

## Preequilibrium $\alpha$ emission in the exciton model

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A new model approach to preequilibrium  $\alpha$  emission is presented which does not contain a preformation parameter or hypothesis. It takes explicit account of a pickup type of emission process. The correlation requirement proposed for the nucleons forming the emitted cluster is solely based on the cluster binding energy, and it fits readily into the framework of the exciton model. Numerical calculations are shown to be in good agreement with experimental data.

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### I. INTRODUCTION

The preequilibrium emission of  $\alpha$  particles has proven much more difficult to describe in terms of semiclassical models than nucleon emission. A number of approaches [1–13] has been tried and found to require an additional assumption and/or an adjustable parameter regarding the probability at which the cluster to be emitted is performed (prior to emission) inside the unequilibrated composite system. Such “preformation parameters,” as they are often called, are differently defined in the various approaches. Typically, the numerical values extracted by best fits to experimental data depend on the composite system involved, as well as the reaction entrance channel and the excitation energy. Furthermore, preformation parameters obtained in different treatments, and for different entrance channels, vary by orders of magnitude and are at variance both with  $\alpha$  spectroscopic factors and  $\alpha$ -decay data, raising the question if a part of the underlying physics is not absorbed into these parameters instead of being treated explicitly by the reaction model. The approach to be presented attempts an explicit treatment without recourse to a free parameter. It has already been shown [22] that angular momentum conservation is a key ingredient for model calculations aiming at a description of preequilibrium  $\alpha$  emission without fit parameters. Section II reports progress along these lines and briefly describes the reaction model used throughout this paper. Section III addresses the case of nucleon-induced  $\alpha$  emission, where angular momentum conservation is unimportant. It is shown that the  $\alpha$  particle emitted may be “allowed” to contain nucleons from below the Fermi surface merely by using appropriate residual level densities. Doing so is shown to give good agreement with experimental data without any need for a preformation parameter. Section IV gives a summary and a brief discussion of possible improvements.

### II. THE REACTION MODEL AND ANGULAR MOMENTUM CONSERVATION

The reaction model used in the calculations to be presented in this paper is the exciton model in the form

given by Gadioli [15]. It groups emission chances according to generations of  $n$ -exciton configurations, and it assumes configuration mixing within each generation [14]. This forms a convenient basis for the model refinements discussed in this paper, and it is worth recalling what configuration mixing implies:

The intermediate  $n$ -exciton states through which the composite system proceeds toward equilibrium may be modeled in different ways. On the one hand, they may be envisioned to be “pure configurations” [14], i.e., states in which each exciton resides at a well-defined single particle energy level,  $e_k$ , so that the  $n$ -exciton state wave function may be written as a product of single particle wave functions with eigenvalues,  $e_k$ , each. On the other hand, the  $n$ -exciton states may be envisioned to be linear combinations of such configurations, and the amplitudes describing the contributions of each component configuration may — in the absence of theoretical or experimental guidance — be assumed to be randomly distributed. The latter concept — called configuration mixing — is employed in the exciton model [15]. It makes it possible to formulate preequilibrium emission in close analogy to evaporation theory [16], i.e., in terms of competing decay rates of  $n$ -exciton composite system states and with only global regard for restrictions pertaining to individual component configurations: As *each*  $n$ -exciton state is assumed to contain as components essentially *all* possible configurations, it may decay (via emission or intranuclear collisions) into *any and all* daughter states allowed by general selection rules such as energy conservation, according to the principle of detailed balance. Consequently, assumptions about the dynamics of the emission process and the prevailing selection rules are easily implemented into the model, as it is only their influence on residual state densities that needs to be considered. Just as in evaporation theory, the coalescence of nucleons to form the cluster — their momentum correlation, angular momentum coupling, etc. — enters only as far as it places constraints on the residual level density accessible. This is to be contrasted to preformation or coalescence models, in which the key ingredient is the probability to form a cluster prior to emission.

At present, there is no sufficient evidence to decide whether pure or mixed configurations are the better ap-

proximation or the more appropriate concept to use. Configuration mixing will be assumed henceforth as a convenient rather than a proven basis. In this framework, imposing angular momentum conservation is straightforward, namely by (i) rewriting the basic model expression for the preequilibrium emission cross section in a form that pertains to an emission leading from a given composite system spin to a given residual spin, (ii) replacing the composite system formation cross section by a partial cross section describing the formation of the composite system with a given spin, (iii) replacing the inverse cross sections by sums over all transmission coefficients that can mediate a transition from a given composite system spin to a given residual spin, (iv) replacing the residual level density by its spin dependent form [17] and evaluating it for the applicable residual spin, (v) summing over all reachable residual spins, and (vi) summing over all populated composite system spins.

The procedure is much like going from a spin independent form of an evaporation calculation [16] to a spin dependent form [18], and the basic equations involved are given in Ref. [22]. For computational ease, the model [15] is used in its most basic form, i.e., (i) employing Ericson type [19] particle hole level densities, (ii) neglecting hole interactions, (iii) using  $A/8 = a = g\pi^2/6$  as level density parameter, (iv) accounting for neutron-proton distinction only by using appropriate factorials in the level densities, rather than applying a proper two component Fermi gas model, (v) using intranuclear collision rates as derived from quasi-free nucleon-nucleon scattering [20] and without involving a mean free path multiplier, and (vi) using emission rates based on standard optical model inverse cross sections and transmission coefficients [21]. Throughout the calculations reported in this paper, the initial exciton number,  $n_0$ , was chosen to be

$$n_0 = A_p + 2. \quad (1)$$

This choice is based on the idea of the projectile being decomposed into its constituent nucleons upon absorption into the target nucleus via a nucleon-nucleon collision, and thus creating one particle-hole pair before preequilibrium emission begins. The terminology followed in equations is summarized in Table I.

In Ref. [22], it was shown that angular momentum conservation changes the predicted preequilibrium  $\alpha$  emission cross sections drastically in reactions involving massive projectiles and thus transferring considerable amounts of angular momentum to the composite system, such as  $(\alpha, \alpha')$ ,  $(^{12}\text{C}, \alpha)$ , or  $(^{14}\text{N}, \alpha)$ . The mechanism causing this change is that the composite system is “trying to get rid” of high angular momenta by favoring preequilibrium  $\alpha$  emission over nucleon emission. The calculations presented in Ref. [22] tended to overestimate this effect, however, as simplifying assumptions about angular momentum coupling and about intranuclear (equilibrating) collisions were used. Moreover, only  $\alpha$  to proton emission cross section ratios were calculated. More recent calculations have been done without such simplifications and have produced absolute cross section predictions. Figure 1 shows a typical result, demonstrat-

TABLE I. Summary of the terminology followed in equations.

Notation	Definition
$e$	Single particle excitation energy above the Fermi energy.
$e_h$	Hole energy measured from the Fermi energy.
$B_\alpha$	Total binding energy of $\alpha$ particle (28.3 MeV).
$S_N$	Separation energy for a nucleon (taken to be the same for all four nucleons composing the emitted $\alpha$ particle).
$S_\alpha$	$\alpha$ particle separation energy.
$\epsilon_\alpha$	Channel energy of emitted $\alpha$ particle.
$E$	Total composite system excitation.
$p$	Number of particles excited in the composite system
$h$	Number of holes excited in the composite system.
$n$	Total exciton number ( $=p+h$ ).
$n_0$	Initial exciton number.
$\omega_{p,h}$	Particle hole level density.
$g$	Single particle level density.
$A$	Composite system mass number.
$A_p$	Projectile mass number.

ing the difference between neglecting and observing angular momentum conservation. No equilibrium contribution was added to the preequilibrium model predictions, no  $\alpha$ -preformation parameter whatsoever was used, and the calculations simultaneously reproduce proton preequilibrium spectra to the quality commonly expected from the exciton model. Taking explicit account of an important part of the reaction physics — angular momentum conservation — greatly improves the agreement between predicted and experimentally observed preequilibrium  $\alpha$  emission cross sections and eliminates the need for a preformation type of free parameter. The under-

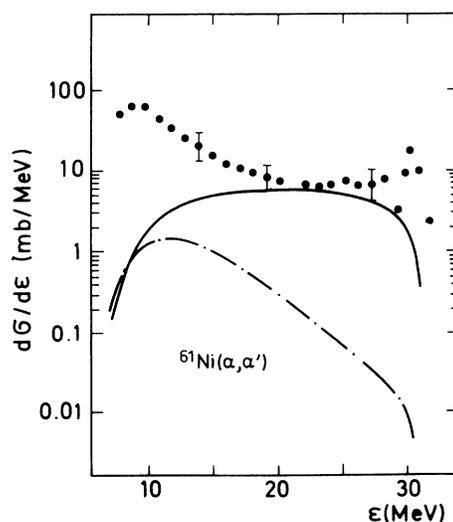


FIG. 1. Measured [23] (dots with some error bars) and calculated exciton model  $\alpha$ -emission cross sections with (full curve) and without (dash-dotted curve) angular momentum conservation. Data and calculated cross sections are given in the center-of-mass system.

lying mechanism is, however, only effective in reactions involving considerable amounts of angular momentum. The domain of reactions proceeding within the range of low angular momenta is addressed in the following section. Another part of the reaction physics hitherto neglected in exciton model calculations will be explicitly taken into account, namely participation of nucleons from below the Fermi surface.

### III. PICKUP TYPE PREEQUILIBRIUM $\alpha$ EMISSION

In nucleon-induced preequilibrium emission of  $\alpha$  particles, angular momentum conservation may safely be neglected, as is readily demonstrated in calculations along the lines described in the previous section. The composite system spins populated in these reactions are not high enough to give any preference of  $\alpha$  over nucleon emission. Yet the experimentally observed nucleon-induced preequilibrium emission of  $\alpha$  particles is appreciable, and it is strongly underestimated by a standard exciton model calculation. This is because most models assume that the  $\alpha$  particle emitted is entirely composed of excitons. This means that a state of  $p$  excited particles and  $h$  holes will produce residual states characterized by  $(p-4, h)$  particles and holes after  $\alpha$  emission. Then, in a nucleon induced reaction, the composite system must pass from a  $2p1h$  initial state to a  $4p3h$  state before  $\alpha$  emission is possible, i.e., it must undergo two thermalizing collisions, which tend to tie up excitation energy in hole degrees of freedom. As a result, the hard part of the  $\alpha$  spectrum is strongly underestimated.

The assumption that preequilibrium  $\alpha$  particles must be composed solely of excitons is, however, quite arbitrary, as plenty of nucleons residing at levels below the Fermi surface are available as "teammates" for excitons to form an  $\alpha$  particle in a pickup type of emission process [24–26]. Consequently,  $\alpha$  emission from, e.g.,  $(5p, 4h)$  configurations can proceed not only as depicted in Fig. 2 but also as depicted in Fig. 3, involving one (or more)

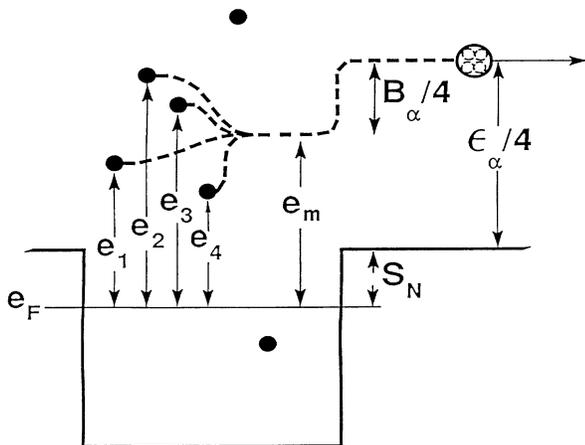


FIG. 2. Preequilibrium  $\alpha$  emission with only excitons participating. For definition of symbols see Table I and Eq. (2).

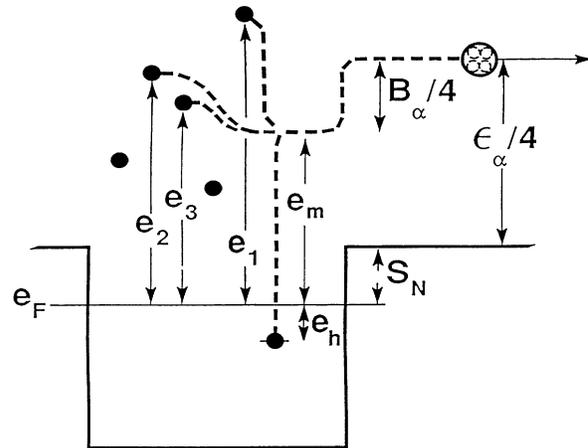


FIG. 3. Pickup type preequilibrium emission.

sub-Fermi nucleons. The possibility of such a pickup type of cluster emission process may readily be taken into account in the framework of the exciton model, and it opens new ways of preequilibrium  $\alpha$  emission: First,  $(p \geq 4, h)$  mother states can decay into  $(p-3, h+1)$ ,  $(p-2, h+2)$ , etc. daughter states in addition to the "traditional"  $(p-4, h)$  daughter states via  $\alpha$  emission. Second,  $(p < 4, h)$  states become capable of  $\alpha$  emission, which they were not under the "traditional" assumption of an excitons-only composition of the emitted cluster.

The quantitative effects of these possibilities on predicted  $\alpha$  emission cross sections in the exciton models are rather drastic, as the inherent [14] model assumption of configuration mixing mandates that any and all daughter states be accessible from any and all mother states, unless energy conservation or the assumed emission mechanism constrains the exit channel phase space. Such constraints arise from (i) the requirement that the cluster constituent *excitons* carry enough single particle excitation, as they must provide hole energy for the holes created in a pickup type emission on top of the exit channel energy, and (ii) a requirement of correlation (if any) imposed on *all constituent nucleons* of the emitted cluster, whether they are excitons or not.

Both types of constraint limit the residual level density by limiting the depth of the hole(s) created in the pickup process. It is important to note that within the model (i.e., under the configuration mixing assumption) it is only these constraints that enter. No cluster preformation probability — be it a parameter or calculated from state density ratios or from momentum correlation or whatever — is required. The transition considered to be the emission process is not  $(n\text{-exciton}) \rightarrow (n\text{-exciton} + \alpha) \rightarrow \text{residue} + \alpha$  but  $(n\text{-exciton}) \rightarrow \text{residue} + \alpha$ .

The correlation requirement proposed here is very simple: It pertains only to the single particle excitation of the nucleons involved, and it requires them to be "within the  $\alpha$  particle binding energy (28.3 MeV) of one another" in the following sense: The four nucleons forming the  $\alpha$  particle share the excitation energy  $\epsilon_\alpha + S_\alpha = \epsilon_\alpha + 4S_N - B_\alpha$  and thus have a mean single particle excitation

$$e_m = (\epsilon_\alpha + S_\alpha)/4 . \quad (2)$$

The correlation requirement proposed may then be stated as

$$\sum_i |e_i - e_m| \leq B_\alpha , \quad (3)$$

with the sum extended over all four nucleons. It means that nucleons are eligible as cluster constituents only if they have a single particle excitation reasonably close to the mean single particle excitation,  $e_m$ , required for emission; so close that their excitation (on average, taken over the four constituents) differs from  $e_m$  by no more than  $B_\alpha/4$ .

Consider the case that one sub-Fermi nucleon is picked up to become a constituent of the  $\alpha$  cluster emitted: The picked-up nucleon must be elevated to  $e_m$  in the emission process (cf. Fig. 3), and Eq. (3) implies that the energy by which it is elevated be smaller than  $B_\alpha$ , i.e.,

$$e_h + e_m \leq B_\alpha . \quad (4)$$

The effect of the correlation requirement is to limit the residual state density by limiting the depth of the hole created in the pickup process. The three participating excitons must share a combined excitation of

$$E_\alpha = \epsilon_\alpha + S_\alpha + e_h \leq E \quad (5)$$

for energy conservation, which constitutes another limit to the hole depth. The residual level density accessible by pickup  $\alpha$  emission is then easily calculated to be

$$\omega_{\text{res,pickup}} = \int \omega_{p-3,h}(E - E_\alpha) g de_h \quad (6)$$

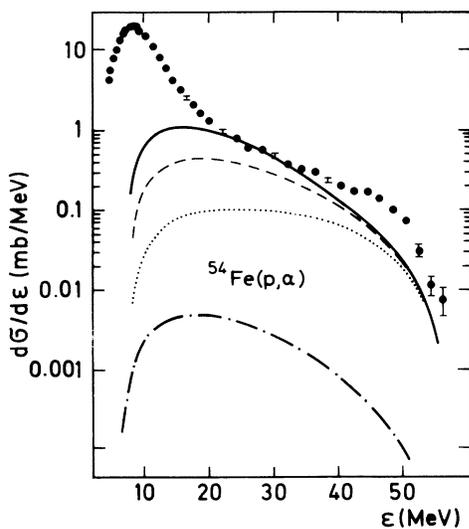


FIG. 4. Exciton model calculations with (full curve) and without (dash-dotted curve) pick up  $\alpha$  emission as compared to experimental data from Ref. [27] at an incident energy of 62 MeV. No evaporation component was added.

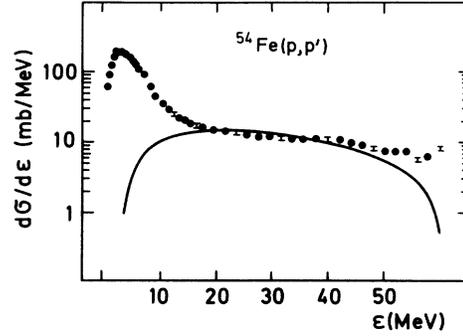


FIG. 5. Proton spectrum resulting from the same calculation. Data from Ref. [27].

with the integral extending from  $e_h = 0$  to the limit imposed by Eq. (4) or (5), whichever is more stringent. Equation (4) may be rewritten as

$$0 \leq e_h \leq B_\alpha - (\epsilon_\alpha + S_\alpha)/4$$

and yields an upper spectral limit for  $1N$  pickup emission contributions amounting to

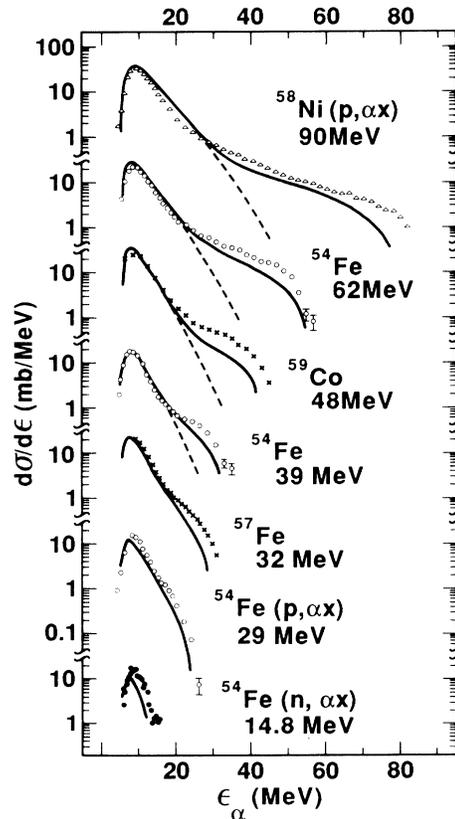


FIG. 6. Calculated preequilibrium (plus evaporative)  $\alpha$ -particle spectra compared to experimental data in the mass 55 region. The data points depicted by the triangles were taken from Ref. [28], the open circles from Ref. [27], the crosses from Ref. [8], the dots from Ref. [29], and the squares from Ref. [11]. The equilibrium components are indicated by dashed lines.

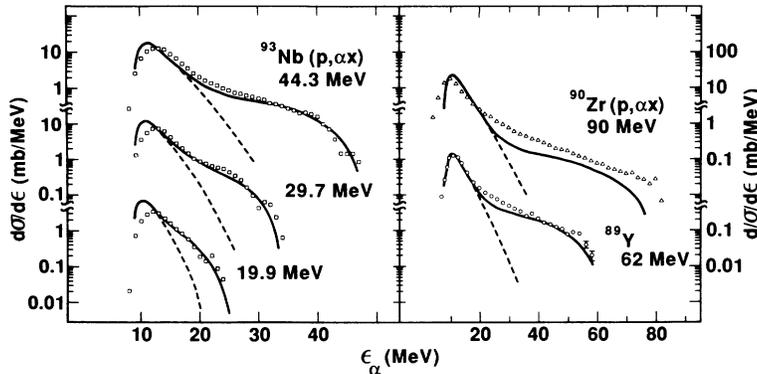


FIG. 7. Same as Fig. 6, but for the mass 90 region.

$$\epsilon_{\alpha} \leq 4B_{\alpha} - S_{\alpha} \quad (7)$$

This offers the possibility of an experimental test.

The treatment is readily extended to the pickup of two or three nucleons, but the resulting restrictions analogous to Eqs. (4), (5), and (7) tend to limit contributions to the softer part of the  $\alpha$  spectrum, where they are entangled with the evaporation component. They were ignored in the calculations presented.

Figure 4 shows the result of an exciton model calculation with one nucleon pickup  $\alpha$  emission along the lines described above. The contribution from 3p2h states—the first in line from the 2p1h entrance channel to allow pickup type emission—is shown separately (thin dotted curve). Including the contribution of 4p3h states yields the thin dashed curve, and including 5p4h contributions (full curve) comes already close to the limit of significance, where the hard part of the spectrum is concerned. The pickup mechanism is obviously very important, and the data are reproduced quite well in terms of absolute cross sections — again without introducing a free parameter. The proton spectrum resulting from the same calculation fits the data with customary exciton model precision (Fig. 5).

A crucial test for all statistical model treatments is whether the results scale correctly as the excitation energy is varied. This question is addressed in Figs. 6 and 7, which show  $\alpha$  spectra observed in reactions spanning some 70 MeV in composite system excitation. It seems that the pickup mechanism passes the energy scaling test rather nicely. Both the shape and the absolute magnitude of the experimentally observed cross sections are reproduced reasonably well without involving preformation parameters.

The data used for comparison in Figs. 4–7 were taken from the literature as indicated. Some are published as angle integrated in the center-of-mass system, some in the laboratory system. Where data were available in the center-of-mass system (Refs. [8,11,23]), the calculated spectra are presented likewise in the figures. Where not, the calculated results were plotted on a channel energy scale, which conforms more nearly — although certainly less than exactly — to the laboratory system. The resulting inaccuracies are insignificant with respect to the conclusions drawn.

#### IV. SUMMARY AND CONCLUSIONS

The exciton model of preequilibrium emission was extended to take explicit account of angular momentum conservation and to allow for a pickup type of cluster emission. It was used in its most basic form, so as to keep the consequences of the model assumptions as transparent as possible, and no fit parameter such as cluster preformation probability was introduced or used. Calculations of  $\alpha$  preequilibrium emission spectra were carried out for a number of nuclear systems and spanning almost an order of magnitude in composite system excitation energy.

Both angular momentum conservation and a pickup of nucleons from below the Fermi surface seem to play an important, if not a commanding role in preequilibrium  $\alpha$  emission. Taking them into account gives about the right spectral shape and the right order of magnitude of emission cross sections without recourse to any adjustable parameter. In particular, no cluster preformation parameter or assumption is needed.

Numerous refinements of the approach described in this paper are conceivable, such as the use of more sophisticated level densities, explicit account of hole interaction, etc. Furthermore, the generalization to ejectiles other than  $\alpha$  particles is straightforward. Whether such improvements and extensions will result in a satisfactory overall description of preequilibrium cluster emission remains to be seen. Given the simplicity of the concept, however, and given — most importantly — the absence of any adjustable parameter, the agreement between data and model predictions seems quite encouraging.

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