"Hidden" world of virtually excited clusters in atomic nuclei and its possible observation in quasielastic knockout of clusters by 1 GeV protons

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A new kind of cluster quasielastic knockout experiment is proposed based on the generalized distorted wave impulse approximation calculation which uses Glauber-Sitenko multiple scattering theory and takes into account deexcitation of virtual excited clusters in the nucleus. Elements of general formalism are presented, including the discussion of the difference between "fast" and "slow" cluster processes. The reaction $^{12}C(p,p\alpha)\,^{8}Be$ is considered in detail. It shows a strong dependence of the "effective momentum distribution" of knocked-out cluster on both the scattering angle of fast proton and orientation angles of recoil momentum $\mathbf{q}$ with respect to the direction of the incident beam and to the scattering plane of a fast proton (i.e., Θ_q and φ_q anisotropies). Experiments of this kind are desirable also for electron-induced cluster knockout. Finally, the possibility of observing Θ_q and φ_q anisotropies in $^{2}H(e,e'p)N^*$ and $^{1}H(e,e'\pi^+)n$ reactions at a few GeV energies due to the quark deexcitation effects is discussed.

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I. INTRODUCTION

The modern era of understanding what is nucleon clustering in atomic nuclei began probably with Wildermuth's paper [1] (see also Ref. [2]), where it was noted that the oscillator shell-model wave function of, say, the ^{8}Be nucleus can be presented as the antisymmetrized wave function of a cluster model (with a common value of the oscillator parameter $\hbar\omega$ for both internal Jacobi coordinates of clusters and $\alpha\alpha$-relative motion ones). This "cluster representation" of the shell-model wave functions has outlined a very close interconnection between two models, i.e., a big "cluster capacity" of the shell model. The resonating group method [2] intimately connected with this approach (the construction of its wave function is similar to that of the above cluster representation) gave a good description of cluster-cluster scattering and reactions. However, the shortcoming of the above concept was that different cluster representations for the same nucleus (e.g., ^{8}Be+ ^{8}Be and $^{12}C+\alpha$ for ^{16}O nucleus) were equally valid for bound systems. Moreover, the straightforward cluster representation exists for some particular cases of nuclear wave functions only. So, there was at that time no quantitative measure of clustering. Such measure was given in [3,4] and in [5], where a completely different approach in comparison to [1,2] was formulated. Specifically, the many-particle fractional parentage technique of the shell model and methods of transformation of one-nucleon coordinates to Jacobi coordinates were used to form a ground-state cluster of b nucleons as the subsystem in the nucleus A and to define the wave function of relative motion $(A-b)-b$. The formulas were written down for cluster spectroscopic factors $S^{b}_{i,f}(A)$ which just measure the aforementioned probability to find the cluster in the nucleus [3–5] [say, $S^{\alpha}_{0,0}(^{16}O)\simeq 0.3$ for the virtual decay of ^{16}O nucleus to $^{12}C_{g.s.}+\alpha$]. Some specific sum rule was found [6]—the total "effective number" of α particles in, e.g., the ^{16}O shell-model nucleus $N_\alpha(^{16}O)$ equals 13.7 (it is in qualitative agreement with inclusive high-energy knockout experiment results for the ^{12}C nucleus [7] [where $N_\alpha(^{12}C)\simeq 7$] if the final state absorption is taken into account [5]). In the 1970s this shell-model formalism was extended and refined [8–11] and the values of $S^{b}_{i,f}(A)$ were tabulated [12,13]. All these theoretical predictions were successfully confined by a large amount of experimental data on cluster stripping and pickup reactions [14,15]. The exclusive process of cluster quasielastic knockout by protons [16] was also very informative here [17,18]; however, the energies of bombarding protons in experiments [17,18] were rather low, 100–200 MeV only. So, all the mentioned reactions [14,15,17,18] have had more or less surface character. Three independent aspects of the above verification can be noted: the shape of cluster momentum distributions, the ratio of components with different orbital momenta (if it is allowed by selection rules), and the absolute values of spectroscopic factors $S^{b}_{i,f}(A)$. The theory was confirmed in all three points by using the extended distorted wave Born approximation (DWBA) and distorted wave impulse approximation (DWIA) calculations.

From the conceptual point of view it is important to note that, in the shell-model formalism under discussion, the nucleon numbers in cluster b in the outgoing channel are fixed (see below) and the identity of nucleons is taken into account by a factor of $A!/(A-b)!b!$ in the expression for $S^{b}_{i,f}(A)$ [the spectroscopic factor is the synthetic quantity, depending on properties of both initial and final states); the identity factor is just the origin of high value of $N_\alpha(^{16}O)$]. We will clarify below that the final-state antisymmetrization (a subject of intense discussion in connection with the α decay of heavy nuclei, etc. [19]) must not be essential, i.e., the cluster transfer or knockout process must be fast ("instantaneous" cluster is measured). So, the kinetic energy of a knocked-out cluster b must be high enough, $E_b \gg \hbar\omega$.

Almost all these theoretical developments were connected with the taciturn assumption that only the ground-state virtual cluster is essential. This assumption seems reasonable if the process is just of a surface character [17].

The internally excited virtual clusters were discussed by Rotter *et al.* [20] who have considered, e.g., the reaction $^{12}C(^{10}B,^{6}Li)^{16}O^*(2^-)$ with the spin transfer $S = 1$, i.e., with the transfer of an excited unbound $^4He^*$ nucleus having the orbital Young scheme $[f] = [31]$, $L^\pi = 1^-$, $T = 0$, and $S = 1$ (the Young scheme selection rules [21,6] permit here the transfer of any possible Young scheme $[f]$=[4],[31],[22]). Of course, two-step *t-p* transfer can occur for the above reaction, too [15]. So, the general approach to the multinucleon transfer reaction theory was formulated in the first two papers [20] where all the possible excited states of a virtual cluster are taken into account; however, the problem of consistent inclusion of corresponding poles into the dynamic theory of the above process is still not solved.

Approximately at the same time the idea was originated by our group to formulate the microscopic theory of cluster quasielastic knockout by fast protons, the elegant analytical theory of multiple scattering of fast hadrons on nuclei given by Glauber [22] and Sitenko [23] appeared to be very suitable here. We came to understand that this microscopic theory of cluster knockout (see Refs. [24–26] and also [27] for the plane-wave version of it) has one striking property: the big contribution of nondiagonal (inelastic) amplitudes of fast hadron scattering on the virtual cluster in the nucleus $\langle O|\Omega|\gamma\rangle$ connecting various internally excited states $|\gamma\rangle$ of virtual cluster with its final ground state $|0\rangle$ (see below). Calculation of spectroscopic factors of virtually excited clusters [11,26] (carried out in connection with the above property of scattering amplitudes) has shown unexpectedly that the sum of these factors exceeds remarkably the ground state ones. For instance, for the above example of the ^{16}O nucleus we have the sum of spectroscopic factors of virtually excited α clusters in the channels $^{16}O_{g.s.} \to {}^{12}C_{g.s.} + \alpha^* N_{\alpha^*,0}(^{16}O) \simeq 21.5$ [7,26] [compare it to the value $S^\alpha_{00}(^{16}O) \simeq 0.3$ given above; this is in fact the "effective number" of $^{12}C_{g.s.}$ nuclei in the ^{16}O nucleus]. So, the problem of virtually excited clusters (subsystems) actually appears to be a general problem of nuclear physics.

The influence of virtually excited clusters has resulted in nonfactorized complicated formulas for cross sections of cluster quasielastic knockout reactions, in the first predicted anisotropies of cluster momentum distributions on orientation angles Θ_q and φ_q of spectator $(A-b)$ recoil momentum $\mathbf{q}$ with respect to the initial beam direction $\mathbf{p}_0/p_0$ [25,26] and to the scattering plane $(\mathbf{p}_0;\mathbf{p}_0')$ [28,26] correspondingly, etc.

It is fortunate that, if in accordance with Glauber-Sitenko theory, the proton energy is chosen to be high enough, $E_0 \approx 1$ GeV, then the cluster knockout process is of volume character more than at lower energies [29]. It creates the chance to reveal the essential influence of the rich world of the internally excited clusters (which are usually "hidden" under the nuclear surface due to their large binding energies in the nucleus) if the verification of these opportunities will be done by means of the "realistic" distorted-wave impulse approximation (DWIA) calculation. We shall see here that this verification, indeed, gives a positive answer. So it becomes possible to outline the character of desirable experimental investigation, bearing in mind that, say, the energy resolution of modern $(p,2p)$ experiments at 1 GeV bombarding energy $\Delta E \simeq 2$–3 MeV [30] is just good enough to separate individual levels of the ^8Be nucleus in the knockout reaction $^{12}C(p,p\alpha)^8Be$.

Our paper is organized as follows. In the second section we present the microscopic DWIA formalism generalized to include the nondiagonal amplitudes of internal cluster rearrangement. Some estimates of the final-state antisymmetrization effect [19] are given here, too. The third section demonstrates the numerical results for the most popular reaction $^{12}C(p,p\alpha)^8Be$. All the original features of cluster knockout amplitudes obtained previously within the plane-wave approximation remain quite visible (albeit sometimes of a smaller magnitude) in these calculations. Finally, the concluding section contains a qualitative view of possible applications of our present ideas to different areas of nuclear physics, which can be exemplified by medium and high energy reactions $^{16}O(\gamma,dd)\ ^{12}C$, $A(e,e'b)A-b$ and $^2H(e,e'p)N^*$ (this last reaction with the excited baryon N^* as a spectator is oriented to investigate the quark degrees of freedom).

II. ELEMENTS OF FORMALISM

Using Glauber-Sitenko multiple scattering operator Ω,

$$\Omega = 1 - \prod_{j=1}^{b}[1 - \omega(\boldsymbol{\rho} - \boldsymbol{\rho}_j')], \tag{1}$$

$$\omega(\boldsymbol{\rho}) = \frac{1}{2\pi i p_0}\int d^2p \exp(i\mathbf{p}\cdot\boldsymbol{\rho}) f(\mathbf{p}), \tag{2}$$

where $f(\mathbf{p})$ is the nucleon-nucleon scattering amplitude and $(\boldsymbol{\rho}_j', z_j')$ means cylindrical coordinate of target nucleon j; we intend to illuminate the general idea by the simplest case of plane wave approximation for maximal scattering multiplicity [25,26].

The multiplicity can be fixed experimentally by the kinematics of free b-fold p-b scattering inserted into the coincidence geometry of $p+b$ registration in our quasielastic knockout reaction. Omitting in the schematic illustration the angular momentum algebra we can write the target nucleus wave function as

$$|A\alpha\rangle = \sum_{\beta\gamma\mu}\langle A\alpha|A-b,\beta;\mu;b\gamma\rangle|A-b,\beta\rangle\Phi_\mu(\mathbf{R})|b\gamma\rangle. \tag{3}$$

Here the cluster fractional parentage technique of the translationally invariant shell model is used [11,26] (so the numbers of nucleons constituting cluster b are fixed); $\mathbf{R} = \mathbf{R}_{A-b} - \mathbf{R}_b$ means relative motion coordinate. Then the scattering amplitude

$$M_{if}(\mathbf{p},\mathbf{q}) = \frac{ip_0'}{2\pi}\int d^2\rho \exp(i\mathbf{p}\cdot\boldsymbol{\rho})\int d\tau\, \psi_f^* \Omega \psi_i \quad (4)$$

with

$$\psi_i \equiv |A\alpha\rangle, \quad \psi_f \equiv |A-b,\beta\rangle \varphi_b(\mathbf{r})\exp(i\mathbf{Q}\cdot\mathbf{R}_b)\exp(-i\mathbf{q}\cdot\mathbf{R}_{A-b})$$

can be written as (b-fold scattering only)

$$\begin{aligned} M_{if}(\mathbf{p},\mathbf{q}) = {} & \frac{ip_0'}{2\pi}(-1)^b \sum_{\gamma\mu}\langle A\alpha|A-b,\beta;\mu;b\gamma\rangle \begin{pmatrix} A \\ b \end{pmatrix}^{1/2} \left(\frac{1}{2\pi i p_0}\right)^b \\ & \times \int d^2\rho \exp(i\mathbf{p}\cdot\boldsymbol{\rho}) \prod_{j=1}^{b}\left\{\int d^2 p_j \exp[-i\mathbf{p}_j\cdot(\boldsymbol{\rho}-\boldsymbol{\rho}_j')] f(\mathbf{p}_j)\right\} \\ & \times |b\gamma\rangle \Phi_\mu(\mathbf{R})\exp(-i\mathbf{Q}\cdot\mathbf{R}_b)\exp(i\mathbf{q}\cdot\mathbf{R}_{A-b})\varphi_b(\mathbf{r}) d^3R_{A-b} d^3r_1\cdots d^3r_{b-1}. \end{aligned} \quad (5)$$

Integrating over ρ we obtain the factor $2\pi^2\delta(\mathbf{p}-\sum_{j=1}^{b}\mathbf{p}_j)$. Then we transform the exponential factors by introducing Jacobi coordinates $\mathbf{r}_i \equiv \mathbf{R}_s - \mathbf{R}_t$, where $\mathbf{R}_s$ ($\mathbf{R}_t$) is the c.m. coordinate of the group of s (t) nucleons ($s+t \le b$) and the corresponding momenta

$$\mathbf{Q}_i = \mu_{st}\left(\frac{1}{s}\sum_{k=1}^{s}\mathbf{p}_k + \frac{1}{t}\sum_{j=s+1}^{s+t}\mathbf{p}_j\right);$$

the reduced mass is given in nucleon masses. Keeping in mind a smooth character of p dependence of scattering amplitude $f(\mathbf{p})$ and taking into account that $|\mathbf{Q}_i| \ll \frac{1}{b}|\mathbf{p}|$ we can carry out easily the integration over coordinate $\mathbf{R}$ and over the intermediate momenta. It results in the expression

$$\begin{aligned} M_{if}(\mathbf{p},\mathbf{q}) = {} & \frac{2\pi i p_0'}{(2\pi i p_0)^b}\sum_{\gamma\mu}\langle A\alpha|A-b,\beta;\mu;b\gamma\rangle \begin{pmatrix} A \\ b \end{pmatrix}^{1/2} \Phi_\mu(\mathbf{q})\left[f\left(\frac{\mathbf{p}}{b}\right)\right]^b \\ & \times \int d^2Q_1\cdots d^2Q_{b-1}\exp\left(i\sum_{j=1}^{b-1}\mathbf{Q}_j\mathbf{r}_j\right)|b\gamma\rangle \varphi_b^*(r) d^3r_1\cdots d^3r_{b-1} \\ = {} & \begin{pmatrix} A \\ b \end{pmatrix}^{1/2}\left(\frac{2\pi}{i}\right)^{b-1}\frac{p_0'}{p_0^b}\left[f\left(\frac{\mathbf{p}}{b}\right)\right]^b (-1)^b \sum_{\gamma\mu}\langle A\alpha|A-b,\beta;\mu;b\gamma\rangle \\ & \times \Phi_\mu(\mathbf{q})\int dz_1\cdots dz_{b-1}|b\gamma\rangle\varphi_b^*(r)|_{\rho_1=\rho_2=\cdots=0}. \end{aligned} \quad (6)$$

After the integration over momenta $\mathbf{Q}_i$ we obtain the very compact formula

$$M_{if}(\mathbf{p},\mathbf{q}) = \begin{pmatrix} A \\ b \end{pmatrix}^{1/2}\left(\frac{2\pi}{i}\right)^{b-1}\frac{p_0'}{p_0^b}\left[f\left(\frac{\mathbf{p}}{b}\right)\right]^b (-1)^b \sum_{\gamma\mu}\langle A\alpha|A-b,\beta;\mu;b\gamma\rangle \Phi_\mu(\mathbf{q}) C_\gamma^{b(b)} \quad (7)$$

where

$$C_\gamma^{b(b)} = C_{N_0L_00}^{b(b)} = \int \varphi_{b,000}^*(00z)\varphi_{b,N_0L_00}(00z)dz_1\cdots dz_{b-1}. \quad (8)$$

One-dimensional overlap on each Jacobi coordinate here means a large contribution to the reaction amplitude of deexcitation of internal cluster states with $N_0L_0 \neq 0$ but $M_0 = 0$ and the radical change of the physics of cluster quasielastic knockout. The traditional factorization of the differential cross section into the free scattering cross section and recoil momentum distribution [16] becomes lost due to the above nondiagonal scattering terms and resulting interference of different momentum wave-function amplitudes. However, it is convenient to retain formally this factorization to follow the traditional convention in the interpretation of experimental data. So, we introduce the "effective momentum distribution" (EMD) of the recoil spectator nucleus as the generalized form factor and represent the differential cross section by

$$\frac{d^3\sigma}{d\Omega_a d\Omega_b dE_a} = \frac{m_p}{h^2}\frac{1}{(2J+1)}\overline{|\mathcal{F}_{if}(q)|^2}\left(\frac{d\sigma_{ab}}{d\Omega a}\right)_{\text{free}}, \tag{9}$$

$$\begin{aligned}\frac{1}{(2J+1)}\overline{|\mathcal{F}_{if}(q)|^2} &= \langle T_1 M_{T_1} T_0 M_{T_0}|TM_T\rangle^2 \begin{pmatrix} A \\ b \end{pmatrix}\frac{1}{4\pi} \\ &\times \sum_{\mathcal{L}jl}[(2J_1+1)(2j+1)(2L+1)(2S+1)]^{1/2}\begin{Bmatrix} L_1 & S_1 & J_1 \\ \mathcal{L} & S_0 & j \\ L & S & J\end{Bmatrix}(-1)^{\mathcal{L}+l}\mathbf{P}_l(\cos\Theta_q) \\ &\times \sum_{\substack{n\Lambda n'\Lambda' \\ L_0 L_0'}} \langle A a N L S T | A-b, \beta; N_1 L_1 S_1 T_1 \\ &\times n\Lambda, bN_0L_0S_0T_0 : \{\mathcal{L}\}\rangle\langle AaNLST|A-b,\beta;N_1L_1S_1T_1;n'\Lambda', bN_0'L_0'S_0T_0:\{\mathcal{L}\}\rangle \\ &\times \begin{Bmatrix} \Lambda' & \Lambda & l \\ L_0 & L_0' & \mathcal{L}\end{Bmatrix} R_{n\Lambda}(q)R_{n'\Lambda'}(q)C^{b(b)}_{N_0L_0}C^{b(b)}_{N_0'L_0'}/(C_0^{b(b)})^2. \end{aligned} \tag{10}$$

This expression results from Eqs. (6)–(8) and includes all the angular momentum algebra.

One new original property of EMD is its dependence on polar angle Θ_q which can be very strong under some specific kinematical conditions (see below). The φ_q anisotropy is absent here due to the limitation $M_0 = 0$.

Turning now from b-fold scattering to smaller proton scattering angles, i.e., to multiplicities less than b and to their interference, we find some new effect. Namely, the limitation $M_0 = 0$ disappears. For instance, when triton is knocked out, the double scattering overlap integrals look like [28,26]

$$C^{3(2)}_{N_0L_0M_0}(\mathbf{p}) = \int \exp\left[i\frac{\mathbf{p}}{2}\cdot\left(\frac{\rho_2}{3}-\frac{\rho_1}{2}\right)\right]\delta^2\left(\frac{\rho_1}{2}+\rho_2\right)\varphi^*_{t,0}(\mathbf{r}_1,\mathbf{r}_2)\varphi_{t,N_0L_0M_0}(\mathbf{r}_1,\mathbf{r}_2)d^3r_1d^3r_2 \tag{11}$$

[compare with Eq. (8)]. As a result, φ_q anisotropy of EMD appears side by side with the remaining Θ_q anisotropy [28,26]. So, EMD depends on the fast proton scattering angle [26,28] (at lower energies $E_p \simeq$ 100–200 MeV this dependence is practically absent [29,18]). In the Glauber-Sitenko theory the dependence of scattering amplitudes on spin-isospin quantum numbers is usually neglected [22,23]; scattering operators are symmetric with respect to the spatial permutations and as a result our deexcitation amplitudes are diagonal on the quantum numbers S_0, T_0, $[f]_0$. For instance, the α-particle excited states taken into account are the following: $N_0 = 1$; $N_0 = 2$, $(\lambda\mu)_0 = (20)$, $L_0 = 0, 2$; $N_0 = 3$, $(\lambda\mu)_0 = (30)$, $L_0 = 1, 3$; $N_0 = 4$, $(\lambda\mu)_0 = (40)$, $L_0 = 0, 2, 4$. The situation can be very different for electron-induced cluster knockout (see below).

The plane-wave formulas will be given below, but here we present the generalized DWIA expression for the cross section [26], which is the basis of our quantitative calculations. It contains the interference terms between different scattering multiplicities, p dependence of form factors, Θ_q and φ_q anisotropies, etc.

This formula looks like

$$\begin{aligned}\frac{d^3\sigma}{d\Omega_p d\Omega_b dE_p} &= \frac{m_p}{\hbar^2}\begin{pmatrix} A \\ b\end{pmatrix}\langle T_1\tau_1, T_0\tau_0|T\tau\rangle^2 \sum \begin{pmatrix} L_1 & S_1 & J_1 \\ \mathcal{L} & S_0 & j \\ L & S & J\end{pmatrix} (\text{FPC})\ (\text{FPC})' \\ &\times B^{b000[4]}_{N_0L_0M_0[f_0]}(p)\left[B^{b000[4]}_{N_0'L_0'M_0'[f_0']}(p)\right]^* (4\pi)^3\mathcal{F}_{n\Lambda l l_1}(k,p)\mathcal{F}^*_{n'\Lambda' l' l_1'}(k,p)(2l'+1) \\ &\times (2l_1'+1)(-1)^{l+l_1'-\tilde{l}+\Lambda'+L_0-\mathcal{L}}\left[\frac{(2\tilde{L}+1)(2\Lambda+1)(2\Lambda'+1)(2l+1)(2l'+1)}{(2L_0'+1)}\right]\langle l_1 0\Lambda 0|l0\rangle \\ &\times \langle l_1'0\Lambda'0|l'0\rangle\langle l0l_10|\tilde{l}0\rangle\langle l_1'0l_10|\tilde{l}_10\rangle\begin{Bmatrix} l & l_1 & \Lambda \\ l' & l_1' & \Lambda' \\ \tilde{l} & \tilde{l}_1 & \tilde{L}\end{Bmatrix}\begin{Bmatrix} \Lambda' & \Lambda & \tilde{L} \\ L_0 & L_0' & \mathcal{L}\end{Bmatrix} \\ &\times \sum_{\tilde{M}\tilde{m}\tilde{m}_1}\langle \tilde{L}\tilde{M}L_0M_0|L_0'M_0'\rangle\langle\tilde{l}_1\tilde{m}_1\tilde{l}\tilde{m}|\tilde{L}\tilde{M}\rangle Y_{\tilde{l}\tilde{m}}(\mathbf{k})Y_{\tilde{l}_1\tilde{m}_1}(\mathbf{p})\end{aligned} \tag{12}$$

with the summation over all intermediate angular momenta. Here, FPC (fractional parentage coefficient expressed in terms of cluster Jacobi coordinates, i.e., within the identity factor of the cluster spectroscopic amplitude) means $\langle A\alpha NLSTJ|A-b, \beta N_1, L_1S_1T_1J_1; n\Lambda, bN_0L_0S_0\{\tilde{L}\tilde{J}\}; JT\rangle$, $\mathbf{k} = (m_{A-b}/m_A)\mathbf{p} + \mathbf{q}$,

$$\mathcal{F}_{n\Lambda l l_1}(k,p) = \int R_{n\Lambda}(R)f_l(k,R)j_{l_1}\left(\frac{m_{A-b}}{m_A}pR\right)R^2dR, \tag{13}$$

and the radial wave function $R_{n\Lambda}(R)$ of virtual cluster b^* c.m. motion in the initial nucleus is defined from the corresponding Woods-Saxon potential [18,29] (see below). Furthermore, the knocked-out cluster distorted wave is expanded over partial waves as

$$\chi^{(-)}(k,R)=\sum_{lm}(-1)^l f_l^*(k,R)Y_{l,m}^*(\mathbf{n}_R)Y_{l,m}(\mathbf{n}_k); \tag{14}$$

it must be calculated by means of the optical potential [18,29,31] (see below). This partial wave expansion method [26] was verified in Refs. [31,32] confirming its good convergence and stability. In reality, 25–30 partial waves are necessary to produce the stable numerical results. The distortions of proton waves are neglected as long as the corresponding corrections to proton plane wave results modeled by using Glauber-Sitenko rescattering amplitudes of fast proton are small numerically, ~5–10% [33]. Due to this circumstance all the plane-wave expressions for overlap integrals [8], [9], etc. remain valid, and they are incorporated into the virtual cluster deexcitation amplitude

$$\begin{aligned}B^{b000[4]}_{N_0L_0M_0[f_0]}(\mathbf{p})&=(2\pi)^2\int d^3r_1\cdots d^er_{b-1}\langle b,N_0L_0M_0,S_0T_0|\\&\times\sum_{s=1}^{b}(-1)^{s+1}\left(\frac{1}{2\pi i p_0}\right)^s\sum_{(s)}^{(b)}\int\left[\prod_i^{(s)}d^2p_i f(\mathbf{p}_i)\exp(i\mathbf{p}_i\cdot\boldsymbol{\rho}_i)\right]\exp(-i\mathbf{p}\cdot\mathbf{R}_b)\\&\times\delta\left(\mathbf{p}-\sum_i^{(s)}\mathbf{p}_i\right)|b,000,S_0T_0\rangle\end{aligned} \tag{15}$$

which is also the same as for the plane-wave approximation. Here, $s\le b$ is the scattering multiplicity; (s) is some definite subset of s numbers chosen from the set $1,2,\ldots,b$; and i runs over all numbers of the subset (s).

A detailed version of the general expression (15) for α-cluster knockout ($b=4$) is given in Appendix A (its schematic description is presented in Ref. [32]), the case of $b=2$ and 3 is illuminated in Refs. [25,28,32].

Here, we present a broad panorama of the potentialities of our generalized DWIA theory containing the deexcitation amplitudes by means of a thorough investigation of the cluster quasielastic knockout process at various kinematical situations. We can manipulate by kinematics to optimize, say, the contribution of deexcitation scattering amplitudes (or, opposite, to arrange the maximal contribution of diagonal ground-state to ground-state ones). The plane-wave formulas presented in Appendix B can serve here as some qualitative guide. We investigate both the q dependence of the form factors along different "rays" characterized by some fixed Θ_q,φ_q values and the anisotropies of form factors when the q value is fixed and either Θ_q or φ_q angle is varied. This picture depends on the proton scattering multiplicity, i.e., on the momentum transfer value p. So, we see, actually, a rich landscape. The reaction $^{12}C(p,p\alpha)^8Be$ is considered as the most favorable from an experimental point of view.

Nuclear shell model wave functions of LS coupling are used in formula (9) for simplicity, but all the calculations are done in the intermediate coupling scheme (see Appendix A). The results are presented below.

To conclude this section of our paper it is reasonable to comment on the effects of nucleon exchange in the final state. We can expand the final state cluster distorted wave $f_l^*(k,R)Y_{lm}^*(n_R)Y_{lm}(n_k)$ in the region of overlap of nuclei $A-b$ and b on the oscillator wave functions [34]

$$f_l^*(k,R)Y_{lm}^*(\mathbf{n}_R)Y_{1m}(\mathbf{n}_k)=\sum_n C^k_{nlm}\Phi_{nlm}(\mathbf{R}), \tag{16}$$

where

$$C^k_{nlm}\equiv\langle f^*_{lm}(\mathbf{k},R)Y_{lm}(\mathbf{n}_k)|R_{nl}(R)\rangle, \tag{17}$$

and $R_{nl}(R)$ is the radial part of the oscillator wave function $\Phi_{nlm}(\mathbf{R})$. It is not difficult to demonstrate that the main components of expansion (16) in the region of overlap of nuclei $A-b$ and b are characterized by n values centered around $n\simeq n_0+k^2/2k_0^2-\frac{3}{2}$ [35] where $k_0=\sqrt{\hbar M_\alpha\omega}\simeq 200$ MeV/c and $n_0=4$. So, due to the high energy of final cluster b this expansion is characterized by high excitations $n\simeq 8$. The final state exchange effects modify the function $f_l^*(k,R)Y_{lm}(\mathbf{n}_R)Y_{lm}(\mathbf{n}_k)$ to [19]

$$\tilde{f}_l^*(k,R)Y_{lm}^*(\mathbf{n}_R)Y_{lm}(\mathbf{n}_k)=\int d^3R'K(R,R')f_l^*(k,R')Y^*_{lm}(\mathbf{n}_{R'})Y_{lm}(\mathbf{n}_k) \tag{18}$$

where $K(R,R')$ is the exchange kernel. If $\hbar\omega_{A-b}\simeq\hbar\omega_b\simeq\hbar\omega_{\text{r.m.}}$ (real for the light nuclei) then the oscillator functions become eigenfunctions of the kernel $K(R,R')$ [19], i.e.,

$$\int d^3R'K(\mathbf{R},\mathbf{R}')\Phi_{nlm}(\mathbf{R}')=\lambda_n\Phi_{nlm}(\mathbf{R}), \tag{19}$$

where

$$\lambda_n=\langle A-b;b;\Phi_{nlm}(\mathbf{R})|\hat{A}|A-b;b;\Phi_{nlm}(\mathbf{R})\rangle, \tag{20}$$

$$\tilde{f}_l^*(k,R)Y_{lm}^*(\mathbf{n}_R)Y_{lm}(\mathbf{n}_k)=\sum_n\lambda_n C^{\mathbf{k}}_{nlm}\Phi_{nlm}(\mathbf{R}). \tag{21}$$

However, $\lambda_N\simeq 1$ for $n\ge 6$ [19] and there is no difference between $\chi^{(-)}(\mathbf{k},R)$ and $\tilde{\chi}^{(-)}(\mathbf{k},R)$.

III. ^{12}C$(p,p\alpha)$ ^{8}BE REACTION EXAMPLE

The sole experimental result [36] in our area of interest is shown in Fig. 1. It corresponds to $E_p = 600$ MeV, $\Theta_p = 36°$, and $\Theta_\alpha = -65°$ within the coplanar geometry. Energy resolution in this energy sharing experiment is about 9 MeV. The reaction ^{12}C$(p,p\alpha)^8$Be(0^+) dominates at small q values; however, the influence of the ^{12}C$(p,p\alpha)$ ^{8}Be(2^+) transition can be visible in the form factor tail region which we do not discuss here. The proton energy is not large enough here to neglect the spin terms in the nucleon-nucleon scattering amplitude, so this illustration is mainly of qualitative character. Keeping this in mind we see that our theoretical description of this experiment is quite reasonable; nevertheless, the choice of geometry is not very favorable to reveal the deexcitation of virtual clusters—its influence increases the cross section twice within the above kinematics but with no change of momentum distribution shape.

However, being armed by the general formula (12) and by its plane-wave simplification (see Appendix B) we can optimize the contribution of virtually excited clusters, making it rather pronounced.

In Figs. 2–4 the "effective momentum distributions" (EMD) of ^{12}C$(p,p\alpha)^8$Be reaction are presented for three levels 0^+, 2^+, and 4^+ of the ^{8}Be nucleus. The kinematics chosen corresponds to double plus triple scattering of the fast proton on the nucleons of the knocked-out α cluster $[p^2 \equiv |\mathbf{p}_0 - \mathbf{p}_0'|^2 = 1\ (\mathrm{GeV}/c)^2]$ and to one of the optimal orientations of the recoil momentum $\mathbf{q}$ $\Theta_q = 71°$, $\varphi_q = 5°$ (the z axis is directed along vector $\mathbf{p}_0$). We see that both absolute magnitudes and shapes of EMD's are very sensitive to the deexcitation effects (the logarithmic scale is used).

In accordance with the above conclusion, Figs. 5 and 6 show very remarkable evolution of EMD for the partial reaction ^{12}C$(p,p\alpha)^8$Be(0^+) when both the multiplicity of proton scattering $[p^2 = 1.6\ (\mathrm{GeV}/c)^2$ corresponds to the interference of triple and quadruple scattering] and the orientation of vector $\mathbf{q}$ are changed (compare to

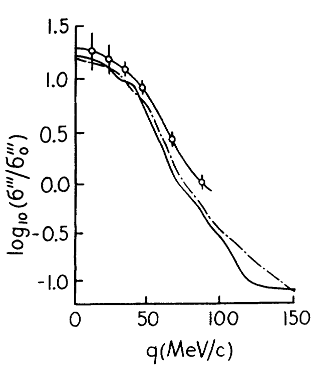

FIG. 1. Cross section of the ^{12}C$(p,p\alpha)$ ^{8}Be(0^+) reaction at $E_p = 600$ MeV within the coplanar geometry [36] in comparison to our calculations (solid line) and to the eikonal approximation for cluster distorted waves [33] (dot-dashed line). The theoretical curve with no deexcitations is of a similar shape but lies 2 times lower. σ''' means $d^3\sigma/d\Omega_1 d\Omega_2 dE$, σ_0''' is 1 μb MeV^{-1} sr^{-2}.

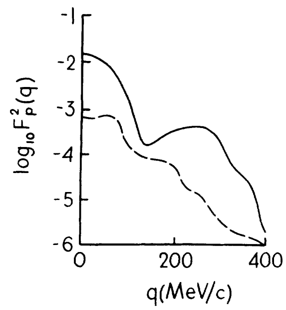

FIG. 2. "Effective momentum distribution" (form factor) $F_p^2(q)$ of recoil nucleus ^{8}Be for the reaction ^{12}C$(p,p\alpha)^8$Be(0^+) at $p^2 = 1\ (\mathrm{GeV}/c)^2$, $\Theta_q = 71°$, and $\varphi_q = 5°$. Solid curve, generalized DWIA with deexcitations; dashed curve, DWIA with no deexcitations.

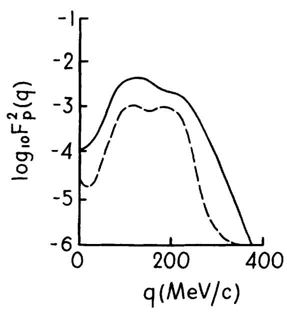

FIG. 3. The same as in Fig. 2 but for the 2^+ level of ^{8}Be nucleus.

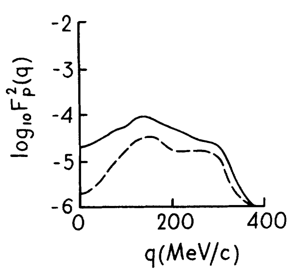

FIG. 4. The same as in Fig. 2 but for the 4^+ level of ^{8}Be nucleus.

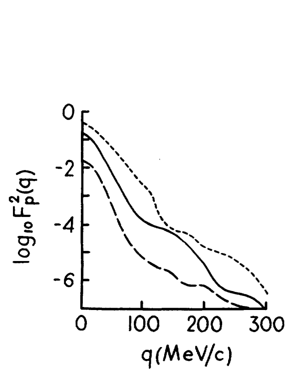

FIG. 5. The same as in Fig. 2 but at $p^2 = 1.6\ (\mathrm{GeV}/c)^2$, $\Theta_q = 36°$, $\varphi_q = 41°$. Dotted curve is the plane-wave IA with deexcitations.

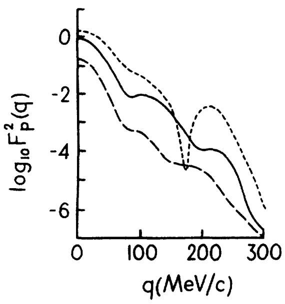

FIG. 6. The same as in Fig. 5 at $\Theta_q = 90°$, $\varphi_q = 0°$.

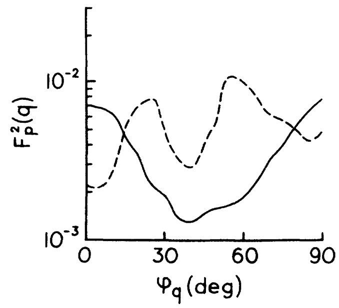

FIG. 9. Θ_q anisotropy of form factor $F_p^2(q)$ for the ${}^{12}C(p,p\alpha)\,{}^8Be(4^+)$ reaction at medium momentum transfer $p^2 = 0.5$ $(GeV/c)^2$. Solid line: $q = 90$ MeV/c, $\varphi_q = 60°$ (the inelastic component with $N_0 = 4$, $L_0 = 4$, $M_0 = 4$ dominates). Dotted line: $q = 80$ MeV/c, $\varphi_a = 20°$ ($N_0 = 4$, $L_0 = 4$, $M_0 = 4$, too). Dashed line: $q = 70$ MeV/c, $\varphi_q = 10°$ ($N_0 = 4$, $L_0 = 4$, $M_0 = 2$).

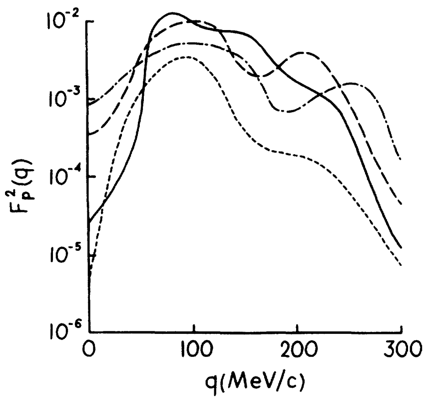

FIG. 7. Form factors $F_p^2(q)$ of recoil nucleus ^{8}Be for the reactions ${}^{12}C(p,p\alpha){}^8Be(2^+)$ and ${}^{12}C(p,p\alpha){}^8Be(4^+)$ at $p^2 = 1.6$ $(GeV/c)^2$, $\Theta_q = 90°$, $\varphi_q = 0°$. Solid curve, 2^+ level with deexcitations calculated in the eikonal approximation of DWIA according to the routine of this paper. Dotted curve, the same but with the oscillator wave function of the α particle in the ^{12}C nucleus ($\hbar\omega = 16$ MeV). Dash-dotted curve, 4^+ leve' with deexcitations calculated in the eikonal approximation of DWIA according to the routine of this paper. Dashed curve, the same but with the oscillator wave function of the α particle in the ^{12}C nucleus ($\hbar\omega = 16$ MeV).

Fig. 2). All the above results were also calculated within the eikonal version of the DWIA procedure. Good agreement was found.

In Fig. 7 the form factors for levels 2^+ and 4^+ of the ^{8}Be nucleus at $p^2 = 1.6$ $(GeV/c)^2$ along the "ray" $\Theta_q = 90°$, $\varphi_q = 0°$ are calculated (within the eikonal version of the DWIA only) under two assumptions: (a) the α-particle wave function in the ^{12}C nucleus corresponds to the potential description [29] adopted here; (b) the oscillator wave function is used ($\hbar\omega = 16$ MeV). We see the very remarkable difference testifying against the use of oscillator functions here.

The q dependence of EMD's is presented in Figs. 8–12 by the Θ_q and φ_q anisotropies at different values of p^2 and q, which is inspired by some concepts of meson physics (Treiman-Yang anisotropy, etc. [37]). However, the composite particles and their internal rearrangements were not considered there.

The predominating deexcitation amplitudes for the 4^+ level of the ^{8}Be nucleus are indicated in the figure captions. This level is very convenient for revealing these amplitudes as far as the "old" form factor (with no deexcitation) is small at $q \leq 100$ MeV/c. We see that the

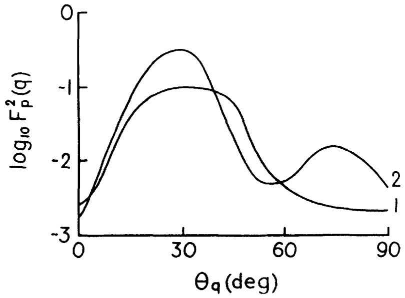

FIG. 8. Θ_q anisotropy of the form factor $F_p^2(q)$ for the ${}^{12}C(p,p\alpha){}^8Be(0^+)$ reaction at small momentum transfer $p^2 = 0.2$ $(GeV/c)^2$: (a) $q = 90$ MeV/c, $\varphi_q = 20°$; (b) $q = 110$ Mev/c, $\varphi_q = 53°$.

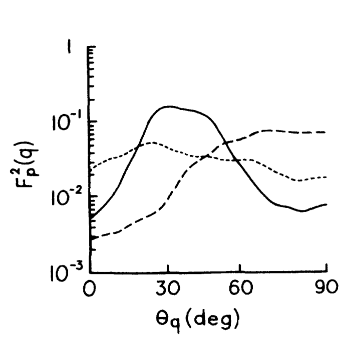

FIG. 10. φ_q anisotropy of form factor $F_p^2(\mathbf{q})$ for the ${}^{12}C(p,p\alpha){}^8Be(4^+)$ reaction at medium momentum transfer $p^2 = 0.5$ $(GeV/c)^2$. Solid line: $q = 100$ MeV/c, $\varphi_q = 15°$ ($N_0 = 4$, $L_0 = 4$, $M_0 = 2$ dominates). Dashed line: $q = 55$ MeV/c, $\varphi_q = 45°$ ($N_0 = 4$, $L_0 = 4$, $M_0 = 4$ dominates).

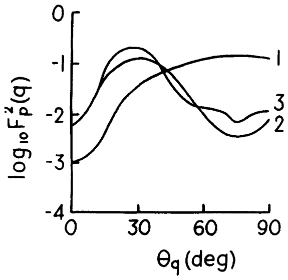

FIG. 11. Θ_q anisotropy of form factor $F_p^2(q)$ for the $^{12}C(p,p\alpha)^8Be(4^+)$ reaction at high momentum transfer $p^2 = 1.6$ (GeV/c)^2: (a) $q = 82$ MeV/c, $\varphi_q = 10°$ (the component $N_0 = 4$, $L_0 = 4$, $M_0 = 2$ is still quite visible); (b) $q = 90$ MeV/c, $\varphi_q = 20°$ (here $N_0 = 4$, $L_0 = 4$, $M_0 = 4$); (c) $q = 110$ MeV/c, $\varphi_q = 53°$ ($N_0 = 4$, $L_0 = 4$, $M_0 = 4$, too).

anisotropies under discussion can be very strong: the ratio of maximum to minimum exceeds sometimes 50. It must be noted here that the large rotation angle of vector $\mathbf{q}$ is connected with rather small mutually consistent changes of vectors $\mathbf{p}_0'$ and $\mathbf{Q}$.

In total, Figs. 2, 5, 6, and 8 give some partial reconstruction of three-dimensional EMD (form factor) for the reaction $^{12}C(p,p\alpha)^8Be(0^+)$ and Figs. 4, 7, 9, 10, 11, and 12 for the reaction $^{12}C(p,p\alpha)^8Be(4^+)$. In all the above calculations, the bound-state α-cluster potential of the Woods-Saxon shape have the parameters $V_0 = -90.0$ MeV, $R_0 = 2.46$ fm, and $a = 0.75$ fm [29]. Distorted waves for the knocked-out α particles are described by the complex potential [29] with the parameters $V_0 = -89$ MeV, $R_0 = 1.98$ fm, $a = 0.81$ fm of the energy-independent real part and with the energy-dependent imaginary part. Its radial parameters $R_0' = 6.0$ fm and $a' = 0.58$ fm are fixed; meanwhile, the amplitude W is growing with energy by the usual way: $W = 4.9$, 19.7, and 32.9 MeV for $E_\alpha = 20$ MeV, 40, and 70 MeV, respectively.

The experimental verification of the above concepts and of numerical results seems to be a very interesting problem.

Proton beams with energies above 800 MeV can be used here and our calculations can be extended in accordance with the conditions of experiment (lower proton energies probably need spin corrections).

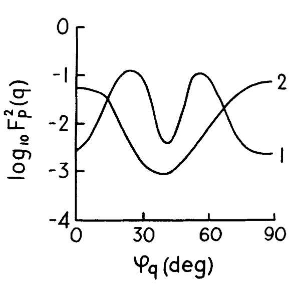

FIG. 12. φ_q anisotropy of form factor $F_p^2(q)$ for the $^{12}C(p,p\alpha)^8Be(4^+)$ reaction at high momenum transfer $p^2 = 1.6$ (GeV/c)^2: (a) $q = 60$ MeV/c, $\Theta_q = 44°$ (component $N_0 = 4$, $L_0 = 4$, $M_0 = 4$ gives the maximal contribution); (b) $q = 125$ MeV/c, $\Theta_q = 15°$ ($N_0 = 4$, $L_0 = 4$, $M_0 = 2$ are of maximal importance).

IV. CONCLUDING REMARKS

There are a few opportunities to extend the experience accumulated in the course of this research.

First, it is expected also [38] that the direct process $^{16}O(\gamma, dd)^{12}C(0^+;2^+;4^+)$ at the photon energies of ~ 80 MeV is connected with the disintegration of the virtually excited α cluster, which should result in approximately the same shape of recoil momentum distributions for all three above levels of ^{12}C nucleus.

Second, investigation of electron-induced cluster knockout from light nuclei at electron energies of 500–600 MeV [39] is interesting, too. The specific property of this reaction, however, is that the electron collision here is always a single one (with the essential role of spin-isospin variables). So, in contrast to Glauber-Sitenko theory, the electron-cluster scattering amplitude decreases very fast with increasing transferred momentum $p = |\mathbf{p}_0 - \mathbf{p}_0'|$ and, say, the knocked-out α cluster cannot be supplied really by the kinetic energy higher than 30–40 MeV. So, it seems very reasonable to investigate here, say, Θ_q and φ_q anisotropies near the maximal available energies of knocked-out clusters to avoid the situation resembling the cluster quasielastic knockout by the 100–200 MeV protons discussed above. However, the number of actual excited states of virtual cluster is rather large here which can result in the smearing of the anisotropies in comparison to the proton-induced knockout. The simplest deexcitation process $^{12}C(e,e'd)^{10}B^*(0^+, T = 1)$ with the spin-isospin rearrangement of virtual cluster $e + d_s \to e' + d$ was just observed a few years ago [40].

Third, ten times increase of electron energy to the value of $E_0 \sim 5$ GeV offers the opportunity to investigate the structure of quark configurations in the lightest nuclei by means of exclusive knockout reactions like $^2H(e,e'p)N^*$ with the investigation of excitations of baryon-spectator N^* [41] (the probability is $\sim 10^{-3}$). Here, if N^* corresponds to one of negative parity states ($J^\pi = 3/2^-$ or $1/2^-$), there are two ways to produce it. The first of them is $e + N \to e + N$ (elastic knockout amplitude) which corresponds to the distribution of two oscillator quanta of quark configuration s^4p^2 over Jacobi coordinates like $N(1p)N^*$ [the symbol $(1p)$ characterizes the relative motion of two baryons]. The second way is connected with the wave-function component $N^*(0s)N^*$ with deexcitation knockout amplitude $e + N^* \to e' + N$.

In the $^1H(e,e'\pi^+)n$ process, evidently, the above anisotropies can result from, say, the nondiagonal knockout amplitude $e + (1^+) \to e' + \pi$ if the virtual mesons with the nonzero value of internal orbital momentum are present in the mesonic cloud of the nucleon.

ACKNOWLEDGMENTS

The authors express their gratitude to Prof. Y. Abe, Prof. I. S. Shapiro, and Prof. Yu. F. Smirnov for stimulating remarks. This work was supported, in part, by a Soros Foundation Grant awarded by the American Physical Society.

APPENDIX A

The reaction amplitude is, as usual [25,26],

$$M_{i\to f}(\mathbf{p},\mathbf{q}) = \frac{ip_0'}{2\pi}\int d\boldsymbol{\rho}\, e^{i\mathbf{p}\cdot\boldsymbol{\rho}} \int d\tau\, \psi_f^* \hat{\Omega} \psi_i. \tag{A1}$$

Here, the initial shell-model wave function in the intermediate coupling can be written as [12,13]

$$\psi_i = \sum_{[f],L,S} a_{[f]LS}^{A,JT} |A\alpha[f]NLST : J, M_J, M_T\rangle. \tag{A2}$$

Introducing cluster spectroscopic amplitudes [11,26] [see formulas (10) and (12) above] we represent (A2) as

$$\begin{aligned}\psi_i = &\sum_{[f],L,S} a_{[f]LS}^{A,JT} \sum_t \langle A\alpha[f]NLST|A-b\alpha_1[f_1]N_1L_1S_1T_1; n\Lambda; b[f_0]N_0L_0S_0T_0\{\mathcal{L}\}\rangle \\ &\times\sqrt{(2J_1+1)(2j+1)(2L+1)(2S+1)} \left\{\begin{matrix} L_1 & S_1 & J_1 \\ \mathcal{L} & S_0 & j \\ L & S & J \end{matrix}\right\} (-1)^{\Lambda+L_0+S_0+j} \\ &\times\sqrt{(2\mathcal{L}+1)(2J_0+1)} \left\{\begin{matrix} \Lambda & L_0 & \mathcal{L} \\ S_0 & j & J_0 \end{matrix}\right\} (T_1M_{T_1}, T_0M_{T_0}|TM_T) \sum_M (L_0M_{L_0}, S_0M_{S_0}|J_0M_{J_0}) \\ &\times(\Lambda M_\Lambda, J_0M_{J_0}|\mathcal{L}M_\mathcal{L})(J_1M_{J_1}, \mathcal{L}M_\mathcal{L}|JM_J)|A-b\alpha_1[f_1]N_1L_1S_1T_1 : J_1, M_{J_1}, M_{T_1}\rangle \\ &\times|n\Lambda M_\Lambda\rangle|b\alpha_0[f_0]N_0L_0S_0T_0M_{L_0}M_{S_0}M_{T_0}\rangle,\end{aligned} \tag{A3}$$

where the composite index t means N_1, α_1, L_1, S_1, J_1, T_1, N_0, α_0, L_0, S_0, J_0, T_0, Λ, $\mathcal{L}$, j, $n = N - N_1 - N_0$, and $M \equiv M_{S_0}$, M_{L_0}, M_Λ, M_{J_0}, M_{J_1}, $M_\mathcal{L}$. The final state wave function looks like [35]

$$\psi_f^{(-)} = \exp[i(\mathbf{Q}-\mathbf{q})\cdot\mathbf{R}_A]\hat{A}\left\{\chi^{(-)}(\mathbf{k},\mathbf{R})\psi_f^{A-b}|b\alpha_0^f[f_0^f]N_0^fL_0^fS_0^fT_0^fM_{L_0^f}M_{S_0^f}M_{T_0^f}\rangle\right\}, \tag{A4}$$

where ψ_f^{A-b} is [12,13]

$$\psi_f^{A-b} = \sum_{[f_f],L_f,S_f} a_{[f_f]L_fS_f}^{A-b,J_fT_f} |A-b\alpha_f[f_f]N_fL_fS_fT_f : J_f, M_{J_f}, M_{T_f}\rangle. \tag{A5}$$

As long as the interaction operator Ω is symmetric with respect to the permutations (see below) and the wave functions ψ_i and ψ_f are antisymmetric, we can remove the operator $\hat{A}$, replacing it by the factor$\binom{A}{b}^{1/2}$. Taking into account that

$$\exp[i(\mathbf{Q}-\mathbf{q})\cdot\mathbf{R}_A] = \exp(i\mathbf{p}\cdot\mathbf{R}_b)\exp\left(i\mathbf{p}\cdot\frac{m_{A-b}}{m_A}\mathbf{R}\right), \tag{A6}$$

we introduce the subsidiary integral

$$W_{n\Lambda M_\Lambda}(\mathbf{k},\mathbf{p}) = \int d\mathbf{R}\exp\left(-i\mathbf{p}\frac{m_{A-b}}{m_A}\mathbf{R}\right)[\chi^{(-)}(\mathbf{k},\mathbf{R})]^*|n\Lambda M_\Lambda\rangle\ . \tag{A7}$$

The final state distorted wave is

$$\chi^{(-)}(\mathbf{k},\mathbf{R}) = 4\pi\sum_l i^l f_l^*(\mathbf{k},\mathbf{R})\sum_m Y_{lm}(\hat{\mathbf{R}})Y_{lm}^*(\hat{\mathbf{k}})\ , \tag{A8}$$

$$\exp\left(-i\mathbf{p}\frac{m_{A-b}}{m_A}\mathbf{R}\right) = 4\pi\sum_{l_1}(-i)^{l_1}j_{l_1}\left(\frac{m_{A-b}}{m_A}pR\right)\sum_{m_1}Y_{l_1m_1}^*(\hat{\mathbf{R}})Y_{l_1m_1}(\hat{\mathbf{p}})\ , \tag{A9}$$

$$|n\Lambda M_\Lambda\rangle = \varphi_{n\Lambda}(R)Y_{\Lambda M_\Lambda}(\hat{\mathbf{R}})\ . \tag{A10}$$

Taking into account that

$$\int Y_{\Lambda M_\Lambda}(\hat{\mathbf{R}})Y^*_{lm}(\hat{\mathbf{R}})Y^*_{l_1m_1}(\hat{\mathbf{R}})d\hat{\mathbf{R}} = \sqrt{\frac{(2l+1)(2l_1+1)}{4\pi(2\Lambda+1)}}(l0,l_10|\Lambda 0)(lm,l_1m_1|\Lambda M_\Lambda) \ , \tag{A11}$$

and using the notation

$$F_{n\Lambda ll_1}(p,k) = \int_0^\infty f_l(k,R)j_{l_1}\left(\frac{m_{A-b}}{m_A}pR\right)\varphi_{n\Lambda}(R)R^2dR, \tag{A12}$$

we can write down integral (A7) as

$$W_{n\Lambda M_\Lambda}(\mathbf{k},\mathbf{p}) = (4\pi)^2\sum_{l,m,l_1,m_1} i^{l+l_1}\sqrt{\frac{(2l+1)(2l_1+l)}{4\pi(2\Lambda+1)}}(l0,l_10|\Lambda 0)(lm,l_1m_1|\Lambda M_\Lambda)F_{n\Lambda ll_1}(p,k)Y_{lm}(\hat{\mathbf{k}})Y_{l_1m_1}(\hat{\mathbf{p}}), \tag{A13}$$

and the matrix element of operator (A1) looks like

$$\begin{aligned}
M_{i\to f}(\mathbf{p},\mathbf{q}) = {}& \frac{ip_0'}{2\pi}(4\pi)^2\begin{pmatrix}A\\b\end{pmatrix}^{1/2}\sum_{\substack{[f],L,S\\ [f_f],L_f,S_f}} a^{A,JT}_{[f]LS}a^{A-b,J_fT_f}_{[f_f]L_fS_f}\\
&\times\sum_t\langle A\alpha[f]NLST|A-b\alpha_1[f_1]N_1L_1S_1T_1;n\Lambda;b[f_0]\alpha_0N_0L_0S_0T_0\{\mathcal{L}\}\rangle\\
&\times\sqrt{(2J_1+1)(2j+1)(2L+1)(2S+1)}\begin{Bmatrix}L_1 & S_1 & J_1\\ \mathcal{L} & S_0 & j\\ L & S & J\end{Bmatrix}(-1)^{\Lambda+L_0+S_0+j}\sqrt{(2\mathcal{L}+1)(2J_0+1)}\\
&\times\begin{Bmatrix}\Lambda & L_0 & \mathcal{L}\\ S_0 & j & J_0\end{Bmatrix}(T_1M_{T_1},T_0M_{T_0}|TM_T)\sum_M(L_0M_{L_0},S_0M_{S_0}|J_0M_{J_0})(\Lambda M_\Lambda,J_0M_{J_0}|\mathcal{L}M_{\mathcal{L}})\\
&\times(J_1M_{J_1},\mathcal{L}M_{\mathcal{L}}|JM_J)\sum_{l,m,l_1,m_1}(-i)^{l+l_1}\left(\frac{(2l+1)(2l_1+1)}{4\pi(2\Lambda+1)}\right)^{1/2}\\
&\times(l0,l_10|\Lambda 0)(lm,l_1m_1|\Lambda M_\Lambda)F_{n\Lambda ll_1}(p,k)Y_{lm}(\hat{k})Y_{l_1m_1}(\hat{p})B^{b000[4]}_{N_0L_0M_0[f_0]}(p)\\
&\times\langle A-b\alpha_f[f_f]N_fL_fS_fT_f:J_f,M_{J_f},M_{T_f}|\hat{\Omega}|A-b\alpha_1[f_1]N_1L_1S_1T_1:J_1,M_{J_1},M_{T_1}\rangle
\end{aligned} \tag{A14}$$

where $B^{b000[4]}_{N_0L_0M_0[f_0]}(p)$ is expressed by formula (15). After standard manipulations of angular momentum algebra we obtain

$$\begin{aligned}
\frac{1}{2J+1}\overline{|M_{i\to f}(\mathbf{p},\mathbf{q}))|^2} = {}& \frac{p_0'^2}{2}\begin{Bmatrix}A\\b\end{Bmatrix}\sum_g a^{A,JT}_{[f]LS}\alpha^{A,JT}_{[f']L'S'}a^{A-b,J_fT_f}_{[f_f]L_fS_f}\alpha^{A-b,J_fT_f}_{[f_f']L_f'S_f'}\\
&\times\sum_h\langle A\alpha[f]NLST|A-b\alpha_f[f_f]N_fL_fS_fT_f;n\Lambda;b\alpha_0[f_0]N_0L_0S_0T_0\{\mathcal{L}\}\rangle\\
&\times\langle A\alpha[f']NL'S'T|A-b\alpha_f[f_f']N_fL_f'S_f'T_f;n'\Lambda';b\alpha_0'[f_0]N_0'L_0'S_0T_0\{\mathcal{L}\}\rangle\\
&\times(-1)^{\Lambda'+L_0+L_0'+J_0'+\mathcal{L}}(T_fM_{T_f},T_0M_{T_0}|TM_T)^2(2J_f+1)(2j+1)\\
&\times(2L_f+1)(2L_f'+1)\frac{(2\mathcal{L}+1)}{2\Lambda'+1}\sqrt{(2L+1)(2S+1)(2L'+1)(2S'+1)(2J_0+1)}\\
&\times\begin{Bmatrix}L_f & S_f & J_f\\ \mathcal{L} & S_0 & j\\ L & S & J\end{Bmatrix}\begin{Bmatrix}L_f' & S_f' & J_f\\ \mathcal{L} & S_0 & j\\ L' & S' & J\end{Bmatrix}\begin{Bmatrix}\Lambda & L_0 & \mathcal{L}\\ S_0 & j & J_0\end{Bmatrix}\begin{Bmatrix}\Lambda' & L_0' & \mathcal{L}\\ S_0 & j & J_0'\end{Bmatrix}\\
&\times\sum_{\substack{l,l',l_1,l_1'\\ \tilde{l},\tilde{l}_1,\tilde{L}}}(-1)^{\tilde{l}+\tilde{l}_1-\tilde{L}}i^{l+l_1+l_1'+l'}(2l+1)(2l_1+1)(2l'+1)(2l_1'+1)\\
&\times(2\tilde{L}+1)^{3/2}(l0,l_10|\Lambda 0)(l'0,l_1'0|\Lambda'0)(l0,l'0|\tilde{l}0)(l_10,l_1'0|\tilde{l}_10)\\
&\times\begin{Bmatrix}l & l_1 & \Lambda\\ \tilde{l} & \tilde{l}_1 & L\\ l' & l_1' & \Lambda'\end{Bmatrix}\begin{Bmatrix}\Lambda & J_0 & \mathcal{L}\\ J_0' & \Lambda' & \tilde{L}\end{Bmatrix}F_{n\Lambda ll_1}(p,k)F^*_{n'\Lambda'l'l_1'}(p,k)B^{b000[4]}_{N_0L_0M_0[f_0]}(p)\\
&\times\left[B^{b000[4]}_{N_0'L_0'M_0'[f_0]}(p)\right]^*\sum_{M_{J_0},M_{J_0'},M_L}(J_0M_{J_0},\tilde{L}\tilde{M}|J_0'M_{J_0'})(Y_{\tilde{l}}(\hat{\mathbf{k}})Y_{\tilde{l}_1}(\hat{\mathbf{p}}))_{\tilde{L}\tilde{M}},
\end{aligned} \tag{A15}$$

where $g \equiv [f], L, S, [f'], L', S', [f_f], Lf, S_f, [f'_f], L'_f, S'_f$ and $h \equiv N_0, \alpha_0, L_0, J_0, \Lambda, \mathcal{L}, j, N'_0, \alpha'_0, L'_0, J'_0, \Lambda', n = N - N_0 - N_f, n' = N - N'_0 - N'_f$. Exploiting the microscopic structure of operator Ω [22,23]

$$\hat{\Omega} = \sum_{i=1}^{4} \hat{\omega}_i - \sum_{j>i=1}^{4} \hat{\omega}_i \hat{\omega}_j + \sum_{k>j>i=1}^{4} \hat{\omega}_i \hat{\omega}_j \hat{\omega}_k - \hat{\omega}_1 \hat{\omega}_1 \hat{\omega}_2 \hat{\omega}_3 \hat{\omega}_4, \tag{A16}$$

$$\hat{\omega}_i \equiv \frac{1}{2\pi i p_0} \int d\mathbf{p}_i e^{-i\mathbf{p}_i \cdot (\rho - \rho_i)} f(\mathbf{p}_i), \tag{A17}$$

it is possible to reduce step-by-step expression (15) for the cluster deexcitation amplitude to the set of elementary expressions. First,

$$\begin{aligned} B^{b000[4]}_{N_0 L_0 M_0 [f_0]}(p) = \tbinom{A}{b} \langle b\alpha_0^f = 0[4] N_0^f = 0 L_0^f = 0 S_0^f = 0 T_0^f = 0 : 0,0,0| \\ \times \Bigg\{ \frac{(2\pi)^2}{2\pi i p_0} \sum_{j=1}^{4} \int d\mathbf{p}_j f(\mathbf{p}_j) \delta(\mathbf{p} - \mathbf{p}_j) e^{i\mathbf{p}_j \cdot \rho_j - i\mathbf{p} \cdot \mathbf{R}_b} - \frac{(2\pi)^2}{(2\pi i p_0)^2} \\ \times \sum_{k>j=1}^{4} \int d\mathbf{p}_k d\mathbf{p}_j f(\mathbf{p}_k) f(\mathbf{p}_j) \delta(\mathbf{p} - \mathbf{p}_k - \mathbf{p}_j) e^{i(\mathbf{p}_j \cdot \rho_j + \mathbf{p}_k \cdot \rho_k) - i\mathbf{p} \cdot \mathbf{R}_b} \\ + \frac{(2\pi)^2}{(2\pi i p_0)^3} \sum_{l>k>j=1}^{4} \int d\mathbf{p}_l d\mathbf{p}_k d\mathbf{p}_j \delta(\mathbf{p} - \mathbf{p}_l - \mathbf{p}_k - \mathbf{p}_j) f(\mathbf{p}_l) f(\mathbf{p}_k) f(\mathbf{p}_j) \\ \times e^{i(\mathbf{p}_l \cdot \rho_l + \mathbf{p}_k \cdot \rho_k + \mathbf{p}_j \cdot \rho_j) - i\mathbf{p} \cdot \mathbf{R}_b} - \frac{(2\pi)^2}{(2\pi i p_0)^4} \int d\mathbf{p}_1 d\mathbf{p}_2 d\mathbf{p}_3 d\mathbf{p}_4 f(\mathbf{p}_1) f(\mathbf{p}_2) f(\mathbf{p}_3) \\ \times f(\mathbf{p}_4) \delta(\mathbf{p} - \mathbf{p}_1 - \mathbf{p}_2 - \mathbf{p}_3 - \mathbf{p}_4) e^{i(\mathbf{p}_1 \cdot \rho_1 + \mathbf{p}_2 \cdot \rho_2 + \mathbf{p}_3 \cdot \rho_3 + \mathbf{p}_4 \cdot \rho_4) - i\mathbf{p}\mathbf{R}_b} \Bigg\} \\ \times |b\alpha_0 [f_0] N_0 L_0 S_0 T_0 : M_{L_0}, M_{S_0}, M_{T_0}\rangle \equiv I_1 - I_2 + I_3 - I_4. \end{aligned} \tag{A18}$$

Note, that the quantities ρ_i in expressions (A17) and (A18) are ordinary c.m. coordinates of nucleons, which form a virtual α cluster. The exponential terms in (A18), for example, $\exp(i\mathbf{p}_1 \cdot \boldsymbol{\rho}_1 - i\mathbf{p} \cdot R_b)$, can be easily rewritten in terms of cluster Jacobi coordinates, as was shown in Ref. [28].

Defining the internal Jacobi coordinates of the α particle

$$\mathbf{x}_1 = \mathbf{r}_1 - \mathbf{r}_2,$$

$$\mathbf{x}_2 = \mathbf{r}_3 - \mathbf{r}_4,$$

$$\mathbf{x}_3 = \frac{\mathbf{r}_1 + \mathbf{r}_2}{2} - \frac{\mathbf{r}_3 + \mathbf{r}_4}{2},$$

and conjugate momenta

$$\mathbf{Q}_1 = \tfrac{1}{2}(\mathbf{P}_1 - \mathbf{P}_2),$$

$$\mathbf{Q}_2 = \tfrac{1}{2}(\mathbf{P}_3 - \mathbf{P}_4),$$

$$\mathbf{Q}_c = \tfrac{1}{2}(\mathbf{P}_1 + \mathbf{P}_2 - \mathbf{P}_3 - \mathbf{P}_4),$$

we can write down I_1, I_2, and I_4 as (see also [33])

$$\begin{aligned} I_1 = \langle b0[4]0000 : 0,0,0| \frac{(2\pi)^2}{2\pi i p_0} f(\mathbf{p}) \Big\{ e^{i\mathbf{p} \cdot (\rho_{x_1} + \rho_{x_3})/2} + e^{i\mathbf{p} \cdot (\rho_{x_3} - \rho_{x_1})/2} \\ + e^{i\mathbf{p} \cdot (\rho_{x_2} - \rho_{x_3})/2} + e^{i\mathbf{p} \cdot (-\rho_{x_3} - \rho_{x_2})/2} \Big\} |b\alpha_0 [f_0] N_0 L_0 00 : M_{L_0}, 0, 0\rangle, \end{aligned} \tag{A19}$$

$$I_2 = \langle b0[4]0000:0,0,0|\frac{(2\pi)^4}{(2\pi i p_0)^2}\left[f\left(\frac{\mathbf{p}}{2}\right)\right]^2\left\{e^{i\mathbf{p}\cdot\boldsymbol{\rho}_{x_3}/2}\delta(\boldsymbol{\rho}_{x_1})+e^{i\mathbf{p}\cdot(\boldsymbol{\rho}_{x_1}+\boldsymbol{\rho}_{x_2})/4}\right.$$
$$\times\delta\left(\frac{\boldsymbol{\rho}_{x_1}-\boldsymbol{\rho}_{x_2}}{2}+\boldsymbol{\rho}_{x_3}\right)+e^{i\mathbf{p}\cdot(\boldsymbol{\rho}_{x_1}-\boldsymbol{\rho}_{x_2})/4}\delta\left(\frac{\boldsymbol{\rho}_{x_1}+\boldsymbol{\rho}_{x_2}}{2}+\boldsymbol{\rho}_{x_3}\right)+e^{i\mathbf{p}\cdot(-\boldsymbol{\rho}_{x_1}+\boldsymbol{\rho}_{x_2})/4}$$
$$\left.\times\delta\left(\frac{-\boldsymbol{\rho}_{x_1}-\boldsymbol{\rho}_{x_2}}{2}+\boldsymbol{\rho}_{x_3}\right)+e^{i\mathbf{p}\cdot(-\boldsymbol{\rho}_{x_1}-\boldsymbol{\rho}_{x_2})/4}\delta\left(\frac{-\boldsymbol{\rho}_{x_1}+\boldsymbol{\rho}_{x_2}}{2}+\boldsymbol{\rho}_{x_3}\right)+e^{-i\mathbf{p}\cdot\boldsymbol{\rho}_{x_3}/2}\delta(\boldsymbol{\rho}_{x_2})\right\}$$
$$\times|b\alpha_0[f_0]N_0L_000:M_{L_0},0,0\rangle, \tag{A20}$$

$$I_4 = \langle b0[4]0000:0,0,0|\frac{(2\pi)^2}{(2\pi i p_0)^4}\left[f\left(\frac{\mathbf{p}}{4}\right)\right]^4\delta(\boldsymbol{\rho}_{x_1})\delta(\boldsymbol{\rho}_{x_2})\delta(\boldsymbol{\rho}_{x_3})|b\alpha_0[f_0]N_0L_000:M_{L_0},0,0\rangle. \tag{A21}$$

Concerning I_3, by means of new variables

$$\mathbf{p}_j+\mathbf{p}_k+\mathbf{p}_l=\mathbf{p}\equiv\mathbf{Q}_1,$$

$$\tfrac{1}{2}(\mathbf{p}_j-\mathbf{p}_k)=\mathbf{Q}_2,$$

$$\tfrac{1}{3}(\mathbf{p}_j+\mathbf{p}_k)-\tfrac{1}{3}\mathbf{p}_l=\mathbf{Q}_1,$$

we express it as

$$I_3 = \langle b0[4]0000:0,0,0|\frac{(2\pi)^5}{(2\pi i p_0)^3}\left[f\left(\frac{\mathbf{p}}{2}\right)\right]^3\left\{e^{i\mathbf{p}\cdot(\boldsymbol{\rho}_{x_2}+\boldsymbol{\rho}_{x_3})/6}\delta(\boldsymbol{\rho}_{x_1})\right.$$
$$\times\delta\left(\boldsymbol{\rho}_{x_3}-\frac{\boldsymbol{\rho}_{x_2}}{2}\right)+e^{l\mathbf{p}\cdot(-\boldsymbol{\rho}_{x_2}+\boldsymbol{\rho}_{x_3})/6}\delta(\boldsymbol{\rho}_{x_1})\delta\left(\boldsymbol{\rho}_{x_3}+\frac{\boldsymbol{\rho}_{x_2}}{2}\right)+e^{i\mathbf{p}\cdot(\boldsymbol{\rho}_{x_1}-\boldsymbol{\rho}_{x_3})/6}$$
$$\times\delta\left(\frac{\boldsymbol{\rho}_{x_1}-\boldsymbol{\rho}_{x_2}}{2}+\boldsymbol{\rho}_{x_3}\right)\delta\left(\frac{\boldsymbol{\rho}_{x_1}+3\boldsymbol{\rho}_{x_2}}{4}+\frac{\boldsymbol{\rho}_{x_3}}{2}\right)+e^{i\mathbf{p}\cdot(-\boldsymbol{\rho}_{x_1}-\boldsymbol{\rho}_{x_3})/6}\delta\left(\frac{-\boldsymbol{\rho}_{x_1}-\boldsymbol{\rho}_{x_2}}{2}+\boldsymbol{\rho}_{x_3}\right)$$
$$\left.\times\delta\left(\frac{-\boldsymbol{\rho}_{x_1}+3\boldsymbol{\rho}_{x_2}}{4}+\frac{\boldsymbol{\rho}_{x_3}}{2}\right)\right\}|b\alpha_0[f_0]N_0L_000M_{L_0},0,0\rangle. \tag{A22}$$

This is the second step of simplifications. The third step is connected with the opportunity of using the nonsymmetrized internal cluster wave functions, expressed by means of products of wave functions corresponding to separated Jacobi coordinates (see Ref. [25] for a justification of this trick through consideration of the symmetry constraints). So, we introduce the coefficients ($b=4$)

$$K^{b\alpha_0}_{[f_0]N_0L_0}[\nu_1\lambda_1,\nu_2\lambda_2,\nu_3\lambda_3]=\langle b\alpha_0[f_0]N_0L_0|\nu_1\lambda_1,\nu_2\lambda_2,\nu_3\lambda_3\rangle, \tag{A23}$$

where $\nu_1+\nu_2+\nu_3=N_0$. Then

$$|b\alpha_0[f_0]N_0L_0M_{L_0}\rangle=\sum_{(\nu\lambda)}K^{b\alpha_0}_{[f_0]N_0L_0}[\nu_1\lambda_1,\nu_2\lambda_2,\nu_3\lambda_3]$$
$$\times\sum_{\mu_1,\mu_2,\mu_3,\mu}(\lambda_1\mu_1,\lambda_2\mu_2|\lambda\mu)(\lambda\mu,\lambda_3\mu_3|L_0M_{L_0})|\nu_1\lambda_1\mu_1\rangle|\nu_2\lambda_2\mu_2\rangle|\nu_3\lambda_3\mu_3\rangle, \tag{A24}$$

$(\nu\lambda)\equiv\nu_1\lambda_1\nu_2\lambda_2\nu_3\lambda_3$. The unified approach to the calculation of all the four integrals I_i consists in the transformation of variables from the spherical system of coordinates to the Cartesian one (see Ref. [15]):

$$|\nu\lambda\mu(\mathbf{r})\rangle=\sum_{n_x+n_y+n_z=\nu}A(\nu\lambda\mu|n_xn_yn_z)\Phi_{n_x}(x)\Phi_{n_y}(y)\Phi_{n_z}(z), \tag{A25}$$

where $\Phi_{n_x}(x)$ stands for the usual one-dimensional oscillator wave function:

$$\Phi_{n_x}(x)=\frac{1}{[2^{n_x}n_x!\sqrt{\pi}x_0]^{1/2}}H_{n_x}\left(\frac{x}{x_0}\right)e^{-(x/x_0)^2/2} \tag{A26}$$

To simplify the formula we introduce the notations

$$D_{0_xn_x}=\int_{-\infty}^{\infty}\Phi^*_{0_x}(x)\Phi_{n_x}(x)dx, \tag{A27}$$

$$J_{p_x n_x}\left(\frac{p_x}{2}\right) = \int_{-\infty}^{\infty} \Phi^*_{0_x}(x)\Phi_{n_x}(x)e^{ip_x x/2}dx. \quad \text{(A28)}$$

In terms of these notations the single scattering deexcitation amplitude I_1 [see formula (A18) above] looks like

$$I_1 = \sum_{(\nu\lambda)} K^{b\alpha_0}_{[f_0]N_0L_0}[\nu_1\lambda_1,\nu_2\lambda_2,\nu_3\lambda_3] \sum_{\mu_1,\mu_2,\mu_3,\mu}(\lambda_1\mu_1,\lambda_2\mu_2|\lambda\mu)$$
$$\times(\lambda\mu,\lambda_3\mu_3|L_0M_{L_0})\sum_n\prod_{j=1}^{3}A(\nu_j\lambda_j\mu_j n_{x_j}n_{y_j}n_{z_j})D_{0n_{z_1}}D_{0n_{z_2}}D_{0n_{z_3}}J_{0n_{x_3}}\left(\frac{p_x}{2}\right)$$
$$\times J_{0n_{y_3}}\left(\frac{p_y}{2}\right)\left\{\left((-1)^{n_{x_1}+n_{y_1}}\right)D_{0n_{x_2}}D_{0n_{y_2}}J_{0n_{x_1}}\left(\frac{p_x}{2}\right)J_{0n_{y_1}}\left(\frac{p_y}{2}\right)+(-1)^{n_{x_3}+n_{y_3}}\right.$$
$$\left.\times\left(1+(-1)^{n_{x_2}+n_{y_2}}\right)D_{0n_{x_1}}D_{0n_{y_1}}J_{0n_{x_2}}\left(\frac{p_x}{2}\right)J_{0n_{y_2}}\left(\frac{p_y}{2}\right)\right\}\frac{(2\pi)^2}{2\pi ip_0}f(\mathbf{p}), \quad \text{(A29)}$$

where the summation index n means $n_{x_1}+n_{y_1}+n_{z_1}=\nu_1$, $n_{x_2}+n_{y_2}+n_{z_2}=\nu_2$, $n_{x_3}+n_{y_3}+n_{z_3}=\nu_3$. Using the abbreviation

$$C_{n_{x_i}n_{x_j}}\left(\frac{p_x}{2}\right)=\int_{-\infty}^{\infty}\Phi^*_{0_{x_i}}(x)\Phi_{n_{x_i}}(x)\Phi^*_{0_{x_j}}(x)\Phi_{n_{x_j}}(x)e^{ip_x x/2}dx, \quad \text{(A30)}$$

we can write down the triple scattering amplitude I_3 as

$$I_3=\frac{(2\pi)^5}{(2\pi ip_0)^3}\left[f\left(\frac{\mathbf{p}}{2}\right)\right]^3\sum_{(\nu\lambda)}K^{b\alpha_0}_{[f_0]N_0L_0}[\nu_1\lambda_1,\nu_2\lambda_2,\nu_3\lambda_3]$$
$$\times\sum_{\mu_1,\mu_2,\mu_3,\mu}(\lambda_1\mu_1,\lambda_2\mu_2|\lambda\mu)(\lambda_3\mu_3|L_0M_{L_0})\sum_n\prod_{j=1}^{3}A(\nu_j\lambda_j\mu_j|n_{x_j}n_{y_j}n_{z_j})$$
$$\times D_{0n_{z_1}}D_{0n_{z_2}}D_{0n_{z_3}}\left\{\Phi^*_{0_{x_1}}(0)\Phi_{n_{x_1}}(0)\Phi^*_{0_{y_1}}(0)\Phi_{n_{y_1}}(0)\left(1+(-1)^{n_{x_2}+n_{y_2}}\right)\right.$$
$$\times C_{n_{x_2}n_{x_3}}\left(\frac{p_x}{2}\right)C_{n_{y_2}n_{y_2}}\left(\frac{p_y}{2}\right)+\Phi^*_{0_{x_2}}(0)\Phi_{n_{x_2}}(0)\Phi^*_{0_{y_2}}(0)\Phi_{n_{y_2}}(0)$$
$$\left.\times\left(1+(-1)^{n_{x_1}+n_{y_1}}\right)C_{n_{x_1}n_{x_3}}\left(\frac{p_x}{2}\right)C_{n_{y_1}n_{y_3}}\left(\frac{p_y}{2}\right)\right\}. \quad \text{(A31)}$$

The simplest expression is that for the maximal scattering amplitude I_4 where p dependence is trivial [25,33]

$$I_4=\frac{(2\pi)^6}{(2\pi ip_0)^4}\left[f\left(\frac{p}{4}\right)\right]^4\sum_{\nu\lambda}K^{b\alpha_0}_{[f_0]N_0L_0}[\nu_1\lambda_1,\nu_2\lambda_2,\nu_3\lambda_3](\lambda_1 0,\lambda_2 0|\lambda 0)$$
$$\times(\lambda 0,\lambda_3 0|L_0 0)\delta_{L_{M_0},0}\sum_n\prod_{j=1}^{3}A(\nu_j\lambda_j 0|n_{x_j}n_{y_j}n_{z_j})\prod_{i=1}^{3}\Phi^*_{0_{x_i}}(0)\Phi_{n_{x_i}}(0)\Phi^*_{0_{y_i}}(0)\Phi_{n_{y_i}}(0)D_{0n_{z_i}}, \quad \text{(A32)}$$

and the most complicated is I_2:

$$I_2=\frac{(2\pi)^4}{(2\pi ip_0)^2}\left[f\left(\frac{\mathbf{p}}{2}\right)\right]^2\sum_{(\nu\lambda)}K^{b\alpha_0}_{[f_0]N_0L_0}[\nu_1\lambda_1,\nu_2\lambda_2,\nu_3\lambda_3]$$
$$\times\sum_{\mu_1,\mu_2,\mu_3,\mu}(\nu_1\lambda_1,\nu_2\lambda_2|\lambda\mu)(\lambda\mu,\nu_3\lambda_3|L_0M_{L_0})\sum_n\prod_{j=1}^{3}A(\nu_j\lambda_j\mu_j|n_{x_j}n_{y_j}n_{z_j})$$
$$\times D_{0n_{z_1}}D_{0n_{z_2}}D_{0n_{z_3}}\left\{J_{0n_{x_3}}\left(\frac{p_x}{2}\right)J_{0n_{y_3}}\left(\frac{p_y}{2}\right)\left[\Phi^*_{0_{x_1}}(0)\Phi_{n_{x_1}}(0)\Phi^*_{0_{y_1}}(0)\Phi_{n_{y_1}}(0)\right.\right.$$
$$\left.\times D_{0n_{x_2}}D_{0n_{y_2}}+(-1)^{n_{x_3}+n_{y_3}}\Phi^*_{0_{x_2}}(0)\Phi_{n_{x_2}}(0)\Phi^*_{0_{y_2}}(0)\Phi_{n_{y_2}}(0)D_{0n_{x_1}}D_{0n_{y_1}}\right]$$

$$+\int_{-\infty}^{\infty}\Phi^*_{0_{x_1}}(x_1)\Phi_{n_{x_1}}(x_1)\Phi^*_{0_{x_2}}(x_2)\Phi_{n_{x_2}}(x_2)\Phi^*_{0_{x_3}}\left(\frac{x_2-x_1}{2}\right)\Phi_{n_{x_3}}\left(\frac{x_2-x_1}{2}\right)$$
$$\times e^{ip_x(x_1+x_2)/4}dx_1dx_2\int_{-\infty}^{\infty}\Phi^*_{0_{y_1}}(y_1)\Phi_{n_{y_1}}(y_1)\Phi^*_{0_{y_2}}(y_2)\Phi_{n_{y_2}}(y_2)\Phi^*_{0_{y_3}}\left(\frac{y_2-y_1}{2}\right)$$
$$\times\Phi_{n_{y_3}}\left(\frac{y_2-y_1}{2}\right)e^{ip_y(y_1+y_2)/4}dy_1dy_2+\int_{-\infty}^{\infty}\Phi^*_{0_{x_1}}(x_1)\Phi_{n_{x_1}}(x_1)\Phi^*_{0_{x_2}}(x_2)\Phi_{n_{x_2}}(x_2)$$
$$\times\Phi^*_{0_{x_3}}\left(\frac{x_1+x_2}{2}\right)\Phi_{n_{x_3}}\left(\frac{x_1+x_2}{2}\right)e^{ip_x(x_2-x_1)/4}dx_1dx_2\int_{-\infty}^{\infty}\Phi^*_{0_{y_1}}(y_1)\Phi_{n_{y_1}}(y_1)$$
$$\times\Phi^*_{0_{y_2}}(y_2)\Phi_{0_{y_2}}(y_2)\Phi^*_{0_{y_3}}\left(\frac{y_1+y_2}{2}\right)\Phi_{n_{y_3}}\left(\frac{y_1+y_2}{2}\right)e^{ip_y(y_2-y_1)/4}dy_1dy_2$$
$$+\int_{-\infty}^{\infty}\Phi^*_{0_{x_1}}(x_1)\Phi_{n_{x_1}}(x_1)\Phi^*_{0_{x_2}}(x_2)\Phi^*_{n_{x_2}}(x_2)\Phi^*_{0_{x_3}}\left(\frac{-x_1-x_2}{2}\right)\Phi_{n_{x_3}}\left(\frac{-x_1-x_2}{2}\right)$$
$$\times e^{ip_x(x_1-x_2)/4}dx_1dx_2\int_{-\infty}^{\infty}\Phi^*_{0_{y_1}}(y_1)\Phi_{n_{y_1}}(y_1)\Phi^*_{0_{y_2}}(y_2)\Phi_{n_{y_2}}(y_2)\Phi^*_{0_{y_3}}\left(\frac{-y_1-y_2}{2}\right)$$
$$\times\Phi_{n_{y_3}}\left(\frac{-y_1-y_2}{2}\right)e^{ip_y(y_1-y_2)/4}dy_1dy_2+\int_{-\infty}^{\infty}\Phi^*_{0_{x_1}}(x_1)\Phi_{n_{x_1}}(x_1)\Phi^*_{0_{x_2}}(x_2)\Phi_{n_{x_2}}(x_2)$$
$$\times\Phi^*_{0_{x_3}}\left(\frac{x_1-x_2}{2}\right)\Phi_{n_{x_3}}\left(\frac{x_1-x_2}{2}\right)e^{-ip_x(x_1+x_2)/2}dx_1dx_2\int_{-\infty}^{\infty}\Phi^*_{0_{y_1}}(y_1)\Phi_{n_{y_1}}(y_1)\Phi^*_{0_{y_2}}(y_2)$$
$$\times\Phi_{n_{y_2}}(y_2)\Phi^*_{0_{y_3}}\left(\frac{y_1-y_2}{2}\right)\Phi_{n_{y_3}}\left(\frac{y_1-y_2}{2}\right)e^{-ip_y(y_1+y_2)/2}dy_1dy_2\Big\}, \tag{A33}$$

where the subsidiary integrals D_{0n} and J_{0n} can be expressed as

$$D_{0n}=\frac{x_{0n}}{[2^n n! x_0 x_n]^{1/2}}\frac{n!}{(n/2)!}\left[\left(\frac{x_{0n}}{x_n}\right)^2-1\right]^{n/2}, \tag{A34}$$

$$J_{0n}\left(\frac{p}{2}\right)=\sqrt{\frac{x_{0n}}{2^n n! x_0}}\left[1-\left(\frac{x_{0n}}{x_n}\right)^2\right]^{n/2}e^{-px_{0n}^2/16}H_n\left(\frac{ipx_{0n}}{8[1-(x_{0n}/x_n)^2]^{1/2}}\right). \tag{A35}$$

x_0 and x_n here mean the parameters of oscillator radius of the ground-state final α particle and of the excited virtual α particle, respectively (if it is necessary to distinguish them), $x_{0n}=(2x_0^2x_n^2)^{1/2}\times(x_0^2+x_n^2)^{1/2}$, and $H_n(x)$ is the Hermitian polynomial. When calculating I_2 we use the well-known formula

$$\int_{-\infty}^{\infty}e^{-x^2}e^{i\mathbf{p}\cdot\mathbf{x}}H_N(Ax)H_M(Ax)dx$$
$$=\sqrt{\pi}e^{-p^2/4}\sum_{k=0}^{\min(N,M)}2^k k!\binom{M}{k}\binom{N}{k}(1-A^2)^{(M+N)/2-k}H_{M+N-2k}\left(\frac{iAp}{2\sqrt{1-A^2}}\right), \tag{A36}$$

$$\int_{-\infty}^{\infty}e^{-x^2}H_N(Ax)H_M(Ax)H_L(Ax)dx=\sum_{k=0}^{\min(N,M)}k!\binom{M}{k}\binom{N}{k}\sum_{p=0}^{\min(L,M+N-2k)}p!$$
$$\times\binom{L}{p}\binom{M+N-2k}{p}\int_{-\infty}^{\infty}e^{-x^2}H_{L+M+N-2k-2p}(Ax)dx. \tag{A37}$$

APPENDIX B

Keeping in mind the final quantum numbers of the detected α particle, $S=0$, $T=0$, $N=0$, $L=0$, $[f]=[4]$, we obtain rather simple plane-wave approximation formulas:

$$\frac{1}{2J+1}\overline{|M_{i\to f}(\mathbf{p},\mathbf{q})|^2} = \frac{2p_0'^2}{\sqrt{\pi}}\begin{pmatrix} A \\ b \end{pmatrix}\sum_{g'} a^{A,JT}_{[f]LS}\alpha^{A,JT}_{[f]L'S'}a^{A-b,J_fT_f}_{[f_f]L_fS}\alpha^{A-b,J_fT_f}_{[f_f']L_f'S'}$$
$$\times\sum_{h'}\langle A\alpha[f]NLST|A-b\alpha_f[f_f]N_fL_fST;n\Lambda';b\alpha_0[f_0]N_0L_000\{\mathcal{L}\}\rangle$$
$$\times\langle A\alpha[f']NL'S'T|A-b\alpha_f[f_f']N_fL_f'S''T;n'\Lambda';b\alpha_0'[f_0]N_0'L_0'00\mathcal{L}\rangle$$
$$\times i^{\Lambda-\Lambda'}(-1)^{\Lambda+\Lambda'+L+L'+L_0'+S+S_0'+\mathcal{L}}(2J_f+1)\left[\frac{(2L+1)(2L'+1)(2\Lambda+1)(2\Lambda'+1)}{2L_0+1}\right]^{1/2}$$
$$\times\begin{Bmatrix} L_f & L & \mathcal{L} \\ J & J_f & S \end{Bmatrix}\begin{Bmatrix} L_f' & L' & \mathcal{L} \\ J & J_f & S' \end{Bmatrix}\begin{Bmatrix} \Lambda & \Lambda' & \tilde{L} \\ L_0' & L_0 & \mathcal{L} \end{Bmatrix}(\Lambda 0,\Lambda' 0|\tilde{L}0)\varphi_{n\Lambda}(q)\varphi_{n'\Lambda'}(q)B^{b000[4]}_{N_0L_0M_0[f_0]}(p)$$
$$\times\left[B^{b000[4]}_{N_0L_0M_0[f_0]}(p)\right]^*\sum_{M_{L_0},M_{L_0'}}(\tilde{L}\tilde{M},L_0M_0|L_0'M_{L_0'})Y_{\tilde{L}\tilde{M}}(\hat{\mathbf{q}}),$$

where $g'\equiv[f],L,S,[f'],L',S',[f_f],L_f,[f_f'],L_f'$ and $h'\equiv N_0,\alpha_0,L_0,\Lambda,n,[f_0],\mathcal{L},N_0',\alpha_0',L_0',\Lambda',n',[f_0'],\tilde{L}$. Furthermore, $\varphi_{n\Lambda}(q)$ is an ordinary Fourier transform of the mutual motion function $\varphi_{n\Lambda}(q)$,

$$\varphi_{n\Lambda}(q)\equiv\int_0^\infty R^2\varphi_{n\Lambda}(R)j_\Lambda(qR)dR.$$

The totality of spherical harmonics $Y_{\tilde{L}\tilde{M}}(\hat{\mathbf{q}})$ in the above formulas is just the source of Θ_1 and φ_q anisotropies, and we can trace easily its connection with the M_0 dependence of amplitudes $B^{b000[4]}_{N_0L_0M_0[f_0]}(p)$ (if $M_0=0$ only, $\tilde{M}=0$ and the φ_q anisotropy disappears, etc.).

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