Consequences of neutron-proton interactions on backbending

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The influence of neutron-proton correlations on backbending is investigated in a single-j-shell model. It is shown that these correlations are quenched if the neutrons and protons fill a high-j intruder orbital asymmetrically. This is due to the repulsive neutron-proton correlation energy in these configurations and is responsible for the observed forking of the ground state band into proton and neutron S bands.

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I. INTRODUCTION

The standard cranked shell model (CSM) assumes that neutrons and protons move independently in a rotating deformed mean field. Within such an independent-particle approach the backbend, e.g., due to the neutron S band, occurs independently of the configuration of the other quasiparticles (qp's). In particular, its crossing frequency and the amount of the aligned angular momentum do not depend on whether there has already been an alignment of protons. Such a simplistic independent-qp picture describes surprisingly well the proton and neutron backbends observed in the rare earth and actinide regions where neutrons and protons occupy different high-j intruder orbitals that generate the S bands [1,2].

In lighter nuclei ($A \leq 130$) the neutron and proton S bands come from the *same* high-j orbitals ($h_{11/2}$ or $g_{9/2}$). It has been pointed out [3,4] that for the case of equal occupancy of protons and neutrons in a high-j shell a rather different alignment pattern may emerge. The neutron-proton (n-p) interaction causes a 50:50 mixing of the proton and neutron S bands resulting in a $T=0$ and a $T=1$ S band with the latter lying about 500 keV above the former. A clear confirmation of this prediction still awaits detailed high-spin data on $N=Z$ nuclei, which will become available from the radioactive beam facilities being constructed or planned [5]. The nature of the backbend can be inferred most directly from a g-factor measurement of states in the crossing region. The case nearest to $N=Z$ for which this has been achieved is ^{84}Zr [6]. In this case, however, the results for the first crossing are consistent with a purely proton alignment.

On the other hand there are many data on nuclei with asymmetric filling of the *same* high-j shell before the crossing. Here again though there is little evidence for strong mixing of the proton and neutron S bands (see for example, Refs. [7–9] for the $A=130, 100$, and 80 regions, respectively). The most striking example is ^{128}Ba [10], which has, roughly speaking, two protons (particles) and two neutron holes in the $h_{11/2}$ shell. The analysis of the energies and $E2$-branching ratios in terms of a three-band model (g, S_ν, S_π) sets an upper limit of 0.5 keV for the effective interaction between the neutron and proton S bands.

In the present paper we study the mechanism that attenuates the mixing of the neutron and proton S bands when the neutron and proton occupation of a j shell becomes increasingly asymmetric. The model used is the same as in [3,4] considering an $h_{11/2}$ instead of a $g_{9/2}$ orbital.

II. SINGLE-J-SHELL MODEL

We start with a model Hamiltonian consisting of a cranked deformed one-body term and a scalar two-body interaction,

$$H' = h' + V$$
$$= -\sum_i \left[4\left(\frac{4\pi}{5}\right)^{1/2} \kappa Y_{20}(\hat{r}_i) + \omega j_i^{(x)}\right] + \sum_{i<j} V_{ij}. \quad (1)$$

The interaction of the nucleons in the $h_{11/2}$ valence shell with the core nucleons is approximately described by the

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quadrupole field generated by them. The deformation energy κ is related to the usual deformation parameter β by $\kappa = 51.5A^{1/3}\beta$ in units of G [11] (an axial shape is assumed). The sums on i and j in Eq. (1) run over all the nucleons in the $h_{11/2}$ shell. The eigenstates of H' have good isospin. The two-body interaction V may be rewritten in terms of the two-body isospin T and angular momentum J,

$$V = \frac{1}{2}\sum_{JT} E_{JT}\hat{J}\hat{T}\left(A^{\dagger}_{JT} \otimes \tilde{A}_{JT}\right)_{00}, \quad (2)$$

with $\hat{x} = \sqrt{(2x+1)}$ and $A^{\dagger}_{JT} = (a^{+}_{j1/2} \otimes a^{\dagger}_{j1/2})_{JT}$. The conjugate operator $\tilde{A}$ is defined by

$$\tilde{A}_{JM;TT_3} = (-1)^{J-M+T-T_3} A_{J-M;T-T_3}. \quad (3)$$

In this form the interaction is manifestly scalar and isoscalar. [It is, of course, simpler to perform numerical calculations using matrix elements in an (m, T_3) scheme.]

It is, however, more useful for some of our subsequent discussion to express the Hamiltonian in Eq. (1) in terms of separate neutron (ν) and proton (π) components, since we wish to discuss the case where we have a few protons in the shell and a few neutron holes. Thus we write

$$H' = h'_{\pi} + h'_{\nu} + V_{\pi\pi} + V_{\nu\nu} + V_{\pi\nu}, \quad (4)$$

where the subscripts indicate how the summations in Eq. (1) are restricted to the corresponding proton or neutron indices. For the two-body proton-proton ($V_{\pi\pi}$), neutron-neutron ($V_{\nu\nu}$), and neutron-proton ($V_{\pi\nu}$) interactions, we choose the δ force

$$V_{mn} = -g_{mn}\sum_{i<j}\delta(\mathbf{r_i} - \mathbf{r_j}), \quad (5)$$

with $m(n) = \pi, \nu$. The matrix elements of the δ function may be expressed in terms of the energy

$$G_{mn} = \frac{g_{mn}}{4\pi}\int_0^{\infty} R^4(r)r^2 dr, \quad (6)$$

where R is the single-particle radial matrix element.

We now consider the effects of particle-hole conjugation on each of these terms separately. Since a hole in a state $|\mu\rangle$ in a full shell may be filled by a nucleon in the time-reversed state $|\bar{\mu}\rangle$, we obtain the well-known relation for a single-particle operator F,

$$\langle\mu_1^{-1}|F|\mu_2^{-1}\rangle = \langle\mu_1|F_c|\mu_2\rangle + \langle 0|F|0\rangle\delta_{\mu_1,\mu_2}, \quad (7)$$

where the conjugate operator is [12]

$$F_c = -(\mathcal{T}F\mathcal{T}^{-1})^{\dagger}. \quad (8)$$

The vacuum expectation value vanishes unless F is a scalar. It thus vanishes for both of our two-body operators Y_{20} and $j^{(x)}$. Both of these operators are Hermitian and the former is invariant under time reversal, whereas the latter changes sign. Thus the deformation matrix elements change sign under particle-hole conjugation and the Coriolis matrix elements remain unchanged. The changing sign of the deformation term is an important effect in our subsequent discussions.

Applying similar arguments to our two-body interaction, it is easy to see that (apart from an overall energy shift) the term $V_{\nu\nu}$ is unchanged if we are considering neutron holes, whereas $V_{\pi\nu}$ changes sign.

III. RESULTS WITHOUT DEFORMATION

In order to gain some insight into the nature of the various aligning configurations, we present the calculations with the deformation term switched off [$\kappa = 0$ in Eq. (1)]. The eigenstates have good angular momentum J. This results in sharp crossings when considered as functions of the rotational frequency ω (see Fig. 4). The consequences of the deformation term will be discussed below in a more qualitative way.

Figure 1(a) shows the spectrum of the two-particle case. [For the n-p system, all the states shown are allowed, whereas for the p-p and n-n systems, only the even-J states ($T = 1$) exist.] The spectrum has the feature expected for a short range force: The states with

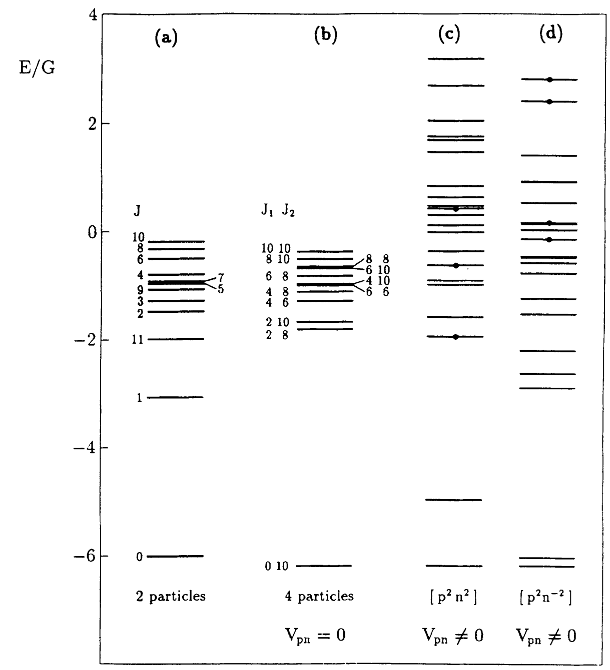

FIG. 1. The left-hand levels show the full spectrum for two nonidentical particles in a $j = 11/2$ shell with a delta-function two-body interaction. For identical nucleons, only the states with $T = 1$ (i.e., J even) are allowed. The levels to the right of that show the $[p^2n^2]$ spectrum with no n-p force starting with the lowest 10^+ state. The levels with $J_1 \neq J_2$ are all doubly degenerate corresponding to an interchange of the two-particle angular momenta (J_1, J_2). Once the isospin-invariant n-p interaction is added these levels are split as shown in the last two sets of levels. The splitting depends very strongly on whether one has neutron particles or holes. The essence of this paper is in the small splitting of the 10^+ levels for the configuration with proton particles and neutron holes. (This splitting may also be obtained from a $[p^2n^2]$ calculation by reversing the sign of the n-p interaction; see Fig. 3.) The dots on the levels in the last two spectra indicate states with large correlations.

spin 0 ($T = 1$) and the maximum spin ($J = 11$, $T = 0$) are most strongly bound, since these wave functions give the maximum nucleon-nucleon overlap. The medium spin states gain only little energy since there is not much overlap. Except for $J = 0$, the odd spin pairs are more strongly bound than the even ones.

The $[p^2n^2]$ and $[p^2n^{-2}]$ ($\equiv [p^2n^{10}]$) spectra without an n-p interaction are shown in Fig. 1(b). (We do not show the ground state with $J_1 = J_2 = 0$.) Without the n-p force, these states do not have good isospin and the spectra are identical (apart from an overall shift). The energies are simply given by the sums of the energies of the even-J states shown in Fig. 1(a) and we label the separate J values giving rise to the states shown. In Figs. 1(c), 1(d) we show the same $[p^2n^2]$ and $[p^2n^{-2}]$ configurations with an isospin invariant force ($V_{\pi\nu} = V_{\nu\nu} = V_{\pi\pi}$). The states now have good isospin and total J and are no longer degenerate. Note in particular that we obtain two distinct 10^+ states, which are the lowest states shown here. The most interesting feature of these figures is that the splitting of these two levels is very different in the cases of neutron particles and neutron holes. This essentially arises since the $[p^2n^{-2}]$ problem is like the $[p^2n^2]$ problem with the sign of $V_{\pi\nu}$ reversed (see Sec. II). (The complete spectra for the two cases are shown in Fig. 2.)

Since there is a direct change of sign of $V_{\pi\nu}$ between these two cases, it is interesting to see how the splitting of the two 10^+ states changes if we vary this in the range $-1 < G_{\pi\nu}/G < 1$ in the $[p^2n^2]$ calculation. We show the results in Fig. 3. The points on this curve correspond to the 10^+ splittings seen in Figs. 1(b),(c),(d) i.e., $G_{\pi\nu} = 0, 1, -1$ correspond respectively to those cases. It is important to understand the marked asymmetry of this curve. (Note that the bare n-p matrix elements are assumed to be negative. Therefore, $G_{\pi\nu} > 0$ corresponds to an attractive n-p interaction and the case with $G_{\pi\nu} < 0$ corresponds to a repulsive one.)

The $G_{\pi\nu}$ dependence of the separation of the lowest two 10^+ states can be qualitatively understood as follows: For attractive interaction ($G_{\pi\nu} > 0$) the states that are highly correlated by the n-p interaction are located in the lower part of the spectrum. For repulsive interac-

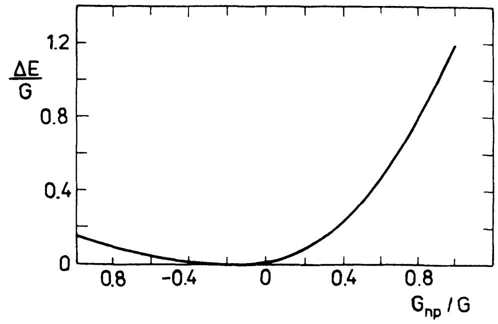

FIG. 3. The splitting of the two lowest 10^+ levels for the $[p^2n^2]$ configuration with a variable n-p strength. The true value is $G_{\pi\nu}/G = 1$. The value $G_{\pi\nu}/G = -1$, however, reproduces the $[p^2n^{-2}]$ spectrum (see text).

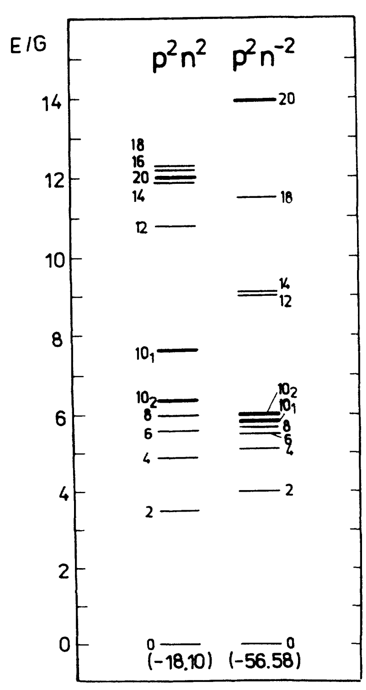

FIG. 2. More complete $[p^2n^2]$ and $[p^2n^{-2}]$ spectra with an n-p force.

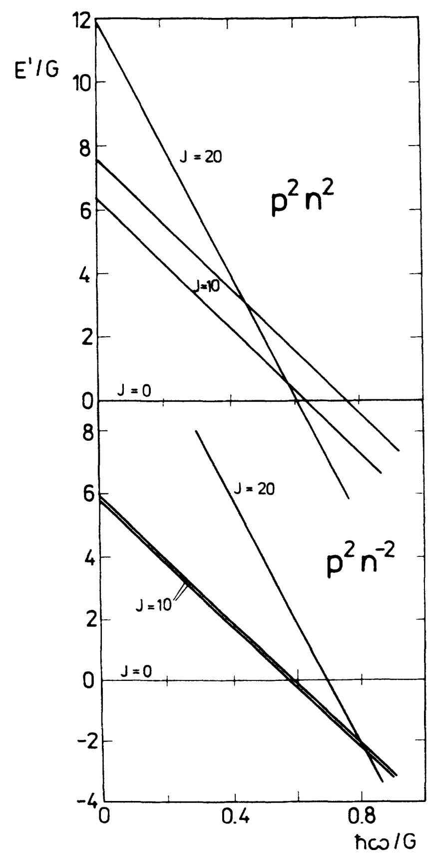

FIG. 4. The crossings of the ground band with the two lowest 10^+ states and the 20^+ state are shown as a function of the rotational frequency ω for $[p^2n^2]$ and $[p^2n^{-2}]$ systems.

tions ($G_{\pi\nu} < 0$) the most strongly correlated states are pushed up into the upper part of the spectrum. This difference is illustrated by Figs. 1(c) and 1(d) where we mark by dots the states for which the absolute values of the amplitudes of all components of the wave function are less than 0.4. (The more the states are correlated the more they are distributed over the basis states.) The difference of the diagonal matrix elements in the lowest two 10^+ basis states is $0.12G$. In the case of the attractive interaction most of the apparent repulsion of the two correlated 10^+ states ($1.21G$) is generated by the couplings of the two 10^+ basis states to the highly correlated states in the spectrum, which lie low in energy. In the case of the repulsive interaction this coupling is much weaker, since the strongly correlated states lie higher. The resulting splitting ($0.16G$) is comparable with the original difference of the diagonal matrix elements.

It is interesting to note here that we have treated the $[p^2n^{-2}]$ system as though it were a $[p^2n^2]$ system with the sign of $V_{\pi\nu}$ reversed. If we take that description literally, it would mean that the force we have would not be isoscalar. Thus the two 10^+ states generated could in principle be mixed by the deformed field. We shall see later that this is indeed the case, since in the true $[p^2n^2]$ problem the states have $T = 0$ and 1 and do not mix, whereas in the true $[p^2n^{-2}]$ problem they both have $T = 4$ and do mix in the deformed field.

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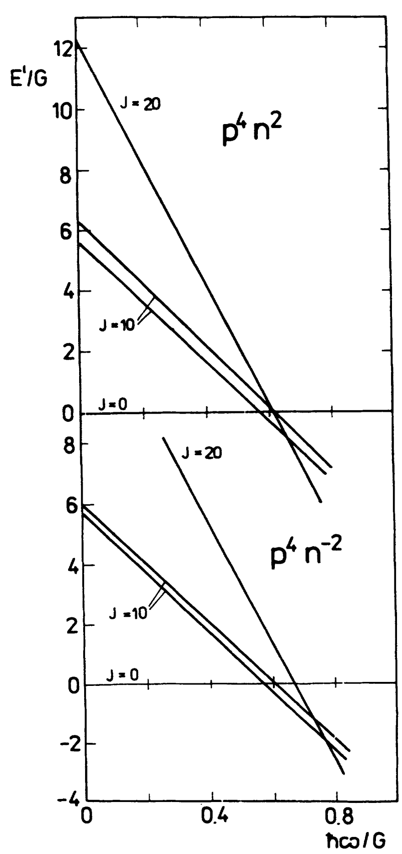

FIG. 5. Same as Fig. 4 but for the $[p^4n^2]$ and $[p^4n^{-2}]$ systems.

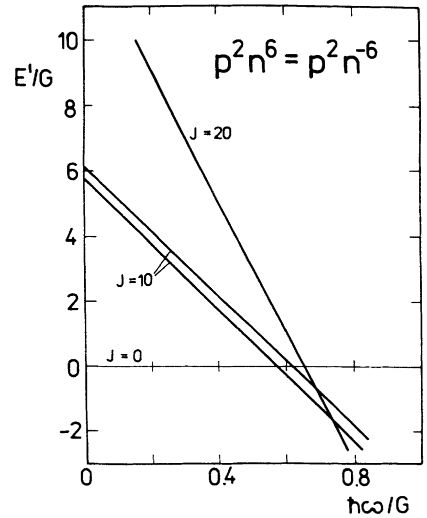

FIG. 6. Same as Fig. 4 but for the $[p^6n^2]$ ($= [p^2n^{-6}]$) systems.

$[p^2n^2]$ and $[p^2n^{-2}]$ systems are shown in Fig. 4. The yrast band with no deformation is the $J = 0$ configuration before $\hbar\omega = 0.6G$ and is crossed by an aligned ($J = 20$) four-particle state (two protons+two neutrons). In the case of $[p^2n^{-2}]$ the yrast band is again the $J = 0$ state until $\hbar\omega = 0.57G$. The two $J = 10$ states are almost degenerate at $\omega = 0$ and consequently cross the ground state band $J = 0$ at $\hbar\omega = 0.57G$ almost simultaneously. Each of the two $J = 10$ states is a two-quasiparticle aligned configuration, i.e., S band. This forking of the ground state band into two S bands has been observed experimentally in the Ba region. The fact that for the $[p^2n^2]$ case the $J = 20$ band crosses earlier than the $J = 10$ band is understood as a consequence of the attractive p-n interaction between the particles with parallel spin [cf. Fig. 1(a) for the $J = 11$ state]. In the $[p^2n^{-2}]$ case the interaction is repulsive and the $J = 20$ band is pushed up relative to the $J = 10$ band.

The results for the $[p^4n^2]$ and $[p^4n^{-2}]$ systems are shown in Fig. 5. It is seen that in the case of $[p^4n^2]$ the first crossing is due to $J = 10$ state (two-particle alignment). For the $[p^4n^{-2}]$ system, the first crossing is again due to the simultaneous alignments of the two $J = 10$ states. However, the difference between the two states has increased as compared to the $[p^2n^{-2}]$ system. This difference is further increased in the case of $[p^2n^6] \equiv [p^2n^{-6}]$ shown in Fig. 6. Thus, the simultaneous alignment of the two $J = 10$ states (forking of the ground state band into two S bands) is more probable for the cases with a few protons (particles) in the lower half and a few neutron holes in the upper half of the $1h_{11/2}$ shell.

IV. CONSEQUENCES OF THE DEFORMATION

The influence of the deformation has been studied in Refs. [3,4] for the $[p^2n^2]$ and $[p^4n^2]$ systems. The deformation term brings in mixing between states of different angular momenta J. The crossings become smoothed but the crossing frequencies do not change so much. For greater differences in particle number, however, there can be significant shifts of crossing frequency [13] which have opposite signs in the different halves of the shell. They are in fact just related to the particle-hole symmetry of the deformed field we have discussed in Sec. II. The important question for our present discussion though is whether there is a *relative* energy shift of the two 10^+ configurations in the $[p^2n^{-2}]$ system. Since these states have good T (=4), they are linear superpositions of aligned protons and aligned neutron holes and the diagonal matrix element (first order in κ) then vanishes. Thus, except for the unrealistic sharpness of the crossings, our spherical calculation should describe the band crossing pattern when the occupation of a high-j shell becomes increasingly asymmetric.

However, there are also important differences between the $[p^2n^2]$ and $[p^2n^{-2}]$ systems concerning their response to deformation. As seen in Eq. (1), the deformed field is isoscalar. For the $[p^2n^2]$ case the first and second $J = 10$ states have isospin $T = 0$ and 1, respectively. Thus they will not be mixed by the deformed field, remaining 50:50 mixtures of proton and neutron two-quasiparticle excitations. In the $[p^2n^{-2}]$ case we find a large matrix element between them, which is of the order κ, since the two states have the same isospin. Since $\kappa \sim 2G$ [3,4] the nondiagonal matrix element of the deformation term is much larger than the distance between the two $J = 10$ states, which amounts to $0.16G$. As a consequence the deformed field will demix the two states into the states $|10_p\rangle$ and $|10_n\rangle$, which are almost pure proton and neutron excitations: To a very good approximation the state $|10_p\rangle$ consists of the proton pair in the $J = 10$ state coupled with the neutron hole pair being with 90% in the $J = 0$ and 4% in the $J = 2$ state, whereas in the state $|10_n\rangle$ the proton and neutron contributions are exchanged. Our earlier comments on the vanishing of the diagonal matrix elements then seem invalid.

However, nuclei with about two $h_{11/2}$ protons and about two $h_{11/2}$ neutron holes are known to be very soft with respect to the triaxiality parameter γ of the deformed field (cf., e.g., Ref. [7]). This means the deformation energy of all other nucleons, which generate the deformed field, is about the same for prolate and oblate shape. Hence, for the state $|10_p\rangle$ the nucleus will take prolate shape ($\kappa > 0$ since $\langle|Y_{20}|\rangle 0$) and for $|10_n\rangle$ oblate shape ($\kappa < 0$ since $\langle|Y_{20}|\rangle < 0$). The gain in energy ($\sim |\kappa|\langle|Y_{20}|\rangle$) is about the same. In other words the two bands drive the deformation in opposite directions, so that the diagonal matrix elements are still the same in each band. (This difference in deformation will in turn reduce the magnitude of the off-diagonal deformation matrix element between the states.) If the moment of inertia of the higher band is somewhat larger than the one of the lower band, the two bands will cross. The interaction of the two crossing bands is just one half of the energy difference between the mixed $J = 10$ states before the deformation term was switched on. In our case it is $V = 0.08G \sim 50$ keV. A situation like the one described is observed in ^{128}Ba in Ref. [10]. The limit for the band-mixing matrix element is found experimentally to be $V < 3$ keV. Thus, there is an additional reduction by one order of magnitude compared to our estimate. This reduction most likely reflects the smallness of the overlap of the collective wavefunction in the γ degree of freedom. In our static discussion of the coupling between the particle and deformation degrees of freedom this overlap is assumed to be one.

V. CONCLUSIONS

We have studied how the pairing properties at high spin change when the occupation of a high-j shell changes from being symmetric ($N = Z$) to being very asymmetric. For the asymmetric case the proton-neutron correlations are strongly suppressed. The reason is that the interaction between protons and neutron *holes* is effectively repulsive. The correlated states are pushed into the higher part of the spectrum resulting in weak proton-neutron correlations in the lowest S bands. The most important point, however, is that (whatever the mechanism) the resulting splitting which the n-p interaction gives between the two good-isospin states is inherently sufficiently small to allow the deformation to play a decisive role. Since the deformation energy of particles and holes has the opposite sign and its magnitude is large compared to the correlation energy, it restores pure proton and neutron excitations. If, in addition, the nucleus is soft with respect to triaxiality, there will be proton and neutron S bands coexisting at about the same energy and having very little coupling. Such a situation is characteristic of nuclei with about the same number of protons and neutron holes in a high-j shell. The very small experimental value of the proton-neutron coupling matrix element in ^{128}Ba cannot be undertstood by the repulsive pairing alone. It seems to indicate an additional reduction due to the very different shapes of the nucleus in the proton and neutron S bands.

Maximal proton-neutron correlations occur for symmetric filling of the j shell, i.e., in $Z = N$ nuclei. This has been pointed out before and we refer to Refs. [14,15]. However, unlike these papers we do not find a preference to build $J = 1$ and/or $J = 2j$ proton-neutron pairs. A decomposition of the $I = 10$ wave functions into a basis of proton-neutron pairs rather shows a preference of the medium spins around $J = 6$ in our single-j-shell model. The consequences of the proton-neutron correlation for the high spin behavior are significant: As already pointed out in Refs. [3,4], the lowest S band is a 50:50 mixture of proton and neutron excitations with $T = 0$. The $T = 1$ S band lies noticably higher. However, the attractive proton-neutron interaction shifts the frequency where the g band is crossed by the four-quasiparticle excitation with two quasiprotons and two quasineutrons aligned ($J = 20$ in our case of the $h_{11/2}$ shell) below the frequency where two quasiparticle $T = 0$ band ($J = 10$ in our case) crosses. As a result, there is a simultaneous alignment of both protons and neutrons twice as large as for the $T = 0$ S band. With increasing asymmetry of the occupation of the high-j shell the alignment of protons and neutrons becomes less and less correlated.

It would also be very interesting to study the consequences of proton-neutron correlations for the high-spin behavior of odd A and odd-odd nuclei in the vicinity of $Z = N$, since one expects modifications of the familiar blocking pattern described, e.g., in Ref. [1]. Generally it appears to be a good way to study the proton-neutron pairing by putting angular momentum into the $Z = N$ nuclei, high enough to break up the correlations. The radioactive beam facilities now under consideration (e.g., [5]) combined with the new generation of multidetector γ-ray arrays will be efficient tools for such studies. They will also allow one to investigate the transition to the familiar pattern of independent-proton and -neutron pairing studied in this paper.

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