Can we do without the Majorana term in the effective nuclear interaction?

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We present phenomenological evidence that the strength of the Majorana term necessary to reproduce collective $M1$-transition strength data could be significantly smaller than is conventionally assumed and therefore more in line with naive microscopic considerations. We also find that g-boson effects are important to the reproduction of the summed $M1$ strength.

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I. INTRODUCTION

Despite the schematic character of the Interacting Boson Model (IBM), it provides a remarkably successful platform for the systematization of the low-energy collective properties of medium-to-heavy nuclei. Among its successes, one that is widely held to be particularly significant is its ability to reproduce collective $M1$-transition strength data associated with the scissors mode discovered after the introduction of the model [1]. In this connection, two previously neglected aspects of the model emerged as crucial: states of "mixed" (as opposed to maximal) proton-neutron symmetry and a repulsive "Majorana interaction," which pushes up states of less proton-neutron symmetry relative to the states of maximal symmetry. Within the Interacting Boson Model, the scissors mode corresponds to the excitation of 1^+ states of mixed proton-neutron symmetry; the energies of these states depend strongly on the strength of the Majorana interaction.

The phenomenological success of the IBM indicates that it is, in the jargon of many-body theory, a description in terms of the appropriate effective degrees of freedom. The ingredients of the corresponding renormalization paradigm have been known in broad terms for some time. Briefly, they comprise truncation of the relevant shell-model space to a collective fermion-pair subspace, followed by the mapping of the fermion-pair states onto boson states. In practice, the reliable calculation of such renormalization effects is very difficult. Applications of the Interacting Boson Model have, therefore, rested on the *ad hoc* introduction of plausible forms of effective operators dependent on free parameters which are then fit to experiment. Despite the potentially arbitrary character of such choices, it is nevertheless possible [2] to obtain good agreement with energy level and $E2$ transition data with an IBM Hamiltonian which adheres closely to the structure suggested by the shell model with parameter values consistent with effective interactions in the shell model [3].

As regards g-factor and $M1$ transition data, the situation is somewhat different. We shall not dwell here on the anomalies [4] in the values of the effective g factors of bosons required to describe static magnetic moments (it may be that these anomalies can be resolved by relaxing the assumption of F-spin purity common to previous studies [5,6]). Instead, our focus is on the Majorana interaction. It is currently accepted [7] that phenomenological studies support the choice of Hamiltonians which include a rather strong Majorana term. There is, however, no direct counterpart of the Majorana interaction in the shell model. The question thus arises are renormalization effects *absolutely essential* for a good description of $M1$ transition data? The issue is all the more intriguing because renormalization effects would seem to be inessential for energy level and $E2$ transition data.

The standard value of the Majorana interaction strength has been obtained by fitting the systematics of the scissors mode centroid [8]. A peculiarity of this work is the use of a Hamiltonian with the nonstandard form

$$-\kappa\,(\hat{Q}_p+\hat{Q}_n)\cdot(\hat{Q}_p+\hat{Q}_n) \tag{1}$$

for the quadrupole-quadrupole piece of the effective interaction instead of the more usual choice $-\kappa\,\hat{Q}_p\cdot\hat{Q}_n$. [Here, $\hat{Q}_p$ ($\hat{Q}_n$) denotes the conventional IBM-2 quadrupole operator for proton (neutron) bosons.] Since the effective interaction between like and unlike nucleons is certainly different, use of the interaction in (1) is questionable from a microscopic point of view. In fact, we believe that the large value of the Majorana strength inferred in [8,9] is a consequence of the artificial symmetry of the interaction in (1).

Anticipating our results below, we can state that, in addition to a Majorana shift, there is a "deformation" contribution to the centroid energy of the scissors mode. Although not made explicit by the present derivation, the deformation term has its origin in the prosaic following

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effect: the attraction exerted by the $\hat{Q}_p \cdot \hat{Q}_n$ interaction increases with the degree of overlap of the neutron and proton density distributions. As a result, the reduction in the energy of a mixed-symmetry state by the $\hat{Q}_p \cdot \hat{Q}_n$ interaction is not so great as the reduction in the energy of the fully-symmetric ground state. It is this difference which gives rise to the deformation contribution in the centroid energy of the scissors mode. It is also apparent that this difference is underestimated if the neutron-proton blind interaction in (1) is adopted.

II. AN IBM-INSPIRED MODEL FOR $M1$ STRENGTH

Our aim, in the first instance, is to obtain a simple (*albeit* approximate) expression for the centroid energy

$$E_c = \frac{\int_{-\infty}^{\infty} \mathrm{d}E\, E\, S_{M1}(E)}{\int_{-\infty}^{\infty} \mathrm{d}E\, S_{M1}(E)} \qquad (2)$$

of the $M1$ excitation strength function $S_{M1}(E)$ which makes transparent the dependence on effective IBM parameters. In the calculation of E_c, we employ sum rules which follow from the result

$$S_{M1}(E) = \sum_{\mu} \left\langle 0^+ \left| \hat{T}^{\dagger}_{\mu}(M1)\, \delta(E - \hat{H})\, \hat{T}_{\mu}(M1) \right| 0^+ \right\rangle ,$$

where $\hat{T}_{\mu}(M1)$ denotes the $M1$ transition operator, $\hat{H}$ the Hamiltonian of the system, and $|0^+\rangle$ the corresponding ground state. The utility of sum rules in elucidating the properites of $M1$ transitions has been demonstrated previously [10–12]. Our model for the strength function $S_{M1}(E)$ treats its ingredients in two contrasting ways. On the one hand, to address problematic aspects of renormalization identified in previous $M1$ strength function studies [12,13], we adopt rather general IBM-2 expressions with an explicit g-boson degree of freedom in the effective operators $\hat{T}_{\mu}(M1)$ and $\hat{H}$. On the other hand, to obtain analytically instructive results, we introduce a simple mean-field approximation to the ground state $|0^+\rangle$. Our adoption of the g-boson degree of freedom is motivated by the observation that the variation in properties of the ground-state band along an isotopic chain can be described within the *sdg* variant of IBM-2 with Hamiltonian parameters which are reasonable from a shell-model point of view under circumstances when this is not possible within sd IBM-2 [14]. (As we shall see, the impact of the g boson on the summed $M1$ strength is non-negligible.)

To facilitate comparison with previous work, we follow [13] in our choice of the $M1$ transition operator and the Hamiltonian. Thus, we adopt for $\hat{T}(M1)$ the one-body operator

$$\hat{T}(M1) = \sqrt{\frac{3}{4\pi}} \sum_{l=2,\,4} \left(g^p_l \hat{L}^p_l + g^n_l \hat{L}^n_l \right) , \qquad (3)$$

where g^p_l (g^n_l) and $\hat{L}^p_l$ ($\hat{L}^n_l$) denote the effective g-factor and angular-momentum operator for proton (neutron) bosons of spin l, respectively, and for the Hamiltonian

$$\hat{H} = \sum_{l=2,\,4} \epsilon_l \left(\hat{n}^p_l + \hat{n}^n_l\right) - \kappa \hat{Q}_p \cdot \hat{Q}_n - f \hat{F} \cdot \hat{F}. \quad (4)$$

In this instance, the Majorana interaction is taken to be the quadratic Casimir invariant formed from the F-spin generators $\hat{F}_+ = s^{\dagger}_p s_n + d^{\dagger}_p \cdot \tilde{d}_n + g^{\dagger}_p \cdot \tilde{g}_n$, $\hat{F}_- = \hat{F}^{\dagger}_+$ and $\hat{F}_0 = \frac{1}{2}\left[\hat{F}_+, \hat{F}_-\right]$ ($s^{\dagger}_{\rho}$ denotes a s-boson creation operator, etc.). As regards the other terms in $\hat{H}$, ϵ_l is the single-particle energy of a spin $l > 0$ boson relative to the s-boson single-particle energy (the single-particle energies are taken to be the same for proton and neutron bosons), $\hat{n}^p_l$ ($\hat{n}^n_l$) the number operator for proton (neutron) bosons of spin l, and $\hat{Q}_{\rho}$ the quadrupole operator ($\rho = p,\, n$)

$$\hat{Q}_{\rho} = d^{\dagger}_{\rho} s_{\rho} + s^{\dagger}_{\rho} \tilde{d}_{\rho} + \chi^{\rho}_{dd} \left[d^{\dagger}_{\rho} \tilde{d}_{\rho}\right]^{(2)} + \chi^{\rho}_{dg} \left[d^{\dagger}_{\rho} g_{\rho} + g^{\dagger}_{\rho} \tilde{d}_{\rho}\right]^{(2)} + \chi^{\rho}_{gg} \left[g^{\dagger}_{\rho} \tilde{g}_{\rho}\right]^{(2)} . \quad (5)$$

Below, we shall employ the notation $\{\chi^{\rho}_{ll'}\}$ to denote the quadrupole operator parameters, it being understood that $\chi^{\rho}_{02} = 1 = \chi^{\rho}_{20}$, $\chi^{\rho}_{24} = \chi^{\rho}_{dg} = \chi^{\rho}_{42}$, etc.

As regards the ground state, we assume from the outset that it has complete neutron-proton symmetry or, in technical terms, that it is a state of good F spin [15] with the F spin taking on its maximal value ($F_{\max}$). Given our interest in the estimation of ground-state expectation values, this is an appropriate idealization. Although there is confusion about the precise extent of $F \neq F_{\max}$ components in low-lying collective states, the extensive numerical and phenomenological studies reported in [7] indicate that, as far as the ground state is concerned, these F-spin impurities probably never occur at more than the few percent level. Accordingly, the contribution to a ground-state expectation value from the maximal F-spin component of the wave function will be dominant if it is nonzero (we shall return to this point below when we apply our results to the Sm isotopes).

The advantage in assuming good maximal F spin is that we can, without any further approximation, substantially simplify the expectation values to be evaluated. The IBM-2 to IBM-1 projection scheme [7] can now be invoked to transcribe the IBM-2 expectation values of interest (in an IBM-2 ground state $|0^+, N_p, N_n\rangle$ containing N_p proton bosons and N_n neutron bosons) in terms of expectation values in a N ($= N_p + N_n$) boson IBM-1 ground state $|0^+, N\rangle$. The computational advantages aside, the projection facilitates the identification of several of the essential features of expectation values.

This last point is well illustrated by the expression for the summed $M1$ strength Σ_{M1} [the denominator in (2)] deduced along these lines. Modulo some two-body terms of negligible magnitude (we return to this point later), one finds that projection yields

$$\Sigma_{M1} \;\equiv\; \int dE\, S_{M1}(E) \;\simeq\; \frac{3}{4\pi}\,\frac{N_p N_n}{N(N-1)} \times \sum_l (\delta g_l)^2\, l(l+1)\, \langle 0^+,\, N|\,\hat{n}_l\,|0^+,\, N\rangle\,, \quad (6)$$

where $\delta g_l \equiv g_l^p - g_l^n$ and $\hat{n}_l$ is the number operator for the spin l bosons of IBM-1. [Formally, the integration in (6) is over all energies; however, comparison of the result with the summed experimental strength is circumscribed by the usual restrictions which apply to the application of the IBM.] Consistent with [16], projection makes explicit the celebrated P-factor dependence [17] of the summed strength ($P = N_p N_n/N$). The novel feature of the result, which may be viewed as a natural generalization of the Ginocchio M1 sum rule [16] on inclusion of the g-boson degree of freedom, is that it makes clear that the summed strength depends on boson occupation numbers *weighted by the corresponding spin squared.* This is at odds with previous studies [12,16], which have included only the s- and d-boson degrees of freedom and which have interpreted the summed strength to be a *model-independent* measure of the d-boson occupation number. In fact, the spin-weighting enhances the significance of the contribution from any g-boson admixture in the ground state relative to the d-boson contribution; in the application to Sm isotopes to be discussed below, we find that the g-boson contribution to the summed strength is at the 20% to 30% level, the contribution increasing with increasing deformation (cf. Table 2). In this respect, rare-earth nuclei are perhaps different from lighter nuclei where there is evidence that G-pair effects are unimportant [18].

In [16], the presence of a g-boson contribution in the summed strength is discounted on the grounds that it would introduce terms which do not scale with N_p and N_n in an empirically acceptable manner (i.e., do not scale as P). We do not find this formal argument compelling: explicit evaluation within the Hartree-Bose approximation (discussed below) shows that terms in Σ_{M1} proportional to the P factor remain dominant on inclusion of the g-boson degree of freedom.

To evaluate the IBM-1 expectation values obtained after projection, we invoke (where necessary) a second approximation, namely the representation of $|0^+,\, N\rangle$ as the Hartree-Bose condensate

$$|x_0,\, x_2,\, x_4\rangle \;=\; \frac{1}{\sqrt{N!}}\,\left(b^\dagger\right)^N\,|-\rangle\,, \quad (7)$$

where $|-\rangle$ denotes the vacuum state and the deformed boson creation operator

$$b^\dagger \;=\; x_0\, s^\dagger \;+\; x_2\, d_0^\dagger \;+\; x_4\, g_0^\dagger, \quad (8)$$

with $\sum_l x_l^2 \;=\; 1$. The wave function $\{x_l\}$ is determined in the usual way by application of the Rayleigh-Ritz variational procedure; the expectation value in the condensate state of the IBM-2 to IBM-1 projection of the Hamiltonian is used. We expect the Hartree-Bose approximation to yield at the very least qualitatively reliable results. For the deformed systems of interest to us, improvements in quantitative accuracy can be achieved by invoking the $1/L_c$ expansion [19], of which this Hartree-Bose approximation is the leading term.

The use of the Hartree-Bose approximation is, in part, a matter of computational convenience and, in part, a matter of taste. Determination of the ground state $|0^+,\, N\rangle$ by exact diagonalization in the large spaces appropriate to deformed systems constitutes a formidable numerical problem [13]. By contrast, numerical solution of the variational problem is straightforward regardless of the boson number N. An added advantage of the mean-field approximation is that various results become more susceptible to physical interpretation. Pertinent to our work is the expression for the sum rule of (6) in the mean-field approximation: the right-hand side of (6) reduces to

$$\frac{3}{4\pi}\;\left(\delta g^2\right)_{\text{eff}}\;\frac{N_p N_n}{N\,(N-1)}\;L_c, \quad (9)$$

where

$$L_c \;=\; N\,\sum_l\, l(l+1)\, x_l^2 \quad (10)$$

is the average angular momentum squared of the condensate and $(\delta g^2)_{\text{eff}}$ is obtained by weighting the $(\delta g_l)^2$'s by the fraction f_l of the average spin squared of a condensate boson carried by a spherical boson of spin l, i.e.,

$$\left(\delta g^2\right)_{\text{eff}} \;=\; \sum_l\, l(l+1)\, x_l^2 (\delta g_l)^2 \Big/ \sum_l\, l(l+1)\, x_l^2\,. \quad (11)$$

Provided $(\delta g^2)_{\text{eff}}$ is insensitive to details of the structure of the ground state (the case if the microscopically plausible values of $g_l^p \simeq 1$ and $g_l^n \simeq 0$ for the boson g factors are used), the summed $M1$ strength acquires the dynamical interpretation of being a measure of the mean angular momentum squared of the ground-state condensate L_c. As the latter depends quadratically on the nuclear deformation parameter δ (through its dependence on x_2 and x_4 which are proportional to δ), (9) provides a natural explanation of the celebrated quadratic increase in Σ_{M1} with δ first reported in [20]. This dependence should apply for all isotope chains where $(\delta g^2)_{\text{eff}}$ is effectively constant.

Our result in (9) for the summed strength resembles the standard sd-IBM result in [16] with the ground-state expectation value of the d-boson number replaced by L_c and the difference in d-boson g factors $(g_2^p - g_2^n)^2$ by $(\delta g^2)_{\text{eff}}$. Since $(\delta g^2)_{\text{eff}}$ will, in general, vary within a chain of isotopes, whereas $(g_2^p - g_2^n)^2$ is usually treated as a constant, introduction of the g-boson degree of freedom leads, in effect, to one additional source of variation in Σ_{M1}, namely the factor $(\delta g^2)_{\text{eff}}$. Variations in $(\delta g^2)_{\text{eff}}$ at the level of 20% or so would resolve the anomalous P dependence of Σ_{M1} identified in [16] for the Gd and Dy isotopes.

III. THE CENTROID OF THE $M1$ STRENGTH DISTRIBUTION

It is reasonable to expect that the centroid energy, being a measure of the mean energy of states excited by the scissors mode, is insensitive to the choice of g factors, and, in practice. we have indeed found this to be so (cf. our comments below). We take advantage of this fact here to simplify our presentation and make more transparent the interpretation of our result. Since exstant microscopic calculations indicate that the spin dependence of effective boson g factors is weak [21], we restrict ourselves to the special case in which the effective boson g factors are strictly independent of the boson spin l, i.e., $g_l^p = g_p$ and $g_l^n = g_n$.

Evaluation within our model of the energy-weighted summed $M1$ strength [the numerator in (2)] begins with determination of the connected double commutator

$$\left[\left(\hat{T}(M1)\right),\, \left[H,\, \hat{T}(M1)\right]\right]^{(0)} \tag{12}$$

followed by IBM-2 to IBM-1 projection of its ground-state expectation value (we invoke the standard relation between energy-weighted sum rules and such double commutators [22]). For the present choice of boson g factors, the double commutator involving the quadrupole piece of the Hamiltonian is particularly simple:

$$\left[\left(\hat{T}(M1)\right),\, \left[\hat{Q}_p\cdot\hat{Q}_n,\, \hat{T}(M1)\right]\right]^{(0)} = \frac{1}{\sqrt{3}}\,\frac{9}{2\pi}\,(g_p-g_n)^2\,\hat{Q}_p\cdot\hat{Q}_n, \tag{13}$$

which is obviously compatible with the expression for this commutator within the restriction of IBM-2 to s- and d-boson degrees of freedom [11]. Another simplication is that the two-body contributions to Σ_{M1} vanish exactly. Our expression for the centroid energy thus reduces to [23]

$$E_c \;=\; f\,N \;+\; 3\,\frac{\kappa\,\langle 0^+,\,N|\,:\hat{Q}(\chi^p)\cdot\hat{Q}(\chi^n):\,|0^+,\,N\rangle}{\sum_l\, l(l+1)\,\langle 0^+,\,N|\,\hat{n}_l\,|0^+,\,N\rangle}. \tag{14}$$

Above, $\hat{Q}(\chi^\rho)$ denotes the quadrupole operator within the sdg variant of IBM-1 [its definition parallels that in (5)] with $\{\chi^\rho_{ll'}\}$ as the choice of quadrupole parameters; the normal ordering is with respect to the spherical s-boson condensate $|x_0 = 1,\, x_2 = 0 = x_4\rangle$.

As asserted in the introduction, our result for the centroid energy comprises two distinct contributions: a shift due to the Majorana interaction (i.e., the $f\,N$ term) and a term involving the ground-state expectation of the quadrupole-quadrupole interaction. We refer to the latter as the deformation term because this expectation value vanishes in the Hartree-Bose approximation if the ground state is spherical.

The remarkable feature of the Majorana shift in (14) is its simplicity. In fact, it coincides with the splitting the Majorana interaction induces between a fully-symmetric state of F spin $F_{\max}$ and a mixed-symmetry state of F spin $F_{\max} - 1$ (recall that $F_{\max} = N/2$). Hence, the same result for the Majorana shift would have been obtained within the crude (but transparent) model of the scissors mode as an excitation of a single mixed symmetry state of F spin $F_{\max} - 1$. The implication is that, with the present choice of Majorana interaction, the Majorana shift does not depend on details of the structure of the scissors mode.

We commented above that we expect *a priori* the centroid energy E_c to depend only rather weakly on the choice of boson g factors. A peculiarity of our result for E_c is that it is *completely* independent of the boson g factors. This is no longer the case when the present restrictions on the l dependence of the boson g factors are relaxed. However, numerical studies suggest that the value taken on by the general expression for the centroid energy (which is too involved to be reproduced here) remains fairly insensitive to the particular choice of g factors. Thus, use of (14) should continue to suffice in practice (i.e., to within an accuracy of ten percent or so).

An important feature of the deformation term in (14) is that it can be computed without having to specify the Majorana interaction strength. The IBM-1 Hamiltonian parameters and IBM-2 quadrupole parameters on which it does depend can be fixed by methods (explicated in [7]) which invoke spectroscopic information on the low-lying members of the nuclear spectrum but which do not require *a priori* knowledge of the Majorana strength. Hence, the deformation term may be viewed as a known contribution to E_c, and below we treat it as such. Coupled with the fact that (14) is independent of g factors, this means that we are in a position to use information on centroid energies to pin down the Majorana interaction strength.

IV. APPLICATION TO THE SM ISOTOPES

It is known empirically that the scissors mode centroid energy is about 3 MeV. It can be argued on general grounds that the deformation term is of comparable magnitude (a few MeV or so). Obviously, the issue of whether a Majorana shift is also needed or not, and if so how large, can only be settled by detailed calculation. Among those nuclei for which there is data on the scissors mode, we consider the samarium isotopes because we can gauge the strengths and weaknesses of our model by comparison with the calculations of [13] for these isotopes (as in the present work, the point of departure in [13] is the *sdg* variant of IBM-2). Specifically, we are able to make a quantative comparison of g factors $g_{2_1^+}$ of 2_1^+ states and a qualitative comparison of summed $M1$ strength for these isotopes. In the presentation below, we first discuss these comparisons before turning to numerical results on the deformation term in the centroid energy E_c. Throughout, we adopt for the Hamiltonian $\hat{H}$ of (4) the parameters used in [13], leaving open, however, the value of the Majorana strength f. (We are at liberty to do this, because, within our model, g factors of 2_1^+ states

TABLE I. The Hartree-Bose ground state.

A	146	148	150	152	154
N (N_n)	7(1)	8(2)	9(3)	10(4)	11(5)
x_0	1	1	0.844	0.753	0.708
x_2	—	—	0.518	0.621	0.660
x_4	—	—	0.137	0.213	0.251
L_c	—	—	18	32	43

and the summed $M1$ strength, like the deformation term in E_c, do not depend on the choice of Majorana interaction strength.)

To calculate the g factors $g_{2_1^+}$, we have employed the basic Hartree-Bose result implied by our assumptions on the ground state, namely [4]

$$g_{2_1^+} = \sum_l f_l \, g_l, \tag{15}$$

where the spin fractions f_l were introduced above in connection with (11) and

$$g_l \equiv \frac{N_p}{N} g_l^p + \frac{N_n}{N} g_l^n. \tag{16}$$

To enable comparison with the $g_{2_1^+}$ results quoted in [13], the following choice of elementary boson g factors is made: $g_d^p = 0.95$, $g_d^n = -0.15$, and $g_g^\rho = r\, g_d^\rho$ with $r = 1/2$ [in the terminology of [13], this is set (c)]. The Hartree-Bose wave functions $\{x_l\}$ found with the choice of Hamiltonian parameters made above and needed to evaluate the spin fractions f_l, are given in Table I. The prominent feature of the Hartree-Bose description of the Sm isotopes is that it displays a spherical-to-deformed shape transition with increasing N_n; within our model, evaluation of 2_1^+ g factors is possible only for members of this chain of isotopes for which the Hartree-Bose state is deformed (i.e., ^{150}Sm, ^{152}Sm, and ^{154}Sm).

Results for the g factors $g_{2_1^+}$ are presented in Table II. As regards our primary concern of the comparison between our results and those of [13], we conclude that the agreement, even at the level of quantitative comparison, is satisfactory: the discrepancy (10% or less) is always comparable with $1/L_c$ (cf. Table I), the order of magnitude of the leading $1/L_c$ correction. Less satisfactory is the comparison between either of these calculations and the experimental data on g factors, but we shall not pursue this matter here (see [6]).

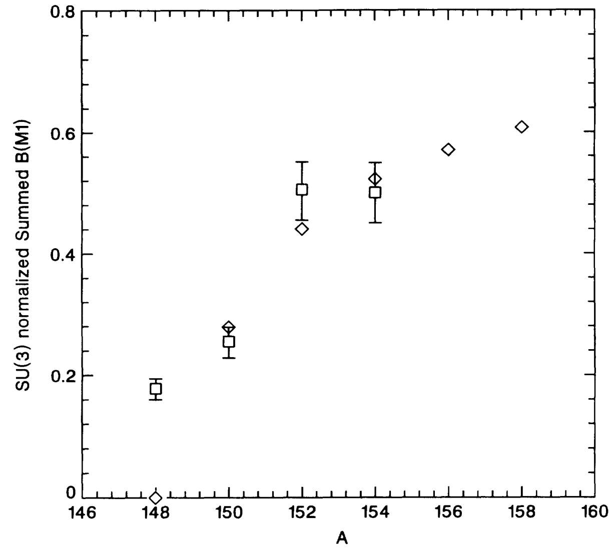

FIG. 1. Summed $B(M1)$ values versus mass number A for the Sm isotopes. The diamonds are the results of our calculations [for g-factor set (a)] and the squares are the experimental results of [20]. The summed $M1$ strength is scaled by the prediction of sdg IBM-2 in the SU(3) limit [24]. To highlight the variation in summed strength associated with a spherical-to-deformed shape transition, we include results for the radioactive isotopes ^{156}Sm and ^{158}Sm.

By contrast, our results on summed $M1$ strength indicate that our model is adequate for the description of the $M1$ strength data of interest to us. A calculation of Σ_{M1} with (9) and the "microscopic" set of boson g factors $g_l^p = 1$, $g_l^n = 0$ (set (a) in [13]) is in excellent agreement with the experimental data of [20] (cf. Fig. 1), the transitional nucleus ^{148}Sm excepted. This level of agreement is achieved *without adjustment of free parameters.* The increase in Σ_{M1} observed with increasing N_n is a clear reflection of the spherical-to-deformed shape transition. Comparison with Fig. 2 of [13] shows that the results of our model bear a strong resemblance to those of [13] (^{148}Sm again excepted). Another similarity is found if the sensitivity of Σ_{M1} to the Majorana strength is considered: in [13] it is observed that Σ_{M1} is essentially unchanged if the sizeable Majorana interaction used (0.1 MeV) is switched off; within our model, the assumption of F-spin purity for the ground state au-

TABLE II. 2_1^+ g factors, summed $M1$ strength, and the deformation term.

	A	148	150	152	154
$g_{2_1^+}$	(Exp.)	0.246(22)	0.345(30)	0.420(25)	0.350(25)
	Our calculation	—	0.53	0.44	0.38
	Calculation of [13]	0.51	0.46	0.41	0.36
Σ_{M1} $[\mu_N^2]$	[Set (a)]	—	1.07	2.05	2.77
	g-boson contribution (%)	—	19	28	33
	Set (c)	—	1.16	2.05	2.63
	2-body contributions [Set (c)]	—	0.050	0.089	0.096
E_c [MeV]	(Exp.)	3.07	3.18	2.98	3.09
	Deformation term	—	2.44	2.46	2.57

tomatically implies that Σ_{M1} is completely independent of the Majorana interaction (this is made explicit by the IBM-2 to IBM-1 projection invoked).

In support of our earlier assertions on properties of the summed strength Σ_{M1}, we include in Table II values typical of the percentage of the g-boson contribution as well as of the magnitude of the two-body contributions discarded in (6). [We adopt g-factor set (c) for the latter since, the two-body contributions vanish identically for set (a).] The negligible change in Σ_{M1} on substituting g factor set (c) for set (a) does not reflect an insensitivity to boson g factors *per se*, but is a consequence of the fact that the g factors of the g bosons in set (c) are chosen so as to compensate for changes arising from the different choice of d-boson g factors [13].

The outcome of the summed strength comparisons is somewhat flattering to our model: given its reliance on a Hartree-Bose ansatz for $|0^+, N\rangle$, it is reasonable to expect the model to work for the deformed nuclei (i.e., ^{152}Sm and ^{154}Sm) but not for the transitional nuclei (i.e., ^{148}Sm and ^{150}Sm). There is another reason to doubt the reliability of our results for ^{148}Sm and ^{150}Sm: the neglect of the effect of nonmaximal F-spin admixtures in the ground state.

The issue of the sensitivity of the summed $M1$ strength of the Sm isotopes to F-spin admixtures has been touched upon in an independent study of their impact on the g factors of 2_1^+ states [6]. It is found that, for ^{154}Sm, the change in the summed strength with increasing F-spin impurity of the ground state is negligible: as the F-spin impurity increases from 1% to 10% (the considerations of [6] indicate that the F-spin admixture is unlikely to be larger), the summed strength varies from $2.49\mu_N^2$ to $2.50\mu_N^2$ [25]. By contrast, in ^{148}Sm, the summed strength does display a marked sensitivity to the degree of F-spin impurity. A similar level of sensitivity has been noted for ^{150}Sm in [12] (information on the isotopes considered in [12] is given in [26]). We believe that this trend is reasonable. Neglect of F-spin admixtures is permissable only when the contribution from the maximal F-spin component to the summed strength is dominant; as the spherical regime is approached, this contribution tends to zero. The conclusion we draw from these results is that our assumption of F-spin purity is adequate as long as we confine ourselves to well-deformed nuclei (as we do below).

Given that the quantitative success of our model for the transitional nucleus ^{150}Sm is fortuitous, a detailed comparison of our evaluation of the deformation term with experimental centroid energies E_c is appropriate only for the deformed nuclei ^{152}Sm and ^{154}Sm. For these two nuclei, it can be seen (cf. Table II) that the deformation term accounts for all but about 0.5 MeV of the experimental centroid energies. The uncertainty in our estimation of the deformation term (because of neglect of F-spin admixtures in the ground state, uncertainties in Hamiltonian parameters and boson g factors) is unlikely to be more than 10% or so. Thus, the discrepancy between the deformation term and the experimental centroid energies is significant. If we attribute this discrepancy entirely to the presence of a Majorana shift, then we arrive at an estimate of the Majorana strength f of about 50 keV independent of the choice of isotope. This is *a factor of four smaller* than the standard range of values of f for these isotopes obtained with the (revised) empirical formula of [9].

V. CONCLUSIONS

In this paper, we have introduced a model for $M1$ strength associated with the scissors mode in deformed nuclei which allows us to explore the consequences of introducing the g-boson degree of freedom in a simple and transparent way and, identify in a clear manner the influence of the Majorana interaction on the summed $M1$ strength and its centroid energy (under the assumption that the ground state is F spin pure).

Despite the introduction of the g-boson degree of freedom, our result for the summed strength [Eq. (9)] can be cast into a form which resembles Ginocchio's sd-IBM sum rule. The principal modification is the appearance of a wave function dependent renormalization of the boson g factors. In a parameter-free calculation of the summed strength for the Sm isotopes, our result yields excellent agreement with experiment. This calculation confirms the existence of a non-negligible g-boson contribution to the summed strength, the importance of which increases with increasing deformation.

Within our model, the effect of the Majorana interaction is confined to an innocuous shift in the centroid energy of the $M1$ strength distribution: fN, where f is the Majorana interaction strength and N the sum of the number of proton and neutron bosons. In addition, there is a deformation term in the centroid energy which stems (in our work) from the attractive $\hat{Q}_p \cdot \hat{Q}_n$ in the Hamiltonian and may be treated as a known contribution. In line with our comments in the introduction, we find that a Majorana interaction strength significantly weaker than the standard value suffices to reproduce experimental centroid energies of the Sm isotopes; the deformation term accounts for all but 0.5 MeV or so of the centroid energy. In fact, our estimate of the deformation term is almost certainly an underestimate. The inclusion of even a weak hexadecupole interaction in the Hamiltonian (a possibility raised by the importance of g-boson effects for the summed $M1$ strength [24]) would have the effect of enhancing the deformation term, thereby marginalizing the contribution of the Majorana interaction still further.

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