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Generator coordinate method calculations for ^{4}He and ^{16}O nuclei

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Generator coordinate method calculations, including the case of two-generator coordinates, which is not usually considered in the relevant scientific literature, are performed for nuclear state energies, as well as momentum and density distributions of the ^{4}He and ^{16}O nuclei. The construction potential used behaves like an harmonic oscillator at large distances from the center of the potential but it has also a strong short-range repulsion, which is expected to simulate, to some extent, effects of the inclusion of short-range correlations in the harmonic-oscillator many-body wave function. The results are compared with available experimental information and discussed.

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I. INTRODUCTION

Many experiments in recent years have concentrated on the study of high-momentum components of the nuclear wave function [1]. We shall mention especially the inclusive- and exclusive-electron scattering on nuclei (e.g., Refs. [2–4]) which show the existence of strong high-momentum components of the total nucleon momentum distribution. An essential spectroscopic information has been also obtained in proton pickup and knockout reactions using polarized deuterons as projectiles (e.g., Refs. [5–7]). Information on the single-particle momentum distributions which is complementary to that from electron- and proton-induced reactions has been obtained from the photoreactions (e.g., Ref. [8]) having unique sensitivity to high-momentum components of the total wave function. The results mentioned above cannot be explained in the framework of the mean-field approximation (MFA) used in nuclear theory. Correlation methods going beyond the limits of MFA and accounting for short-range and tensor nucleon-nucleon correlations have been developed. The review of the results obtained in various correlation methods and their comparison with the experimental data can be found, for instance, in [1d,9].

A correlation approach within the generator coordinate method (GCM) [10–11] using Skyrme-like effective forces [12] has been developed [13,14] and applied to calculate the nucleon momentum and density distributions, the energies and r.m.s. radii of the ground and first monopole excited states of ^{4}He, ^{16}O, and ^{40}Ca nuclei [15]. In addition, the natural orbitals and occupation numbers have been calculated using the GCM one-body density matrix [16], as well as the two-nucleon momentum distributions [17].

In the GCM the trial many-particle wave function $\Psi(\{\vec{r}_i\})$ of a system of A nucleons is written in the form of a linear combination:

$$\Psi(\vec{r}_1,\vec{r}_2,...,\vec{r}_A)=\int f(x_1,x_2,...)\Phi(\vec{r}_1,\vec{r}_2,..,\vec{r}_A;x_1,x_2,...)dx_1\,dx_2...\,, \tag{1}$$

where the generating function Φ depends on the radius-vectors of the nucleons $\{\vec{r}_i\}$ and on the generator coordinates x_1,x_2,.... This function is usually chosen to be a Slater determinant built up from single-particle wave functions corresponding to a given single-particle potential (the so-called "construction potential") parametrized by x_1,x_2,.... It is obvious that in this case the wave function (1) of the system, being a superposition of Slater determinants, goes beyond the limits of the MFA. The so-called "weight," or "generator" function $f(x_1,x_2,...)$ can be determined using the variational principle

$$\delta E[\Psi]=0\,, \tag{2}$$

where

$$E[\Psi]=\frac{\langle\Psi|H|\Psi\rangle}{\langle\Psi|\Psi\rangle} \tag{3}$$

and $\hat{H}$ is the Hamiltonian of the system.

Various applications of the GCM to the nuclear problems are simplified using the effective N-N Skyrme interaction [12]. Monopole, dipole, and quadrupole isoscalar and isovector vibrations of light double-magic nuclei are considered in [13]. The appearance of the effective interaction between two nucleons in a medium, which is different from that between nucleons in vacuum, is due to the effects of other nucleons on the pair of the particles considered. The approach of Skyrme gives a direct parametrization of the effective density-dependent nucleon-nucleon interaction.

The account of the N-N correlation effects within the GCM depends on the choice of the construction potentials, namely, the harmonic oscillator

$$V(r) = -V_0 + \tfrac{1}{2}m\omega^2 r^2 \quad (V_0 > 0) \tag{4}$$

and the square well with infinite walls

$$V(r) = \begin{cases} -V_0\,, & r < x, \quad V_0 > 0 \\ \infty\,, & r > x \end{cases} \tag{5}$$

(where the oscillator parameter $\alpha = (\frac{m\omega}{\hbar})^{1/2}$ and the radius of the well x are the generator coordinates, respectively) which have been used in the approach of Refs. [14–17]. In particular, the use of the harmonic-oscillator construction potential (4) affects strongly the behavior of some physical quantities due to the specific asymptotic behavior of the oscillator functions. In the case of the construction potential (5) the generating function $\Phi(\{\vec{r_i}\}, x)$ corresponds to a state of A nucleons confined in a finite spatial volume (a sphere with radius x). It has been concluded in [15] that the high-momentum components of the nucleon momentum distribution obtained in the latter case are due to the existence of intermediate states (i.e., states with a given value of the generator coordinate x) with high densities in which the distances between the nucleons are small and short-range forces are operative. In the case of GCM with harmonic-oscillator construction potential the high-density intermediate states are not possible and the nucleon momentum distributions do not show high-momentum behavior.

A single-particle potential model which has been suggested for the study of charge form factors and nucleon momentum distribution of light nuclei is the one used in Refs. [18,19]. The main feature of the model is that the single-particle potential contains both an attractive and a repulsive part:

$$V(r) = -V_0 + \frac{1}{2}m\omega^2 r^2 + \frac{B}{r^2}, \quad V_0 > 0, B \geq 0\,. \tag{6}$$

In the particular case $B = 0$ the potential (6) coincides with (4). This potential behaves like an harmonic-oscillator potential for large values of r but it has in addition a repulsive term which is the dominant one at short distances from the origin. It is expected that this term simulates to some extent effects of the inclusion of short-range correlations in the harmonic-oscillator many-body wave function.

In this paper we consider the potential (6) as a construction potential for GCM calculations using the parameters $\beta = (\frac{\hbar}{m\omega})^{1/2} = \frac{1}{\alpha}$ (which determines the strength of the attraction) and B (determining the strength of the repulsion) as generator coordinates in the study of ^{4}He and ^{16}O nuclei. Our aim is to follow several consecutive steps in this study, namely, by first minimizing the expectation value of the nuclear Hamiltonian with a single Slater determinant (SSD) wave function with respect to these two parameters. Secondly, we perform one generator coordinate calculations (OGC) fixing the one of the parameters and varying the other one. Finally, we perform a two generator coordinate calculation (TGC) with the above-mentioned two parameters, which appears to be a rather interesting possibility in view also of the very limited cases of this type of calculations in the literature.

In Sec. II, the basic GCM relations are given. In Sec. III the explicit forms for the single-particle wave functions corresponding to the potential (6) and the analytic expression of the overlap integrals are also presented. In the final section the numerical results are given and discussed.

II. GENERATOR COORDINATE METHOD RELATIONS

We consider the GCM trial many-body wave function (1) with two generator coordinates: $x_1 = \beta$ and $x_2 = B$. The application of the Ritz variational principle [Eqs. (2) and (3)] leads to the Hill-Wheeler integral equation for the weight function:

$$\int [\mathcal{H}(x,x') - E\mathcal{N}(x,x')]\, f(x')dx' = 0\,, \tag{7}$$

where

$$x \equiv (\beta, B) \tag{8}$$

and

$$\mathcal{H}(x,x') = \langle \Phi(\{\vec{r_i}\}, x)|\hat{H}|\Phi(\{\vec{r_i}\}, x')\rangle\,, \tag{9}$$

$$\mathcal{N}(x,x') = \langle \Phi(\{\vec{r_i}\}, x)|\Phi(\{\vec{r_i}\}, x')\rangle \tag{10}$$

are the energy and overlap kernels, respectively, and $\hat{H}$ is the Hamiltonian of the system. The generating function $\Phi(\{\vec{r_i}\}, x)$ is taken to be a Slater determinant [built up here from proton and neutron orbitals $\phi_\lambda(\vec{r}, x)$ corresponding to the construction potential (6)]. In this case the energy kernel (9) has the form [20]

$$\mathcal{H}(x,x') = \langle \Phi(\{\vec{r_i}\}, x)|\Phi(\{\vec{r_i}\}, x')\rangle \cdot \int H(x,x',\vec{r})d\vec{r}\,. \tag{11}$$

In the case of Skyrme effective forces, neglecting the Coulomb and the spin-orbit interaction, the function $H(x,x',\vec{r})$ is given, for equal number of neutrons and protons, by [13]

$$H(x,x',\vec{r}) = \frac{\hbar^2}{2m}T + \frac{3}{8}t_0\rho^2 + \frac{1}{16}(3t_1 + 5t_2)(\rho T + \vec{j}^2) + \frac{1}{64}(9t_1 - 5t_2)(\nabla\rho)^2 + \frac{1}{16}t_3\rho^{2+\sigma}\,. \tag{12}$$

The quantities $t_0, t_1, t_2, t_3, \sigma$ are the Skyrme force parameters and the density ρ, the kinetic energy density T and the current density $\vec{j}$ are defined by

$$\rho(x,x',\vec{r}) = 4\sum_{\lambda,\mu=1}^{A/4}(N^{-1}(x,x'))_{\mu\lambda}\phi_\lambda^*(\vec{r},x)\phi_\mu(\vec{r},x') , \tag{13}$$

$$T(x,x',\vec{r}) = 4\sum_{\lambda,\mu=1}^{A/4}(N^{-1}(x,x'))_{\mu\lambda}\nabla\phi_\lambda^*(\vec{r},x)\nabla\phi_\mu(\vec{r},x') , \tag{14}$$

$$\vec{j}(x,x',\vec{r}) = 2\sum_{\lambda,\mu=1}^{A/4}(N^{-1}(x,x'))_{\mu\lambda}[\phi_\lambda^*(\vec{r},x)\nabla\phi_\mu(\vec{r},x') - (\nabla\phi_\lambda^*(\vec{r},x))\phi_\mu(\vec{r},x')] . \tag{15}$$

In Eqs. (13)–(15) A is the mass number of the nucleus and $(N^{-1})_{\mu\lambda}$ is the inverse of the overlap matrix:

$$N_{\lambda\mu}(x,x') = \int \phi_\lambda^*(\vec{r},x)\phi_\mu(\vec{r},x')d\vec{r} . \tag{16}$$

The overlap kernel (10) has the form

$$\mathcal{N}(x,x') = [\det(N_{\lambda\mu})]^4 . \tag{17}$$

The one-body density matrix for the ground state of the nucleus is given by [1d,9,16]

$$\rho(\vec{r},\vec{r'}) = \int f_0(x)f_0(x')\mathcal{N}(x,x')\rho(x,x',\vec{r},\vec{r'})dxdx' , \tag{18}$$

where

$$\rho(x,x',\vec{r},\vec{r'}) = 4\sum_{\lambda,\mu=1}^{A/4}(N^{-1}(x,x'))_{\mu\lambda}\phi_\lambda^*(\vec{r},x)\phi_\mu(\vec{r'},x') \tag{19}$$

and $f_0(x)$ is the solution of Eq. (7) corresponding to the lowest energy eigenvalue. It follows from (18) that the nuclear density distribution $\rho(\vec{r})$ and the nucleon momentum distribution $n(\vec{k})$ can be expressed as

$$\rho(\vec{r}) = \int f_0(x)f_0(x')\mathcal{N}(x,x')\rho(x,x',\vec{r})dxdx' \tag{20}$$

and

$$n(\vec{k}) = \int f_0(x)f_0(x')\mathcal{N}(x,x')\rho(x,x',\vec{k})dxdx' , \tag{21}$$

where

$$\rho(x,x',\vec{k}) = 4\sum_{\lambda,\mu=1}^{A/4}(N^{-1}(x,x'))_{\mu\lambda}\tilde{\phi}_\lambda^*(\vec{k},x)\tilde{\phi}_\mu(\vec{k},x') \tag{22}$$

and $\tilde{\phi}(\vec{k},x)$ is the Fourier transform of $\phi(\vec{r},x)$.

III. SINGLE-PARTICLE WAVE FUNCTIONS AND OVERLAP INTEGRALS

One of the main advantages of the construction potential (6) is that analytic expressions can be derived for the wave functions and for other useful quantities, some of which are needed in the present investigation. The normalized energy eigenfunctions are

$$\phi_\lambda(\vec{r};x) = \phi_\lambda(r,\theta,\varphi;\beta,B) = R_{nl}(r)Y_l^m(\theta,\varphi) , \tag{23}$$

where Y_l^m are the spherical harmonics and $R_{nl}(r)$ are given by the expression [19b, and references therein]:

$$R_{nl}(r) = \left(\frac{2(n!)}{\Gamma(n+2\lambda_l+\frac{1}{2})\beta^{4\lambda_l+1}}\right)^{1/2} r^{2\lambda_l-1}L_n^{2\lambda_l-\frac{1}{2}}\left(\frac{r^2}{\beta^2}\right)e^{-\frac{r^2}{2\beta^2}} , \tag{24}$$

where

$$\lambda_l = \frac{1}{4}\left(1+\sqrt{(2l+1)^2+\frac{8mB}{\hbar^2}}\right) . \tag{25}$$

$L_n^{2\lambda_l-\frac{1}{2}}$ are the associated Laguerre polynomials. The dependence of $R_{nl}(r)$ on the parameters $x=(\beta,B)$ has been omitted for the sake of simplicity. It may be easily verified that when $B=0$, that is when $\lambda_l=\frac{1}{2}(l+1)$, the above expressions go over to the well-known expressions for the corresponding harmonic-oscillator wave functions. It should be noted that the usual radial Schrödinger equation is satisfied by the functions $\varphi_{nl}(r)=rR_{nl}(r)$.

In the present study we need only the R_{00} and R_{01} and their derivatives. A little more generally, the R_{0l} and its derivative are given by the simple expressions

$$R_{0l}(r)=\left(\frac{2}{\beta\Gamma(2\lambda_l+\frac{1}{2})}\right)^{1/2}\frac{1}{r}\left(\frac{r}{\beta}\right)^{2\lambda_l}e^{\frac{-r^2}{2\beta^2}} \tag{26}$$

and

$$\frac{dR_{0l}(r)}{dr}=\frac{1}{r}\left[(2\lambda_l-1)-\left(\frac{r}{\beta}\right)^2\right]R_{0l} . \tag{27}$$

In addition, the overlap integrals $N_{0l,0l}(\beta,B;\beta',B')$ in the matrix (16) may also be calculated analytically in terms of the Γ function. The result is

$$N_{0l,0l}(\beta,B;\beta',B')=\beta^{2\lambda_l'+\frac{1}{2}}\beta'^{2\lambda_l+\frac{1}{2}}\left(\frac{2}{\beta^2+\beta'^2}\right)^{\lambda_l+\lambda_l'+\frac{1}{2}}\left(\frac{\Gamma^2(\lambda_l+\lambda_l'+\frac{1}{2})}{\Gamma(2\lambda_l+\frac{1}{2})\Gamma(2\lambda_l'+\frac{1}{2})}\right)^{1/2} . \tag{28}$$

The quantity inside the square root is equal also to the ratio of two Beta functions of arguments $(\lambda_l+\lambda_l'+\frac{1}{2},\lambda_l+\lambda_l'+\frac{1}{2})$ and $(2\lambda_l+\frac{1}{2},2\lambda_l'+\frac{1}{2})$, respectively. It may be easily checked out that for $\beta'=\beta$ and $\lambda_l'=\lambda_l$ the overlap integral becomes unity, as expected. In addition, if $\lambda_l=\frac{1}{2}(l+1)$ the above overlap integral goes over to the known expression for harmonic-oscillator wave functions.

The analytic expressions given in this section are useful in obtaining our numerical results and diminish the required computing time.

IV. RESULTS AND DISCUSSION

The GCM formalism described above has been applied to ^{4}He and ^{16}O nuclei for calculating the energy of the ground and first collective excited states as well as the ground-state momentum and density distributions. The SkIII set of Skyrme force parameters [21] has been used in the calculations.

The Hill-Wheeler integral equation (7) has been solved using a discretization procedure similar to that of [13] with respect to $x\equiv(\beta,B)$ and $x'\equiv(\beta',B')$. The resulting matrix eigenvalue problem was solved numerically. The values of the lowest solutions for the energy E_0 (the ground-state energy) and E_1 (the first collective excited state energy) and the difference $\Delta E_1=E_1-E_0$ are presented in Tables I and II. The eigenfunction (i.e., the weight function) $f_0(x)$ corresponding to the lowest eigenvalue E_0 has been used in Eqs. (20) and (21) for calculating the density and momentum distributions. This procedure enables us to study separately the effects of each of the generator coordinates β and B, fixing one of them and varying the other one in the GCM, as well as the effect of varying both of them.

The results given in the first column of Tables I and II (for $B=0$) correspond exactly to the GCM calculations with the harmonic-oscillator construction potential (4) and reproduce well the energies obtained in [13] in the case of SkIII parameter set: E_0=-32.93 MeV, E_1=-5.35 MeV, ΔE_1=27.58 MeV, for ^{4}He and E_0=-140.32 MeV, E_1=-108.65 MeV, ΔE_1=31.67 MeV for ^{16}O.

The variation of the generator coordinates β and B in the present GCM calculations was carried out in the vicinity of the values $\beta=\beta_0$ and $B=B_0$ which minimize the diagonal element of the energy kernel (9):

$$\langle\Phi(\{\vec{r_i}\},\beta_0,B_0)|\hat{H}|\Phi(\{\vec{r_i}\},\beta_0,B_0)\rangle=\min . \tag{29}$$

For ^{4}He $B_0=2$ MeV fm^2, $\beta_0=1.5$ fm and the variational energy of a single Slater determinant (SSD) of single-particle functions corresponding to the potential (6) is $E_V=-26.68$ MeV. For ^{16}O $B_0=0$, $\beta_0=1.8$ fm and $E_V=-138.13$ MeV.

TABLE I. Energies (in MeV) of the ground state (E_0) and the first collective excited state (E_1), and their difference ΔE_1 in the GCM with the construction potential (6) and SkIII effective forces, together with the variational energy E_V of a single Slater determinant (SSD), for the ^{4}He nucleus.

	Variations of β $B=0$	Variations of β $B=2$ (MeV fm^2)	Variations of B $\beta=1.5$(fm)	Variations of β and B
E_V	-26.40	-26.68	-26.68	-26.68
E_0	-33.01	-31.99	-29.81	-43.01
E_1	-5.39	-4.98	$\sim 10^{-10}$	-8.45
$\Delta E_1 =$ E_1-E_0	27.62	27.01	29.81	34.56

TABLE II. Energies (in MeV) of the ground state (E_0) and the first collective excited state (E_1), and their difference ΔE_1 in the GCM with the construction potential (6) and SkIII effective forces, together with the variational energy E_V of a single Slater determinant (SSD), for the ^{16}O nucleus.

	Variations of β $B=0$	Variations of B $\beta=1.8$ (fm)	Variations of β and B
E_V	-138.13	-138.13	-138.13
E_0	-140.43	-139.87	-142.59
E_1	-108.87	-98.74	-110.81
$\Delta E_1 =$ $E_1 - E_0$	31.56	41.13	31.78

The GCM numerical calculations have been performed using a set of regular mesh points with different steps as well as ranges of values of the generator coordinates β and B until the results do not change after decreasing the step size or increasing the range of the generator coordinate values. For example, in the case of the two generator coordinate calculations the results given in Tables I and II are obtained using the following ranges and steps: ^{4}He [$0.6 < \beta < 2.4$ fm, step 0.1 fm; $0 < B < 9$ MeV fm^2, step 0.5 MeV fm^2] and ^{16}O [$1.1 < \beta < 2.5$ fm, step 0.1 fm; $0 < B < 7$ MeV fm^2, step 0.5 MeV fm^2].

We note the following features of the results.

(i) As is expected, the value of $|E_0|$ in the two generator coordinate case (TGC) is larger than that in both one generator coordinate cases (OGC) in ^{4}He and ^{16}O.

(ii) The value of $|E_1|$ in TGC case is larger than that in both OGC cases.

(iii) The values of $|E_0|$ and $|E_1|$ for the OGC case of the harmonic-oscillator construction potential ($B = 0$ and variations of β) are larger than those in the OGC case with variations of B at fixed value of β, for both nuclei.

(iv) The value of ΔE_1 in the TGC case is larger than the OGC calculations with the harmonic-oscillator construction potential ($B = 0$) for ^{4}He and ^{16}O.

(v) As can be seen from Tables I and II the difference between E_0 and E_V is much larger in the case of ^{4}He than in the case of ^{16}O. This is a common feature of the GCM method related to the property of the integral kernels $\mathcal{N}(x, x')$ and $\mathcal{H}(x, x')$ which become much more peaked with the increase of the number of particles.

(vi) The energy diagonal matrix elements do depend on B (as well as on β) in both cases, ^{4}He and ^{16}O. The minimum in the case of ^{4}He is achieved at $\beta_0 = 1.5$ fm and $B_0 = 2$ MeV fm^2. In Table I we give also the value of the minimum with respect to β in the case when $B = 0$ (pure harmonic oscillator). In the case of ^{16}O the values $B = 0$ and $\beta_0 = 1.8$ fm lead to the minimum of the energy diagonal matrix element. This means that the harmonic-oscillator functions are the best ones in the sense of the SSD used. Obviously, this is not the case for the nucleus ^{4}He where the best SSD is not a pure harmonic-oscillator one.

The nucleon momentum distributions in ^{4}He and ^{16}O are calculated using Eq. (21) and the results are presented in Figs. 1 and 2 for OGC and TGC cases as well as for the SSD cases using β_0 and B_0.

We note, firstly, that in the SSD case, with $\beta = \beta_0 = 1.5$ fm and $B = B_0 = 2$ MeV fm^2 for ^{4}He there are substantially high momentum components, which is not the case when $B = 0$. The qualitative behavior of $n(k)$ in this case is as in Ref. [19] and is due to the existence of the repulsive term of the potential. Secondly, the OGC results for $n(k)$ with $B = 0$ (that is with simple harmonic-oscillator functions in the generating function) are far below those of TGC and OGC when B is the generator coordinate and $\beta = \beta_0$. In spite of the considerable improvement, however, the theoretical values are still much lower than the corresponding experimental ones in the region of large k. The main effect of the GCM procedure (in comparison with the SSD [at $\beta = \beta_0 = 1.5$ fm and $B = B_0 = 2$ MeV fm^2)] is to increase the values $n(k)$ in the region 2 fm^{-1} $< k <$ 3 fm^{-1}. For the remaining nucleon correlations one should consider other methods

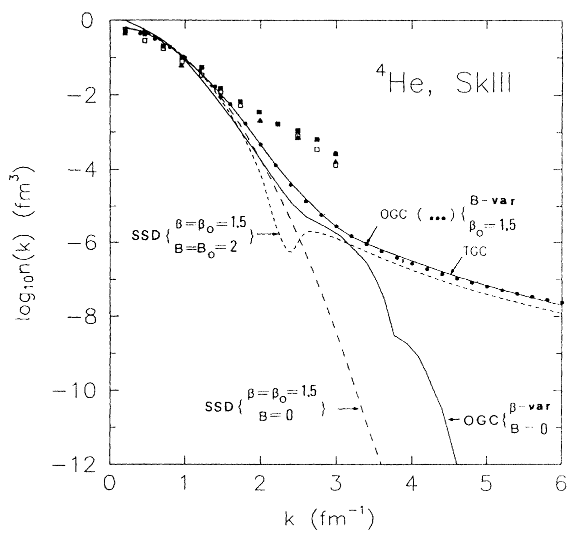

FIG. 1. Nucleon momentum distribution of ^{4}He within the generator coordinate method (GCM) using Skyrme effective forces (SkIII): (i) with one generator coordinate B and $\beta = \beta_0 = 1.5$ fm (OGC) (points); (ii) with one generator coordinate β and $B = 0$ (OGC); (iii) single Slater determinant case with $\beta = \beta_0 = 1.5$ fm and $B = 0$ and $B = B_0 = 2$ MeV fm^2 (SSD); (iv) with two generator coordinates (TGC); the experimental data (□, ▲, ■) are taken from Ref. [22]; the normalization is $4\pi \int n(k) k^2 dk = 1$.

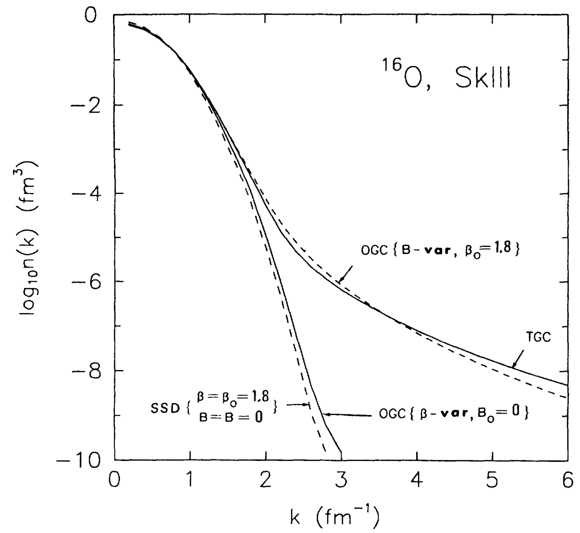

FIG. 2. Nucleon momentum distribution of ^{16}O. The notation is the same as in Fig. 1 with $\beta = 1.8$ fm and $B_0 = 0$. The normalization is as in Fig. 1.

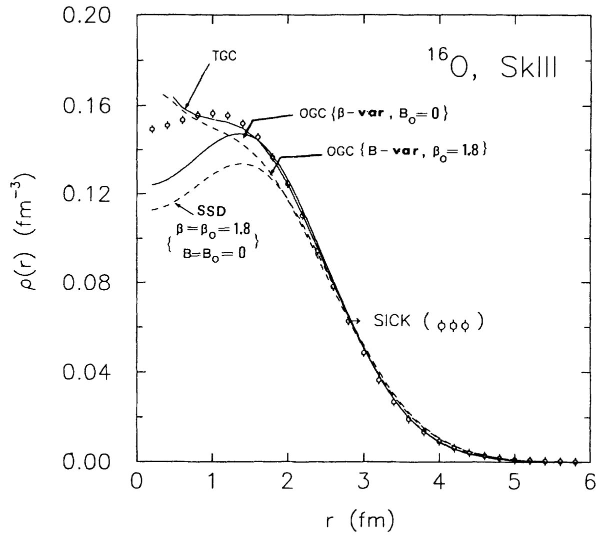

FIG. 3. Density distribution of ^{16}O. The notation is the same as in Fig. 1. with $\beta_0 = 1.8$ fm and $B_0 = 0$. The normalization is $4\pi \int \rho(r) r^2 dr = A = 16$. The experimental data are taken from Ref. [25].

showing up stronger correlation effects, such as coupled cluster theory or Jastrow-type correlations [1d,9,23,24]. The corresponding results of $n(k)$ for ^{16}O in Fig. 2 show in general a rather similar behavior. The OGC results (where β is the generator coordinate and B=0) is quite close now to the SSD results (with $\beta = \beta_0 = 1.8$ fm, $B = 0$).

From Figs. 1 and 2 one might conclude that the results at a variation of B for an appropriate value of β are as good as those in the TGC case. Indeed, the OGC results for $n(k)$ almost coincide with those of TGC calculations for ^{4}He and they are close to those for ^{16}O. However, this is not the case for the density distribution, as can be seen from Fig. 3 where the density distribution $\rho(r)$ for ^{16}O calculated using Eq. (20) is plotted. The consecutive use of β, B, and both of them (β and B) as generator coordinates improves the agreement with the experimental data [25] in the region $r \simeq 1$ fm. The general behavior of $\rho(r)$ for ^{4}He, in the various cases, is rather similar.

To summarize:

(i) In the present work the energy of the ground and first collective excited states, the momentum and density distributions of ^{4}He and ^{16}O are calculated within the generator coordinate method using a construction potential containing harmonic-oscillator and repulsive parts (and consequently two generator coordinates) and Skyrme effective forces.

(ii) The values of the energies of the ground $|E_0|$ and the first excited collective state $|E_1|$ in the TGC case are larger than those in both OGC cases for ^{4}He and ^{16}O. The values of $|E_0|$ and $|E_1|$ for the OGC case of the harmonic-oscillator construction potential are larger than those in the OGC case with variations of B at a fixed value of β for both nuclei. The values of ΔE_1 increase in the TGC case in comparison with the OGC calculations with the harmonic-oscillator construction potential ($B = 0$).

(iii) The calculations of the nucleon momentum distribution in ^{4}He and ^{16}O show an existence of high-momentum components even in the SSD case for ^{4}He when the repulsive part of the potential (6) is included ($B_0 \neq 0$). This is not the case in ^{16}O, where $B_0 = 0$. The high-momentum components manifest themselves better in the GCM case with one generator coordinate B (at a fixed value of β) and in the GCM with two generator coordinates (β and B). The results in the latter cases are close to each other. Although significantly improving the results in the single Slater determinant case with fixed values of $\beta = \beta_0$ and $B = 0$ they are still incapable of describing the experimental data for $n(k)$ at $k \gtrsim 2$ fm^{-1} in ^{4}He.

(iv) The GCM calculations of the density distribution of ^{16}O with one and two generator coordinates show a significant improvement of the agreement with the experimental data in the region $r \simeq 1$ fm in comparison with the single Slater determinant (with fixed values of β and B) case.

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