl

TABLE I. M_{min} for various CN reaction tests.

Experiment	Assumption	M_{min} (κ)
TDB tests [6]	Complete spectroscopic information	3 (0)
TC test [4]	Only ratio of neutron partial widths unknown	4 ($\frac{3}{2}$)

would appear to be the most promising candidate: As many as eight resonance pairs of same spin and parity (1^-) have been identified.

In conclusion, we have presented a simple criterion (in terms of the minimum number of independent measurements which have to be performed) to judge whether a CN reaction test of time reversal will actually set a useful bound on the strength of time-reversal noninvariant interactions.

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