# Dependence of the Landau parameters on the single particle potential

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(Received 28 February 1994)

We investigate the relevance of the single particle potential on the theoretical determination of the Landau parameters within the Brueckner-Bethe-Goldstone theory of symmetric nuclear matter. In particular we study the influence of the choice of the single particle potential on their overall trend as well as the fulfilment of the sum rules. At the two-hole level of approximation the single particle continuous choice and the corresponding effective mass give Landau parameters which best satisfy the sum rules. However, the spin-isospin parameter  $G'_0$  appear small in comparison with phenomenology. A good value can be obtained by a slight enhancement of the effective mass, with respect to the Bruechner value, still keeping the other Landau parameters within the phenomenological boundaries and fulfilling the sum rules fairly well.

PACS number(s): 21.65.+f, 21.30.+y, 21.60.Ev

### I. INTRODUCTION

The low energy properties of a Fermi liquid, in particular nuclear matter, can be described in terms of a set of Landau parameters, which determine the effective interaction of two quasiparticles at the Fermi surface. Phenomenology on finite nuclei provides bounds to the values of these parameters in nuclear matter. On the theoretical side, the microscopic derivation of the Landau parameters is one of the challenging problems in nuclear physics not yet fully solved [1]. Despite a general theoretical framework being available, their precise values depend on many-body effects and details of the nucleonnucleon interaction, which need to be treated accurately. Microscopic calculations of the Landau parameters have been presented in Refs. [2-8], and a review of the field has been given in Ref. [1]. It is well-known that, if one adopts the Brueckner G marix as an effective nucleonnucleon interaction, nuclear matter is unstable since the incompressibility turns out to be negative, namely, the Landau parameter  $F_0$  is less than -1. This drawback of the model can be cured by including the so called "rearrangement term" which comes out naturally from the functional derivative definition of the Landau parameters within the Brueckner approach. This result indicates that screening effects in the effective interaction are essential for a correct microscopic theory of the Landau parameters. It was therefore suggested [3,4] to include the sum to all orders of the RPA (bubble diagrams) insertions, as a substitute for the rearrangement term. This procedure introduces diagrams with an arbitrary number of internal hole lines. Furthermore, if one neglects the momentum dependence of the quasiparticle interaction, this interaction appears in all vertices, and

therefore a self-consistent nonlinear integral equation for the interaction is obtained. This approach was already followed in Refs. [3,4] and further elaborated in Ref. [1]. More general nonlinear self-consistent integral equations for the full particle-hole interaction can be devised along the same lines, but keeping the momentum dependence [6]. The equations were solved by an iterative scheme, assuming a parabolic single particle spectrum, with a constant effective mass, determined self-consistently. However, dispersion effects on the single particle self-energy are known [9] to be relevant and to produce a large enhancement of the effective mass around the Fermi surface, in agreement with phenomenology, that favors a value of the effective mass close to the bare one.

In the present work we follow a simpler approach, based on the hole-line expansion and the functional derivative method, along the lines of the work of Bäckman [2]. There, the functional derivative of the Brueckner energy was used to obtain the quasiparticle interaction, and the single particle spectrum was calculated according to the standard "gap" choice. This choice produces unphysically large differences between particle and hole energies, and it is the origin of the violation of the Pauli sum rule for the Landau parameters. It appears interesting, therefore, to consider the same approach within the "continuous choice," where no discontinuity is present between particle and hole energies, and with more modern bare nucleon-nucleon interactions. Actually, the relevance of the choice of the single particle spectrum was already pointed out by Bäckman and Brown [1].

The theoretical framework is described in Sec. II, results and discussions are presented in Sec. III, and the conclusions are drawn in Sec. IV.

# **II. THE MODEL**

According to the Landau theory of a normal Fermi liquid, the excitation energy  $\delta E$  of the system, obtained by

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changing the occupation number  $n(\vec{k})$  of the quasiparticle

state 
$$\vec{k}$$
 by an amount  $\delta n(\vec{k})$ , can be written

$$\delta E = \sum_{\vec{k}} \epsilon(\vec{k}) \delta n(\vec{k}) + \frac{1}{2} \sum_{\vec{k}\vec{k}'} f(\vec{k},\vec{k}') \delta n(\vec{k}) \delta n(\vec{k}') + \cdots , \qquad (1)$$

where  $\epsilon(\vec{k})$  is the quasiparticle energy, the sum over  $\vec{k}$  includes spin and isospin variables, and the funciton  $f(\vec{k}, \vec{k}')$  is the quasiparticle effective interaction, which, according to the expansion in Eq. (1), is given by

$$f(\vec{k},\vec{k}') = \frac{\delta^2 E}{\delta n(\vec{k})\delta n(\vec{k}')} .$$
 (2)

The spin and isospin dependence of the effective interaction, neglecting the tensor and spin-orbit parts of the interaction, is commonly written as

$$f(\vec{k}, \vec{k}') = F(\vec{k}, \vec{k}') + F'(\vec{k}, \vec{k}')\tau \cdot \tau' + G(\vec{k}, \vec{k}')\sigma \cdot \sigma' + G'(\vec{k}, \vec{k}')\sigma \cdot \sigma'\tau \cdot \tau' .$$
(3)

Equation (3) can be solved for the functions F, F', G, G', using the trace properties of the Pauli matrices, namely,

$$F = \frac{1}{16} \operatorname{tr}_{\sigma,\tau}(f) = \frac{1}{16} [f^{00} + 3f^{01} + 3f^{10} + 9f^{11}] ,$$

$$F' = \frac{1}{48} \operatorname{tr}_{\sigma,\tau} (f\tau_1 \cdot \tau_2) = \frac{1}{16} [-f^{00} + f^{01} - 3f^{10} + 3f^{11}] ,$$
(4)

$$G = \frac{1}{48} \operatorname{tr}_{\sigma,\tau} (f\sigma_1 \cdot \sigma_2)$$
  
=  $\frac{1}{16} [-f^{00} - 3f^{01} + f^{10} + 3f^{11}] ,$ 

$$\begin{aligned} G' &= \frac{1}{144} \mathrm{tr}_{\sigma,\tau} (f \sigma_1 \cdot \sigma_2 \tau_1 \cdot \tau_2) \\ &= \frac{1}{16} [f^{00} - f^{01} - f^{10} + f^{11}] \;, \end{aligned}$$

where  $f^{ST}$  are the matrix elements of f in the coupled spin and isospin representation, and are in agreement with standard expressions found in the literature [7]. Since the trace is an invariant quantity, it is unaffected by use of the particle-particle or particle-hole scheme. This certainly expresses the fact that the quasiparticle interaction of Eqs. (2) and (3) is defined only for momenta at the Fermi surface, and it is the relevant quantity for low energy and momentum nuclear matter excitations. The quantities F, F', G, G' are therefore dependent only on the relative angle between  $\vec{k}$  and  $\vec{k'}$  and the usual expansion in Legendre polynomials holds. The Landau parameters  $F_L, F'_L, G_L, G'_L$  are the coefficients of the expansion, for each partial wave L, multiplied by the density of states  $2m^*p_F/\pi^2\hbar^2$ .

The total binding energy per particle in the Brueckner approximation is given by

$$E = \sum_{\vec{k}} \frac{\hbar^2 k^2}{2m} + \frac{1}{2} \sum_{\vec{k}_1 \vec{k}_2} \langle \vec{k}_1 \vec{k}_2 | \mathcal{G}(\omega) | \vec{k}_1 \vec{k}_2 \rangle_A , \qquad (5)$$

where the single particle momenta are inside the Fermi sea, m is the nucleon mass, and the energy parameter  $\omega = e(k_1) + e(k_2)$  is the sum of the two single particle energies inside nuclear matter, defined as  $e(k) = \hbar^2 k^2 / 2m + U(k)$ . The quantity U is the self-consistent single particle potential, obtained by solving self-consistently the Bethe-Brueckner-Goldstone equation for the  $\mathcal{G}$  matrix simultaneously with the equation

$$U(k_1) = \sum_{\vec{k}_2 k_2 \le k_F} \langle \vec{k}_1 \vec{k}_2 | \mathcal{G}(\omega) | \vec{k}_1 \vec{k}_2 \rangle_A .$$
(6)

In Eqs. (5) and (6) the subscript A indicates antisymmetrization. According to the "continuous choice" the definition of Eq. (6) is valid in the whole range of single particle momentum, below and above  $k_F$ , while according to the "standard choice" the definition is restricted to momenta inside the Fermi surface, and the potential U is set equal to zero outside.

The energy functional in the Brueckner approximation is assumed to be an extension of the definition of Eq. (5), valid for generic single particle occupation numbers n(k), namely,

$$E = \sum_{\vec{k}} n(k) \frac{\hbar^2 k^2}{2m} + \frac{1}{2} \sum_{\vec{k}_1 \vec{k}_2} n(k_1) n(k_2) \langle \vec{k}_1 \vec{k}_2 | \mathcal{G}(\omega) | \vec{k}_1 \vec{k}_2 \rangle_A , \qquad (7)$$

and the functional derivative of Eq. (7) with respect to the occupation numbers, keeping to first order, gives the  $\mathcal{G}$  matrix as the effective quasiparticle interaction. Going to second order in the  $\mathcal{G}$  matrix one gets the contributions to the effective quasiparticle interaction depicted graphically in Fig. 1. The graphs 1(a) and 1(b) represent, at this order, the screening effect of the particle-hole excitations on the bare  $\mathcal{G}$  matrix. They represent the effect of long range correlations on the Landau parameters, and are usually referred to as the rearrangement term. Diagram 1(c) contains two internal hole lines, and in agreement with the hole-line expansion was not considered.

It was suggested in other works [3,4] to insert the bubble summation in graphs 1(a) and 1(b), according to the RPA series, which builds up the so called "induced interaction" [3,4]. However, this summation introduces an arbitrary number of hole lines. In the present calculation, for consistency with the hole-line expansion, the Landau parameters are determined by summing up the direct and the rearrangement terms. This restriction to lowest order suffices for the purpose of the present calculation to



FIG. 1. Contributions of the rearrangement term to the effective quasiparticle interaction.

study the relevance of the single particle spectrum and of the bare NN interaction.

It was also proposed [1] to include a wider set of diagrams by solving a nonlinear integral equation (see Fig. 22 of Ref. [1] where the  $\mathcal{G}$  matrix in the induced interaction is replaced by the quasiparticle interaction f. This procedure is, for realistic forces, quite complex [6], and it is well known that the solution by iteration of nonlinear equations requires a large number of iterations.

From the above considerations, the Landau effective quasiparticle interaction can be written

$$\begin{split} f(\vec{k}, \vec{k}') &= \langle \vec{k}, \vec{k}' | \mathcal{G} | \vec{k}, \vec{k}' \rangle_A \\ &- 2 \sum_{\vec{k}_1 \vec{k}_3} \langle \vec{k}_1 \vec{k} | \mathcal{G} | \vec{k}_3 \vec{k}' \rangle_A \Pi^{(0)}(\vec{k}_1, \vec{k}_3, 0) \\ &\times \langle \vec{k}_3 \vec{k}' | \mathcal{G} | \vec{k}_1 \vec{k} \rangle_A \ , \end{split}$$

where  $|\vec{k}| = |\vec{k}'| = k_F$  and  $\Pi^{(0)}$  is the free particle-hole propagator. Few approximations are required in order to explicitly evaluate the induced interaction of Eq. (8). In principle, the  $\mathcal{G}$  matrices appearing in the vertices are dependent also on the total momentum, which was approximated by an average over the Fermi sphere. Furthermore, the  $\mathcal{G}$  matrices are nonlocal, so they depend on two relative momenta. All these momenta have modulus of the order of  $k_F$ . In such a range the matrices are smooth, and were replaced by diagonal  $\mathcal{G}$  matrices evaluated at the average of the two relative momenta,

$$\langle \vec{k}_1 \vec{k} | \mathcal{G} | \vec{k}_3 \vec{k}' \rangle_A \equiv \langle \vec{k}_1 - \vec{k} | \mathcal{G} | \vec{k}_3 - \vec{k}' \rangle_A \approx \langle \vec{k}_3 - \vec{k} | \mathcal{G} | \vec{k}_3 - \vec{k} \rangle_A \equiv \langle \vec{k}_1 - \vec{k}' | \mathcal{G} | \vec{k}_1 - \vec{k}' \rangle_A .$$

$$(9)$$

In this matrix element we take  $|\vec{k_1}| \approx k_F$ . Consistently, the entry energy  $\omega$  was taken at twice the Fermi energy. This scheme of approximations is equivalent to the one used in Refs. [1-4].

Expanding in partial waves the direct part  $f_d$  of the quasiparticle interaction in the particle-hole representation, one gets

$$f_{d}^{ST}(\vec{k},\vec{k}') = \frac{1}{2\pi} \sum_{ts} (-1)^{t+s} (2t+1) W(\frac{1}{2} \frac{1}{2} \frac{1}{2} \frac{1}{2}; Tt) \\ \times W(\frac{1}{2} \frac{1}{2} \frac{1}{2} \frac{1}{2}; Ss) \\ \times \sum_{Jl} (2J+1) (lsJq|\mathcal{G}^{t}|lsJq) , \qquad (10)$$

where  $\vec{q} = \vec{k}' - \vec{k}$  is the momentum transer, with modulus  $q = 2k_F \sin(\theta_L/2), \theta_L$  is the Landau angle, l, s, J, and t are the two-particle angular momenta, spin, and isospin, S and T are the spin and isospin of the particle-hole pair, and W is the Wigner 6-j symbol. Similarly, for the rearrangement part of the interaction  $f_r$ , one obtains

$$f_{r}^{ST}(\vec{k},\vec{k}') = \frac{1}{\pi} \sum_{tt'ss'} (-1)^{t+t'+1} (2t+1)(2t'+1)W(t't\frac{1}{2}\frac{1}{2};T\frac{1}{2})^2 W(s's\frac{1}{2}\frac{1}{2};S\frac{1}{2})^2 \sum_{LL'l} \frac{\langle Ll00|L'0\rangle^2}{2L'+1} \alpha_l \mathcal{G}_L^{st} \mathcal{G}_{L'}^{s't'} P_l(\cos\Theta)$$
(11)

with

$$\alpha_{l} = \frac{2l+1}{2} \int_{-1}^{1} d\cos\psi \times \int \frac{k_{3}^{2} dk_{3}}{(2\pi)^{3}} \Pi^{(0)}(\vec{k}_{3} + \vec{q}, \vec{k}_{3}; 0) P_{l}(\cos\psi) ,$$
(12)

$$egin{aligned} \mathcal{G}_L^{st} &= rac{2L+1}{2} \sum_{Jl} (2J+1) \ & imes \int_{-1}^1 d\cos heta_L (lsJq|\mathcal{G}^t|lsJq) \ & imes P_L (\cos heta_L) \ , \end{aligned}$$

where  $\Theta = (\pi - \theta_L)/2$  is the angle between  $\vec{q}$  and  $\vec{k}$  (or  $\vec{k}'$ ) and  $\psi$  is the angle between  $\vec{k}_3$  and  $\vec{q}$ . Here the  $\mathcal{G}$  matrices depend on the Landau angle and were expanded in Legendre polynomials, while the particle-hole propagator was expanded in multipolarities. An exact calculation of the free particle-hole propagator  $\Pi^{(0)}$  requires the knowledge of the single particle spectrum. If a constant effective mass is assumed, the spectrum is quadratic and the evaluation can be done analytically. The accuracy and relevance of this assumption will be discussed in the next section.

A test of consistency of the various Landau parameters

can be made by evaluating sum rules obtained from Pauli principle constraints. Following the derivation of Ref. [1], we consider the two sum rules

$$\begin{split} \sum_{L} & \left( \frac{F_L}{1 + F_L(2L+1)} + \frac{F'_L}{1 + F'_L/(2L+1)} \right. \\ & \left. + \frac{G_L}{1 + G_L/(2L+1)} + \frac{G'_L}{1 + G'_L/(2L+1)} \right) = 0 \;, \; (13) \end{split}$$

$$\sum_{L} \left( \frac{F_L}{1 + F_L/(2L+1)} + 3 \frac{G'_L}{1 + G'_L/(2L+1)} \right) = 0 \ . \ (14)$$

Each term appearing in these sum rules represents an approximate expression for a scattering matrix in a given Landau partial wave L. In particular, the first sum rule expresses the fact that singlet odd scattering matrices must vanish at the forward direction. Similary, the vanishing of the triplet scattering matrix at the forward direction gives another sum rule which, combined with the previous one, produces Eq. (14) only dependent on the Landau parameters  $F_L$  and  $G_L$ .

# **III. RESULTS AND DISCUSSION**

Self-consistent Brueckner calculations of nuclear matter properties [10] provide the two-body  $\mathcal{G}$  matrices to be inserted in the evulation of the Landau parameters, according to the scheme described in the previous section. It is well known that the choice of the single particle reference spectrum is essential for the convergence [11,12]of the hole-line expansion. In the present work, we compare the results for the Landau parameters obtained with the standard gap choice and with the continous choice, which assumes the potential of Eq. (6) valid for the whole range of single particle momenta, and therefore does not introduce any discontinuity across the Fermi sphere. In the applications the potential was truncated at 4  $\rm{fm}^{-1}$ [10]. However, for the single particle spectrum appearing in the denominators of the free particle-hole propagator  $\Pi^{(0)}$ , a parabolic form is assumed in both choices, with the appropriate effective mass. In fact, it was shown [8] that the presence of an unphysical gap in the particle-hole spectrum violates badly the sum rules of Eq. (13). The differences between the two choices appear only through the  $\mathcal{G}$  matrices and the corresponding effective masses obtained from the Brueckner self-consistence, and are due only to the different correlations introduced in the  $\mathcal{G}$  matrices.

The sensitivity to the bare nucleon-nucleon interaction is studied considering two realistic NN potentials, the Reid soft core [13] and the Argonne  $v_{14}$  [14]. For the latter all channels up to J = 4 were included. For the former, the J = 3 and J = 4 channels were provided by the  $v_{14}$ , since they are missing in the original Reid potential.

The results are shown in Table I, where the reported effective masses  $m^*/m$  are the output of the Brueckner self-

TABLE I. Landau parameters for the Reid and  $v_{14}$  interactions in the gap and continuous choices. The last line contains Brueckner effective masses, except for the last one.

			<b>A</b>		
	$v_{14}$		Reid		$v_{14}$
	$\mathbf{Gap}$	Cont.	Gap	Cont.	Cont.
$F_0$	-0.267	-0.501	-0.118	-0.482	-0.378
$F_1$	-0.791	-0.575	-0.991	-0.740	-0.625
$F_2$	-0.466	-0.324	-0.392	-0.211	-0.395
$F_3$	-0.172	-0.122	-0.133	-0.072	-0.143
$F_4$	-0.121	-0.085	-0.101	-0.054	-0.102
$F_5$	-0.068	-0.048	-0.054	-0.029	-0.057
$F_6$	-0.055	-0.038	-0.045	-0.024	-0.046
$F_7$	-0.036	-0.026	-0.029	-0.016	-0.030
$G_0$	0.017	0.079	0.019	0.076	0.066
$G_1$	0.212	0.281	0.262	0.336	0.324
$G_2$	0.189	0.197	0.172	0.177	0.238
$G_3$	0.073	0.082	0.059	0.071	0.097
$G_4$	0.048	0.052	0.042	0.046	0.062
$F_0'$	0.438	0.356	0.477	0.403	0.404
$F_1'$	0.606	0.301	0.724	0.351	0.357
$F'_2$	0.247	0.108	0.199	0.034	0.130
$F'_3$	0.110	0.048	0.085	0.012	0.057
$F'_4$	0.067	0.029	0.054	0.009	0.035
$G_0'$	1.363	1.169	1.335	1.084	1.467
$G_1'$	0.442	0.273	0.376	0.159	0.346
$G'_2$	0.061	0.013	0.080	0.019	0.013
$G'_3$	0.042	0.017	0.051	0.019	0.021
$G'_{4}$	0.018	0.005	0.024	0.007	0.005
$m^*/m$	0.625	0.586	0.624	0.564	0.7

consistent calculations at the saturation density, taken at  $k_F = 1.4 \text{ fm}^{-1}$ , and were the ones used in Eqs. (10) and (11) and the density of states for the evaluation of the Landau parameters, except for the last column.

The compressibility parameter  $F_0$  turns out to be negative in all considered cases. As is well known [1-4], the inclusion of the rearrangement term reestablishes the stability condition  $F_0 > -1$ , which otherwise would be violated by the direct term alone. The nuclear matter compression modulus  $K = 3(p_F^2/m^*)(1+F_0)$  is therefore positive and can be readily calculated, and is reported in Table II. In the case of the continuous choice for the Argonne  $v_{14}$ , the value of K is quite close to the one that can be extracted directly from the saturation curve [10] of nuclear matter, which is around 200 MeV. This is an indication of the accuracy of the approximations adopted. It also means that the diagrams with higher numbers of holes, which have been neglected in performing the functional derivative, give only a small contribution to the actual value of K. This K value is also close to the empirical one [15]. In the gap choice  $F_0$  is substantially smaller in absolute value and therefore results in a much larger compression modulus.

The sensitivity of the results to the choice of the single potential is a consequence of the large cancellation between the direct and exchange terms. Also, a modest modification of the  $\mathcal{G}$  matrix results in a large modification of the final value. For instance, in the case of the Reid potential, the direct and rearrangement terms for  $F_0$  are (with the effective masses reported in Table I) -1.97 and 1.85 respectively with the gap choice, but become -1.41 and 0.93 with the continuous choices. Since the rearrangement term depends qudratically on the  $\mathcal{G}$ matrix, it suffers a stronger variation. A similar trend is found for the Argonne  $v_{14}$  interaction.

The spin parameter  $G_0$  is very small in all calculations, in agreement with phenomenology, since no collective spin mode has ever been observed.

The symmetry energy parameter parameter  $F'_0$  is substantially reduced going from the gap to the continuous choice. However, the symmetry energy  $\beta = (p_F^2/6m^*)(1 + F'_0)$  is around 30 MeV, since the smaller value of  $F'_0$  is compensated systematically by a smaller value of the effective mass. Again this value is very close to the one extracted directly from Brueckner calculations for asymmetric nuclear matter with the  $v_{14}$  and the empirical value taken from the mass formula [16].

The spin-isospin parameter  $G'_0$  can be determined phenomenologically from the position of the Gamow-Teller giant resonance, and is expected to be around 1.6. Our

TABLE II. Compression modulus K, symmetry energy  $\beta$ , and effective masses given from the Landau parameters of Table I, calculated with the Reid and  $v_{14}$  potentials in the gap and continuous choices.

	$v_{14}$	Reid			$v_{14}$
	Gap	Cont.	Gap	Cont.	Cont.
K	286.0	207.7	344.8	223.9	217.0
$\boldsymbol{\beta}$	31.2	<b>31.4</b>	32.1	33.7	27.2
$M^*/M$	0.736	0.808	0.670	0.753	0.792

values are smaller, following the general trend of all microscopic calcualtions, which have problems in reproducing such a high value. Some improvement [5] can be expected by including the coupling to the  $\Delta$  isobar, which will contribute an additional repulsion of ~ 0.2. A slightly higher effective mass can also improve the result, as the last column of Table I shows. Here the calculation was redone with the same  $\mathcal{G}$  matrices, but with a modified effective mass  $m^*/m = 0.7$  in the density of states.

The parameter  $F_1$  determines, in principle, the nucleon effective mass  $M^*$  at the Fermi energy,  $M^*/M =$  $1 + F_1/3$ . The value of  $M^*/M$  extracted from  $F_1$  is systematically larger than the self-consistent Brueckner  $m^*/m$ , as reported in Table II. In particular, the continuous choice calculation with the  $v_{14}$  gives  $M^*/M = 0.808$ , in contrast with  $m^*/m = 0.586$ . This is easily understood, since the calculation of  $F_1$  includes correlations that go beyond the Brueckner approach. In fact, the single particle potential, consistent with the graphs of Fig. 1, includes also a rearrangement term, usually referred to as  $M_2$  [9,11]. It has been shown that  $M_2$  produces a substantial enhancement of the effective mass around the Fermi energy, due to its energy dependence [9,11]. It is just this enhancement which we observe in the increase of the effective mass. This also indicates that the effective mass cannot be considered as a constant across the Fermi surface. In fully self-consistent calculations of the Landau parameters [5], the effective mass is also calculated self-consistently, assuming a parabolic single particle spectrum. According to the preceding discussion, this procedure appears questionable. In the present work, the "real" effective mass is the one calculated from  $F_1$ , i.e.,  $M^*$ . The method follows the hole-line expansion, and the effective mass appearing in Eq. (8), obtained from the functional derivative, Eq. (2), is the Brueckner one  $m^*$ . The enhancement of the effective mass is larger for the continuous choice, since in the gap choice the effect of the long range correlation is suppressed, due to the presence of a large particle-hole gap, and it is close to the one observed in actual nuclear matter calculations [11].

The other Landau parameters are not known phenomenologically, and decrease rapidly with the partial wave L.

A test on the method of calculation is provided by the sum rules of Eqs. (13) and (14), which will be denoted as  $\sum_{1}$  and  $\sum_{2}$ . From Table III it is apparent that the sum rules are in any case fairly well satisfied, and systematically better for the continuous choice. As expected, the use of the Brueckner effective mass is essential for the fulfillment of the sum rules. However, in the continuous

TABLE III. Sum rules  $\sum_{1}$  and  $\sum_{2}$  of Eqs. (13) and (14), respectively, calculated with the Reid and  $v_{14}$  potentials in the gap and continuous choices.

	$v_{14}$	Reid			v <sub>14</sub>
	Gap	Cont.	Gap	Cont.	Cont.
$\sum_{i}$	0.662	-0.041	0.664	-0.176	0.477
$\overline{\sum}_{2}^{1}$	0.847	0.012	0.894	-0.177	0.538

choice calculation with the  $v_{14}$  presented in the last column, where the effective mass in the density of states was slightly increase in  $m^*/m = 0.7$ , the sum rules are still reasonably well satisfied.

We also found that the series appearing in the sum rules converge quite slowly. The reason for that is due to the dependence of the  $\mathcal{G}$  matrix on the relative momentum q. If we represent the  $\mathcal{G}$  matrix in the relevant momentum range by a polynomial, we find that the linear component is large. We have used a cubic polynomial. The linear term is proportional to  $\sin(\theta_L/2)$ ,  $\theta_L$  being the Landau angle, and therefore it can be expanded in a Legendre polynomial according to the formula [17]

$$\sin(\theta_L/2) = \sum_L a_L P_L(\cos\theta_L) ,$$

$$a_L = (-1)^L \frac{2\Gamma(\frac{3}{2})^2(2L+1)}{\Gamma(\frac{5}{2}+L)\Gamma(\frac{3}{2}-L)} .$$
(15)

It can be readily checked that the coefficients  $a_L$  decrease with L as  $1/L^2$ , and therefore, if one truncates the series at a maximum partial wave  $L_m$ , the remainder of the series decreases as  $\sim 1/L_m$ . The higher powers of q have a faster rate of convergence. Therefore the contribution to the sum rules of the direct term, in the calculation of the Landau parameters, is expected to converge as  $\sim 1/L_m$ , and this should also give the asymptotic behavior of the whole series for large  $L_m$ . We have therefore calculated the series apearing in the sum rule up to L = 10, and then we fitted the behavior of the summation for the largest values of L with a quadratic polynomial in 1/L. The extrapolated value for  $L \to \infty$ , namely, the constant term of the polynomial, is considered the asymptotic value of the series. The procedure is illustrated in Fig. 2, where  $\sum_{2}$  is reported, together with the extrapolating polynomial, for the case of the Argonne  $v_{14}$  potential in the continuous choice. All the sum rules reported in Table III were calculated according to this procedure. We believe that this procedure is essential for an accurate estimate of the sum rules.



FIG. 2. Sum  $\sum_{2}$  as a function of the largest partial wave L included, open squares. The full line indicates the polynomial used to extract the asymptotic value of the series in the sum rule.

### **IV. CONCLUSIONS**

We have presented in this work a calculation of the Landau parameters for nuclear matter based on the Brueckner theory. The effective guasiparticle interaction was calculated from the functional derivative of the Brueckner total energy and keeping only the diagrams with at most one internal hole line. In this approach, the quasiparticle interaction is the sum of a direct term given by the  $\mathcal{G}$  matrix and a rearrangement term accounting for the screening of the long range particle-hole correlations. The single particle potential was considered in the standard gap and continuous choices. For the bare nucleon-nucleon interaction a modern realistic potential, the Argonne  $v_{14}$ , was used, as well as the Reid soft-core potential. At variance with the work by Bäckman [2], we used a particle-hole spectrum without any gap, since this unphysical discontinuity leads to a strong violation [8] of the Pauli principle sum rules. As a consequence, we found that the sum rules are fairly well satisfied in all cases, especially for the continuous choice used in the calculation of the Brueckner  $\mathcal{G}$  matrix.

This simple approach provides a set of Landau parameters which are in fair agreement with more elaborated approaches [1,5,6], where the quasiparticle interaction was obtained in a self-consistent procedure which includes all orders in the  $\mathcal{G}$  matrix and an arbitrary number of internal hole lines. In Ref. [6], where the Reid potential was also used, values of  $F_0$  and  $F'_0$  which are different by about a factor of 2 were obtained. However, it has to be stressed that the values of most of the Landau parameters are the results of large cancellations between positive and negative contributions. In particular, in the present approach there is a systematic cancellation between the direct and the rearrangement terms. We believe that, within the Brueckner approach, the scheme of approximation followed in the present work is fairly accurate, as can be seen from the above mentioned agreement between the values of the incompressibility K and symmetry energy parameter  $\beta$  extracted from the Landau parameters on one hand, and from a direct Brueckner calculation on the other hand. Improvements can be obtained only going beyond the Brueckner scheme, as the approach of Ref. [6] surely does. In our opinion, however, it is not clear if the inclusion of the screening to all orders is the main missing physical effect, or if other effects are as important, like a better treatment of the single particle spectrum, or others.

As in nuclear matter calculations [9,11], we found an enhancement of the effective mass calculated from the Landau parameter  $F_1$ , with respect to the original Brueckner one. This enhancement is a consequence of the dispersion effect in the nucleon self-energy, and raises questions about the self-consistent procedures where the nucleon effective mass is considered constant.

The calculated parameters are in close agreement with phenomenology. The only exception is  $G'_0$ , which is slightly smaller than the empirical one extracted from the position of the Gamow-Teller giant resonance. A substantial improvement can be obtained by a slight increase of the effective mass in the density of states, still keeping the sum rules satisified with a reasonable accuracy.

Certainly the present approach is not complete; in particular, the problem of the sharp variation of the effective mass around the Fermi surface should be included in the treatment, in a self-consistent way, by inserting in the particle-hole propagator an energy and momentum dependent nucleon self-energy, as required in the functional derivative of the Brueckner total energy. This would require one to go beyond the standard Brueckner approach, including in the single particle potential the  $M_2$  contribution [11,12]. We believe this to be a necessary step towards a completely self-consistent calculation of the Landau parameters. However, this is a difficult task to achieve.

#### ACKNOWLEDGMENTS

This work has been partially supported by the Commission of European Communities, Programme "Human Capital and Mobility," Grant No. CHRX-CT92-0075.

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