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Statistical model of three nucleon pion absorption

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In this paper we present a model for three nucleon pion absorption based on the assumption that pion absorption on three nucleons is a one-step process. In the assumption that the square of the effective matrix element is proportional to the probability that three nucleons are contained inside a certain volume V, this model is equal to the Fermi statistical model. The construction of the volume V is different from the Fermi model. Very good agreement between this simple model and experimental data for the energy dependence of the three-nucleon pion absorption cross section on ^{3}He and ^{3}H opens a new possibility for the explanation of multinucleon effects in pion absorption.

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Pion absorption is a process in which a hadronic boson is absorbed on a nucleus, resulting in the emission of energetic nucleons. Because of momentum and energy conservation, pion absorption is forbidden on a free nucleon and highly suppressed on a single nucleon inside the nucleus. The next possible channel for pion absorption in nuclei is quasifree absorption on two nucleons. There has been extensive experimental study of this channel because the identification of two-nucleon absorption as a quasifree process is relatively easy, requiring the detection of two nucleons which share the total pion energy and are emitted roughly back to back in the respective center-of-mass system. The quasifree two-nucleon absorption process may be distorted by scattering of the incident pion prior to absorption (initial-state interaction) and/or rescattering of nucleons after the absorption (final-state interaction). However, the strength of these processes does not account for the observed total absorption cross section. Speculation on the existence of other reaction dynamics involving more than two nucleons in the nucleus has not been easy to verify. The experimental identification of processes in which more than two nucleons are involved in a coherent way has been difficult in conventional limited solid angle experiments because the data generally lack either kinematic completeness or large phase space coverage. Only in recent years has there been a significant shift from studying pion absorption on two nucleons toward studying reactions in which the pion has been absorbed on more than two nucleons [1]. The status of studies of pion absorption may be found in three recent review papers [2–4].

From many attempts which have been made to prove the existence of absorption processes involving more than two nucleons [5], we will restrict ourselves only to the results of the experiments which measure pion absorption on three-nucleon targets. Such targets allow for kinematically complete experiments with just a two-arm coincidence. Because of the absence of additional nucleons, such nuclei are ideal for studying three-nucleon absorption processes.

Three-nucleon events are the class of events in which three nucleons participate substantially in the sharing of the momentum and energy of the absorbed pion. From this class of events we will exclude events with collinear emission of deuteron-nucleon pairs or events where two nucleons have small relative momentum and the third nucleon is emitted in the opposite direction. It is customary to describe such events as quasifree two-nucleon absorption followed by a final-state interaction of the Watson-Migdal type [6,7]. Their contribution to the total absorption cross section is smaller than the uncertainties which are quoted for the total absorption cross section [5,8]. Experimental results to date suggest that the rest of the three-nucleon events fill the entire kinematically allowed region with a roughly constant three-body phase space density; in the rest of this paper, we will define three-nucleon absorption (3NA) as only this class of events.

The most recent kinematically complete experimental results on three-nucleon absorption in ^{3}He [5,9–12] and ^{3}H [8] agree quite well on the character of the three-nucleon absorption process and the contribution of this process to the total absorption cross section. At the same time these experiments do not report clear evidence that initial-state interactions or final-state interactions are important in pion absorption on these nuclei.

Most theories explain multinucleon absorption as a multistep process. These theories range from the framework of a simple intranuclear cascade model calculation [13,14] to those in which more direct multinucleon absorption mechanisms are included as sequential or successive (nonsequential) Δ-hole processes [15–18], very similar to the well-known double Δ model for four-nucleon absorption [19,20]. Even in the direct multinucleon absorption calculation, the major contribution to three-nucleon absorption comes from a two-step process.

The characteristics of three-nucleon absorption as a statistical phase space distribution of outgoing nucleons forces us to attempt to find a solution in a different direction. Even if it is not excluded *a priori* that a calculation using Feynman diagrams may result in a phase space distribution, it is not easy to relate a phase space distribution to a specific Feynman diagram. There are alternative approaches in the calculation of the pion absorption cross section. For example, models from statistical physics are used in the calculation of pion-nucleus scattering and absorption as a solution of the Boltzmann

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equation [21], and the stochastic method is used for the calculation of scattering and absorption of hadrons on ^{4}He [22]. Other approaches may be found in the review paper by Thies [23].

In this paper a different approach to the calculation of the pion absorption cross section on three nucleons is proposed. In a previous paper [24] it was shown that, in the calculation of isospin effects on three-nucleon pion absorption in light nuclei, there was good experimental agreement with the assumption that no characteristics of the individual nucleons in the three-nucleon system are important and that the process can be reduced to the interaction of a pion with the nucleus or the group of nucleons as a whole, characterized by its total isospin. In neglecting the properties of the individual nucleons, we assumed that a pion is absorbed on a three-nucleon system as a one-step process. As a consequence, it may be possible to treat pion absorption in a three-nucleon system under certain conditions without recourse to any specific theories of nuclear interactions.

The basis for this model is the assumption that three-nucleon pion absorption can take place if the nucleons in the three-nucleon system are contained inside a small volume. This assumption may be supported in two different ways. The first argument is the same as that used for the Fermi statistical model for multiple production of particles in high energy nucleon collisions. If the energy in the two-nucleon collision is released in a very small volume and the interaction is very strong, the distribution of energy will be determined by statistical laws [25–27]. The second argument is that if the pion wavelength is of the order of the dimension of the three-nucleon object, one cannot resolve individual pion-nucleon interactions. It is thus reasonable to expect a statistical distribution of the nucleons since any other distribution will be in contradiction with the resolution. Even if one notices that these two arguments may be in contradiction, they are mentioned here only as possible justifications for a statistical model. We are interested here in the consequences of such a simplified statistical interpretation. This description of three-nucleon absorption is unconventional since it neglects such mechanisms as the excitation of the Δ resonance which is known to dominate pion scattering and two-nucleon absorption. Nevertheless, it agrees with the distributions found in the experiments.

The cross section for the reaction

$$\pi + (3N) \to N_1 + N_2 + N_3 \tag{1}$$

can be written using the relativistic form of Fermi's golden rule for scattering [28,29] as

$$\sigma_{3N} = |M|^2 \frac{S}{4\sqrt{(p_\pi p_{3N})^2 - (m_\pi m_{3N})^2}} \rho_3, \tag{2}$$

where ρ_3 is Lorentz-invariant three-nucleon phase space:

$$d\rho_3 = \frac{1}{(2\pi)^9} \frac{d\vec{p}_1}{2E_1} \frac{d\vec{p}_2}{2E_2} \frac{d\vec{p}_3}{2E_3} (2\pi)^4 \times \delta^4(p_\pi + p_{3N} - p_1 - p_2 - p_3), \tag{3}$$

and M is the transition amplitude, S is a statistical factor, p_i, $i = \pi, 3N, 1, 2, 3$, are the four-momenta of the pion, three-nucleon system, and the three final nucleons, m_π and m_{3N} are the pion and three-nucleon system masses, and E_i and $\vec{p}_i$, i=1,2,3, are the total energies and momenta of the three final nucleons.

As in the case of the Fermi statistical model [25], the assumption is that the square of the effective matrix element $|M|^2$ is proportional to the probability that three nucleons are contained inside a volume V:

$$|M|^2 = \left(\frac{V}{(2\pi)^3}\right)^2. \tag{4}$$

Finally, using Eqs. (2) and (4), we may write the total three-nucleon absorption cross section as

$$\sigma_{3N} = C \frac{V^2}{\sqrt{(p_\pi p_{3N})^2 - (m_\pi m_{3N})^2}} \rho_3, \tag{5}$$

where C is a constant. Up to this point we have exactly followed the formulation of the Fermi statistical model from the point of view of the scattering theory. A detailed description of the method may be found in the review paper by Kretzschmar [30]. To work out the essential points, the same level of approximation as in Ref. [30] is applied, meaning that while conservation of energy and momentum is taken into account in the calculation, conservation of angular momentum is not. It is assumed as in the Fermi statistical model [25] that the results obtained by neglecting conservation of angular momentum differ only by a small numerical factor from those in which angular momentum is conserved.

To understand the dependence of the three-nucleon absorption cross section on kinematic variables, we must first define the volume V. The transverse dimension, normal to the pion beam, of the three-nucleon system must be of the order of magnitude of the range of nuclear force and is taken to be the pion Compton wavelength, i.e., $a \approx m_\pi^{-1}$ [25–27]. In the construction of the longitudinal dimension, our model differs from the Fermi statistical model. The longitudinal dimension must be constructed such that it corresponds to the absorption of a real pion. From the uncertainty relation the time interval for which a three-nucleon system can have an imbalance of energy corresponding to a real pion energy E_π is equal $\Delta t \approx E_\pi^{-1}$. In this time a real pion travels a distance $b \approx |\vec{p}_\pi|/(E_\pi^2)$, where $|\vec{p}_\pi|$ is momentum of the real absorbed pion. In this way we have constructed the volume which is equivalent to the real pion through uncertainty relations. So, in the three-nucleon system, for three-nucleon absorption to occur, three nucleons must be inside a volume V_0:

$$V_0 = \tilde{C} \frac{|\vec{p}_\pi|}{E_\pi^2}. \tag{6}$$

Since the calculation of the cross section is done in the pion–three-nucleon center-of-mass system, the longitudinal dimension is shortened by Lorentz contraction and the volume V is

$$V = V_0 \frac{(m_\pi + m_{3N})}{E_{\text{c.m.}}}, \tag{7}$$

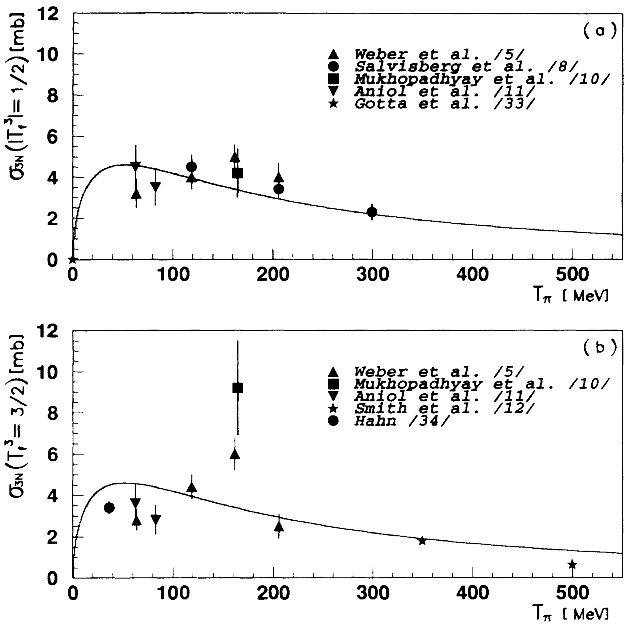

FIG. 1. Comparison of the total three-nucleon–pion absorption cross section calculated using Eq. (9) with the existing experimental data for ^{3}He and ^{3}H: (a) for two protons and a neutron or two neutrons and a proton in the final state, (b) for three protons in the final state.

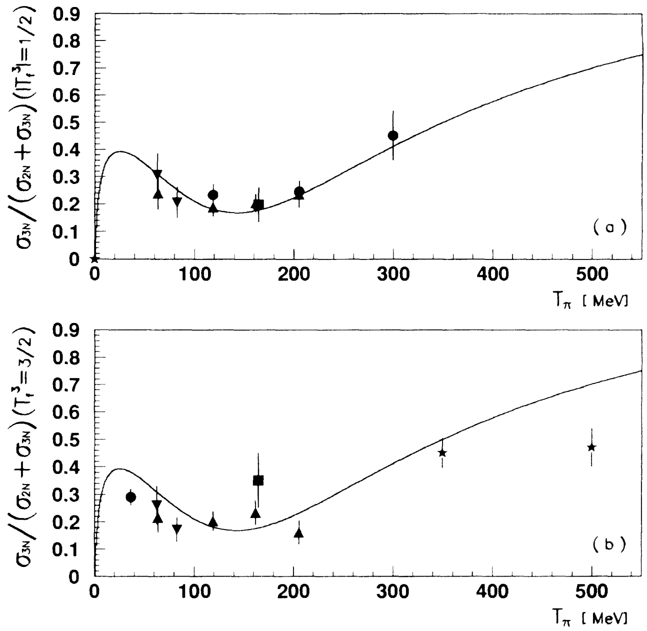

FIG. 2. Comparison of the relative contribution of the three-nucleon absorption cross section calculated using Eqs. (9) and (10) with the existing experimental data for ^{3}He and ^{3}H: (a) for two protons and a neutron or two neutrons and a proton in the final state, (b) for three protons in the final state. Notation is the same as in Fig. 1.

where $E_{\mathrm{c.m.}}$ is the total center-of-mass energy.

Finally, we may write a complete expression for three-nucleon–pion absorption. In the pion–three-nucleon center-of-mass system,

$$\sqrt{(p_\pi p_{3N})^2 - (m_\pi m_{3N})^2} = |\vec{p}_{\pi\mathrm{c.m.}}| E_{\mathrm{c.m.}}, \qquad (8)$$

where $\vec{p}_{\pi\mathrm{c.m.}}$ is the pion momentum in the pion–three-nucleon center-of-mass system. The cross section, as a function of pion energy, is

$$\sigma_{3N} = C \frac{|\vec{p}_\pi|^2}{E_\pi^4} \frac{(m_\pi + m_{3N})^2}{|\vec{p}_{\pi\mathrm{c.m.}}| E_{\mathrm{c.m.}}^3} \rho_3, \qquad (9)$$

where the three-nucleon density of final states ρ_3 can be calculated numerically using, for example, the CERN library routine GENBOD [31]. Comparison of the model prediction with experimental results is shown in Fig. 1. We have adjusted the normalization constant C to scale to the data. In our model there is no isospin dependence on the three-nucleon absorption cross section, and so the same constant is used for both π^+ and π^- absorption. To our knowledge there are no other existing models in which three-nucleon–pion absorption cross section is calculated for ^{3}He or ^{3}H. The models described earlier in the paper have typically been applied to three-nucleon absorption on nuclei heavier than ^{3}He or ^{3}H. When applied to the cases of ^{3}He or ^{3}H, they do not show as good agreement with experiment as our statistical model [3].

The experimental uncertainties coming from data normalization are considerably reduced if instead of σ_{3N} we plot the ratio of the measured three-nucleon absorption cross section to the measured total cross section $\sigma_{3N}/\sigma_{\mathrm{tot}}$. In this case we shall calculate σ_{tot} as

$$\sigma_{\mathrm{tot}} = \sigma_{2N} + \sigma_{3N} = 1.5\sigma_{\mathrm{D}} + \sigma_{3N}, \qquad (10)$$

where we use the Ritchie semiempirical parametrization of the energy dependence of the total cross section for pion absorption on the deuteron below 1 GeV [32] for the determination of σ_{D}. The factor 1.5 is the scaling of the deuteron absorption cross section by the number of deuteronlike pairs in ^{3}He or ^{3}H for quasifree two-nucleon absorption. Using this factor, we ignore other differences between deuteron and three-nucleon targets, such as the difference in density, but for the purposes of our calculation this is a good enough approximation to the results reported by most experiments [5,8–12]. The agreement between our calculation of the relative contribution of the three-nucleon–pion absorption cross section and measured values is shown in Fig. 2. In the case of the state with a final total isospin projection $T_f^{(3)} = \pm\frac{1}{2}$, the agreement between the experimental data and our model is remarkably good.

The statistical model for three-nucleon–pion absorption proposed in this paper is based on some nontrivial assumptions. Even though not meant to completely explain three-nucleon–pion absorption, the agreement with existing data suggests that three-nucleon absorption may be explained as a statistical process and may generally open a new way of looking at multinucleon phenomena in nuclear physics.

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