Strength of the ρ meson coupling to nucleons

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In his recent publication of a NN phase shift analysis below 160 MeV, Henneck reports relatively large values for the mixing parameter ϵ_1. Based on these results, Henneck suggests that the strength of the ρ meson coupling to the nucleon may be weaker than used in present day NN interactions, like the Paris, Nijmegen, or Bonn potentials. We point out that at low energies ($\lesssim$ 100 MeV) there is rather little sensitivity to the strength of the ρ coupling, due to the compensating effect of the second-order tensor term. In order to establish sensitivity, one has to go to energies $\gtrsim$ 200 MeV, where the second-order contribution has gone out. As it happens, the ϵ_1 mixing parameter is well determined in the region of energies 200–300 MeV, and there is agreement with the predictions by the Paris and Bonn potentials; the weak-ρ model is about 50% above the data. This and additional considerations in triplet P waves reconfirm that NN scattering requires the strong ρ, consistent with the $\pi\pi$-$N\bar{N}$ partial-wave analysis by Höhler and Pietarinen.

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Following the measurement of the longitudinal spin correlation coefficient A_{zz} at 67.5 MeV by the Basel group [1], there has been a lot of controversy about the strength of the tensor force in the nucleon-nucleon (NN) interaction and about the strength of the ρ meson coupling.

Henneck has published a recent paper: "Phase-shift analysis of NN scattering below 160 MeV: Indication of a strong tensor force" [2]. In a fundamental sense, it is unreasonable to include "Indication of a strong tensor force" in a title. Since half a century, we all know from the deuteron that there is a strong tensor force.

In both Refs. [1] and [2] the implication is that the coupling of the ρ meson must be weaker than used in present day NN interactions Bonn [3, 4], Nijmegen [5], or Paris [6]; more like the vector-dominance value.

In contradiction to these claims, Klomp *et al.* [7] and Machleidt and Slaus [8] show from phase shift fits that the Bonn and Paris potentials do well in reproducing the S-D mixing parameter ϵ_1, about which the controversy revolves; cf. Fig. 1.

Moreover, a recent paper by Wilburn *et al.* [13] shows Henneck's value of ϵ_1, arrived at from his phase shift analysis, to lie exactly on the Bonn curve at 25 MeV, and the low-energy measurements of ϵ_1 to lie nicely (aside from a deviation the authors do not believe to be real) and to follow well the Bonn curve.

The coupling of ρ mesons to nucleons is described by the Lagrangian

$$\mathcal{L}_{NN\rho} = g_\rho \bar{\psi}\gamma_\mu \boldsymbol{\tau}\psi\cdot\boldsymbol{\varphi}_\rho^\mu + \frac{f_\rho}{4M}\bar{\psi}\sigma_{\mu\nu}\boldsymbol{\tau}\psi\cdot(\partial^\mu \boldsymbol{\varphi}_\rho^\nu - \partial^\nu \boldsymbol{\varphi}_\rho^\mu) \,. \tag{1}$$

Sometimes the strength of the ρ coupling is characterized in terms of $\kappa_\rho \equiv f_\rho/g_\rho$, the ratio of the tensor to vector coupling constant. The value $\kappa_\rho = 3.7$, which is arrived at by using vector dominance [14], is considered weak, while $\kappa_\rho = 6.6 \pm 1.0$ is considered strong. The strong ρ coupling is obtained from a $\pi\pi$-$N\bar{N}$ partial-wave analysis conducted by Höhler and Pietarinen [15]. Later work by Grein [16] basically confirmed the Höhler-Pietarinen result.

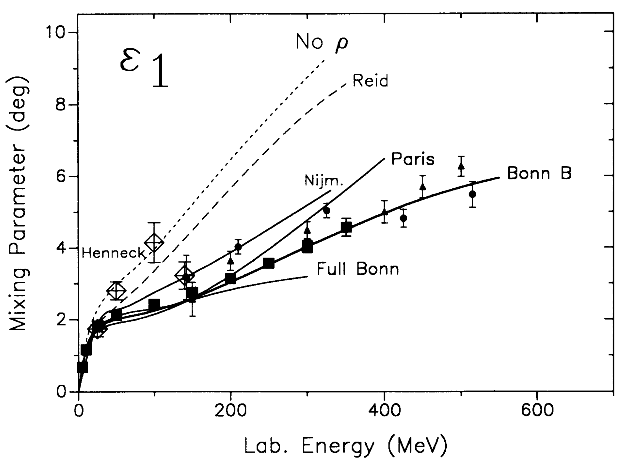

FIG. 1. The ϵ_1 mixing parameter at low and intermediate energies. Predictions by models which use a strong ρ are represented by solid lines, namely: Full Bonn [3], Bonn B [4], Paris [6], and Nijmegen [5] potentials. Furthermore, the predictions by the (weak-ρ) Reid potential [9] (dashed line) and a model that does not include a ρ meson ("No ρ," dotted line) are shown. The phase shift analysis by Henneck [2] is represented by the diamonds; besides this, we display the analyses by Arndt [10] (solid triangles), Bugg and Bryan [11] (solid dots), and the Nijmegen group [12] (solid squares).

A warning is warranted here. Use of just κ_ρ to describe the strength of the ρ coupling makes sense only if a value is used for g_ρ that is close the empirical one, $g_\rho^2/4\pi = 0.6 \pm 0.1$ [15]. If this is not the case, it is better to consider the overall strength of the ρ coupling, as it emerges from the calculation of a one-ρ-exchange Feynman diagram between two nucleons (cf., e.g., Eq. (A.9) or Eq. (A.22) of Ref. [4]); this strength is given by

$$(g_\rho + f_\rho)^2/4\pi = g_\rho^2(1+\kappa_\rho)^2/4\pi \ . \tag{2}$$

Using Höhler-Pietarinen values, one obtains

$$g_\rho^2(1+\kappa_\rho)^2/4\pi = 37 \pm 15 \quad \text{for "strong } \rho\text{."} \tag{3}$$

In the Bonn potential [3], $g_\rho^2/4\pi = 0.84$ and $\kappa_\rho = 6.1$, which yields

$$g_\rho^2(1+\kappa_\rho)^2/4\pi = 42.3 \quad \text{for the Bonn potential.} \tag{4}$$

The Bonn B potential [4], which is a one-boson exchange (OBE) parametrization of the (full) Bonn potential, uses very similar values, namely, $g_\rho^2/4\pi = 0.90$ and $\kappa_\rho = 6.1$, leading to $g_\rho^2(1+\kappa_\rho)^2/4\pi = 45.4$. From our Fig. 1 we see that this version of the Bonn potential gives the best fit to the mixing parameter ϵ_1. The Paris potential is based upon dispersion theory, therefore, by construction it is consistent with the Höhler-Pietarinen values.

The Nijmegen potential [5] is expressed differently. In Table II of their paper [5], the Nijmegen group states that $g_\rho^2/4\pi = 0.795$ and $\kappa_\rho = 4.221$ is used in their potential. This might suggest that the ρ-meson coupling in the Nijmegen potential is weak, because of the small κ_ρ. However, this is not correct. The Nijmegen group multiplies their potential with a form factor $\exp(-\mathbf{k}^2/\Lambda^2)$, with $\Lambda = 964.52$ MeV. At the meson pole (i.e., at $t = -\mathbf{k}^2 = m_\rho^2$, with t Mandelstam variable), this factor has the value $\exp(m_\rho^2/\Lambda^2) = 1.89$ [$m_\rho = 770$ MeV]. Thus, at the meson pole, the coupling constant is $g_\rho^2/4\pi = 0.795 e^{m_\rho^2/\Lambda^2} = 1.504$. This is the value one has to use, since dispersion theory (applied by Höhler and Pietarinen) determines coupling constants at the meson pole. Now, the overall ρ-meson strength comes out to be

$$g_\rho^2(1+\kappa_\rho)^2/4\pi = 41.0 \quad \text{for the Nijmegen potential.} \tag{5}$$

This is definitely a strong ρ coupling.

For comparison, we also give what vector dominance implies; using $g_\rho^2/4\pi = 0.6$ and $\kappa_\rho = 3.7$ one obtains

$$g_\rho^2(1+\kappa_\rho)^2/4\pi = 13.25 \quad \text{for "weak } \rho\text{."} \tag{6}$$

Notice that this is about 1/3 of the strong-ρ value.

We will now discuss in detail how the strength of the ρ meson coupling influences the strength of the nuclear tensor force. The usual argument goes as follows.

The tensor force generated by ρ exchange has the opposite sign of that from pion exchange [17, 4]. Therefore, summing up the ρ exchange will decrease the tensor force. However, this argument is too simplistic as was realized long ago.

If the ρ exchange tensor interaction is weak, on the order of the vector dominance value, then the tensor force will be strong, because not much of the pion contribution is cancelled [see Fig. 8a of Ref. [17]]. In this case, second-order effects of the tensor interaction will be strong [Eq. (2.10) of Ref. [17]] and one has an effective tensor contribution arising from these second-order terms of

$$V_{\text{eff}}(r) = \frac{(3 - 2\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2)}{\bar{E}} 2\mathbf{S}_{12}(V_{\text{tensor}}^{(1)})^2 \tag{7}$$

where the average energy $\bar{E} \approx 200$ MeV. Note that the $\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2$ piece is negative, of opposite sign to the π-exchange tensor. This is, of course, easy to understand, because in the iterated exchange one has a box diagram, and the two pions in the crossed channel must, while the interaction is isovector, be in a P wave because of Bose statistics. Consequently, a tensor interaction with weak-ρ coupling builds up an effective ρ-meson type coupling, strengthening the ρ channel. Once second-order effects are included, the net result is little different for weak- and strong-ρ coupling, although different amounts of the effective ρ (iterated pion exchange) result in the two cases.

All potentials, with weak or strong ρ coupling, were constrained to fit the deuteron. Thus, it is no surprise that Henneck's point at 25 MeV lies on the Bonn curve [13].

Our conclusion is that it is doubtful whether low-energy scattering experiments distinguish between weak- and strong-ρ couplings. In going to higher energies, the second-order term, Eq. (7), will tend to be eliminated. How high in energy does one have to go?

In order to answer this question, let us remember that the main part of the symmetry energy in nuclei comes from the second-order tensor interaction

$$V_{\text{symm}} \approx +\frac{12}{\bar{E}}(V_{\text{tensor}}^{(1)})^2 \ , \tag{8}$$

as one can deduce from Eq. (2.10) of Ref. [17]. Brown, Speth, and Wambach [18] showed that the V_τ so obtained dropped over a scale of energies 100 – 200 MeV, as the incident nucleon energy eats into the principal value integral. Of course, some of the symmetry energy comes from the vector coupling of the ρ meson, but one can see directly from the work of Ref. [18] that this is small; very little of the interaction drops at the slow rate that $V_{\sigma\tau}$, which comes mainly from the Born term, drops.

The work of Ref. [18] employed a strong-ρ tensor coupling, giving a relatively weak $V_{\text{tensor}}^{(1)}$. The calculations are straightforward, and would have given a much greater symmetry energy had a weak-ρ coupling been used.

Our first statement, which can be directly tested by calculations, is that the symmetry energy in nuclear matter calculations will be much too large if a weak-ρ coupling is used. Equivalently, the V_τ used in investigations of isobaric analog states will be much too large.

Calculations of these matters introduce various many-body intermediate steps, which lead to suspicion in the views of experimentalists. However, the work of Ref. [18]

points out that if one wants to see differences between the weak-ρ and strong-ρ scenarios, one should go to scattering energies $E > 100$ MeV, by which time some of the second-order contribution of the tensor interaction is stripped off. Energies of 200 MeV would be better.

Our conclusion is that it would be safest to determine ϵ_1 at energies $\gtrsim 200$ MeV where the second-order contributions from the tensor interaction have been stripped off.

The Henneck ϵ_1 at 50 MeV is about 2.8°, what one would obtain with zero ρ coupling (cf. Fig. 1). The Reid potential [9], with essentially vector dominance coupling, gives $\approx 2.4°$, whereas Nijmegen [5], Paris [6], and Bonn [3, 4] (which all use the strong ρ) give $\approx 2°$. Note that the overall strength of the ρ coupling has increased by a factor of about three in going from vector dominance to strong-ρ coupling, with a change of $\approx 0.4°$ in ϵ_1. This is what we mean by insensitivity at low energy. Quoted error in Henneck is $\pm 0.25°$.

Since Reid fits the deuteron, we believe that this potential gives a good indication of what weak-ρ coupling will give in the 200–300 MeV region, once the second-order contribution to the effective-ρ exchange has gone out. Reid gives $\epsilon_1 \approx 7°$ in this region; meanwhile Paris, Nijmegen, and Bonn predict $\approx 4°$, in perfect agreement with the phase-shift analyses by Arndt [10], Bugg and Bryan [11], and by the Nijmegen group [12] (cf. Fig 1). This confirms that NN scattering requires the strong ρ.

We note that there are phase parameters which are even more suitable than ϵ_1 to pin down the ρ coupling strength, namely, the the triplet P-wave phase shifts. In contrast to ϵ_1, the ${}^3P_J$ phase shifts are reliably determined and there is no controversy among different researchers conducting phase shift analyses. Moreover, the effect of the ρ meson is very large in P waves. For ${}^3P_0$ and ${}^3P_2$, we demonstrate this in Fig. 2, where the predictions by the weak and the strong ρ are shown. It is clearly seen that these P waves require by all means the strong ρ. This is probably the best argument why NN scattering needs the large ρ coupling, consistent with the determination by Höhler and Pietarinen [15].

Höhler [20] has recently reviewed developments since the Höhler-Pietarinen work. He points out that a somewhat different method led to $\kappa_\rho = 6.1 \pm 0.6$ [21]. Furthermore, the large Höhler-Pietarinen value agrees with an unpublished calculation by Gustafson, Nielsen, and Oades [22]. We mentioned the work by Grein [16] using NN forward-dispersion relations, which is compatible with Höhler-Pietarinen. The chief point of Höhler [20] is that the full information on the ρNN coupling is contained in the $\pi\pi$-$N\bar{N}$ P-wave helicity amplitudes and the direct way to extract coupling constants in an approximate description was employed in Ref. [15]. These helicity amplitudes were obtained from πN partial wave amplitudes with imposition of unitarity and analyticity by Höhler and Pietarinen. Work since that time has not consistently enforced these constraints [20].

In conclusion, we have shown that at low energies ($\lesssim 100$ MeV) there is rather little sensitivity to the strength of the ρNN coupling. In order to establish sensitivity, one has to go to higher energies ($\gtrsim 200$ MeV). Phase shift analyses here clearly favor the strong ρNN coupling. Independent of this empirical argument, the strong ρNN coupling was firmly established from detailed knowledge of the $\pi\pi$-$N\bar{N}$ helicity amplitudes [15].

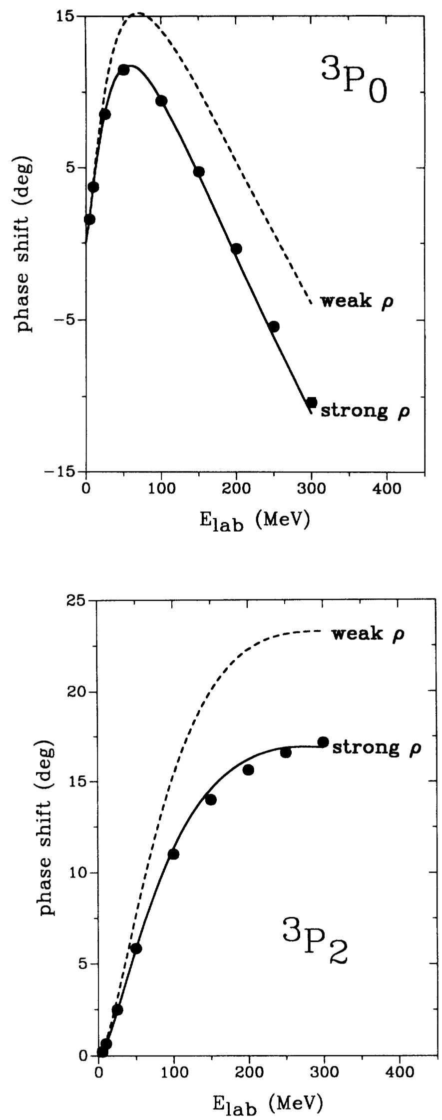

FIG. 2. The ${}^3P_0$ and ${}^3P_2$ phase shifts of proton-proton scattering. The solid line gives the prediction by a meson model that includes the strong ρ, while the dashed line is obtained using the weak ρ. The solid dots represent the Nijmegen pp multi-energy phase shift analysis [12]. (From Ref. [19].)

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