Structure of neutron-deficient Pt, Hg, and Pb isotopes

S. Yoshida,[1,*] S. K. Patra,[1] N. Takigawa,[1] and C. R. Praharaj[2]
[1]*Department of Physics, Faculty of Science, Tohoku University, 980-77 Sendai, Japan*
[2]*Institute of Physics, Bhubaneswar 751 005, India*
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We study the shell closure properties of the even-even neutron-deficient Pt, Hg, and Pb isotopes using a deformed relativistic mean field formalism. The ground state structures are investigated by comparing the prolate, oblate, and spherical solutions. Many of the nuclei are considerably deformed in their ground state configurations. Studies of the quadrupole and the hexadecapole moments, density distributions, and the single-particle level structure clearly indicate that some of the neutron-deficient isotopes having proton number $Z=82$ do not behave as good magic nuclei.
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I. INTRODUCTION

The magic number plays an important role in the description of finite nuclei, although it is one of the oldest objects in nuclear physics. Magic nuclei are known to be considerably more bound than ordinary isotopes. It has been shown that proton magic numbers Z are more prominent than neutron magic numbers N in the description of shell structure properties [1]. The shape of magic Z nuclei (O, Ca, Ni, Sn) in their ground states is spherical for almost all isotopes including those near the proton and the neutron drip lines [2]. Also, all the nuclei having proton magic numbers have more distinct shell gaps than the nonmagic isotopes. On the other hand, though many theoretical and experimental investigations [1] have shown that ^{31}Ca is a magic nucleus, ^{31}Na is deformed and does not show any shell closure properties.

The properties of nuclei having magic Z have attracted considerable experimental and theoretical attention [3,4]. Many theoretical and experimental studies were carried out in neutron-deficient regions [3–9] to examine the magicity of magic nuclei. The exotic characters of the magic number $Z=82$ [3,9,10] near the proton drip line have recently attracted much attention. Toth *et al.* [3] determined the α-decay branches for neutron-deficient Pb isotopes and concluded that midway between $N=82$ and $N=126$ the proton number $Z=82$ is not magic. The magic number $Z=82$ loses its magicity when the neutron number decreases from $N=126$. Thus the proton magic number $Z=82$ seems to be unimportant for neutron-deficient isotopes. Brown [4] also analyzed the partial α-decay half-lives. One of his conclusions is that $Z=82$ is perhaps not a good magic number. In contrast, in a recent paper, Wauters *et al.* [9] reported an experimental study to claim the stability of the $Z=82$ magic shell at the very neutron-deficient side.

Bearing in mind the nonuniqueness in experimental conclusions as well as of theoretical descriptions, it is worthwhile to have a relativistic microscopic calculation to understand the properties of Pb isotopes in the neutron-deficient region.

In the past few years relativistic mean field (RMF) models have been applied with great success [11–17] to learning the structure of finite nuclei. This model is able to describe the shell effect [16] and other nuclear properties, such as binding energies, rms radii, and multipole moments even near the proton and the neutron drip lines. The nonlinear parameter set NL1 has achieved a striking success in reproducing these bulk properties of finite nuclei. It may be noted that the RMF theory is very much successful in view of its in-built spin-orbit interaction and the density dependence in the interaction. In general the ability to reproduce the experimentally obtained nucleon distributions provides a first and foremost criteria for the goodness of any nuclear interaction.

In this paper, we calculate the properties of neutron-deficient Pt, Hg, and Pb isotopes using a relativistic mean field (RMF) approach [14,15,17] in the Hartree approximation. The ground state deformations, binding energies, and single-particle energies are calculated. We discuss the magic structure of these nuclei towards very neutron-deficient regions.

The paper is organized as follows. In Sec. II we present the relativistic Lagrangian and the procedure of numerical calculations. The results for binding energies, quadrupole and hexadecapole moments, and single-particle spectra are discussed in Sec. III. Summary and concluding remarks are given in Sec. IV.

II. THEORY AND CALCULATION

We start with the relativistic Lagrangian density [11,13–15,17] for the interacting nucleon-meson many-body system, consisting of nucleons, scalar (σ), vector mesons ($\omega,\vec{\rho}$), and photon

$$\mathcal{L}=\bar{\psi}_i\{i\gamma^\mu\partial_\mu-M\}\psi_i+\frac{1}{2}\partial^\mu\sigma\partial_\mu\sigma-U(\sigma)-g_s\bar{\psi}_i\psi_i\sigma-\frac{1}{4}\Omega^{\mu\nu}\Omega_{\mu\nu}+\frac{1}{2}m_\omega^2V^\mu V_\mu-g_\omega\bar{\psi}_i\gamma^\mu\psi_iV_\mu-\frac{1}{4}\vec{B}^{\mu\nu}\cdot\vec{B}_{\mu\nu}$$
$$+\frac{1}{2}m_\rho^2\vec{\rho}^\mu\cdot\vec{\rho}_\mu-g_\rho\bar{\psi}_i\gamma^\mu\vec{\tau}\psi_i\cdot\vec{\rho}_\mu-\frac{1}{4}F^{\mu\nu}F_{\mu\nu}-e\bar{\psi}_i\gamma^\mu\frac{(1-\tau_{3i})}{2}\psi_iA_\mu\ . \tag{2.1}$$

*Electronic address: yoshida@nucl.phys.tohoku.ac.jp

The scalar meson is assumed to move in a nonlinear potential [18]

$$U(\sigma)=\tfrac{1}{2}m_\sigma^2\sigma^2+\tfrac{1}{3}g_2\sigma^3+\tfrac{1}{4}g_3\sigma^4 , \quad (2.2)$$

ψ_i are the Dirac spinors for nucleons, whose third component of the isospin is denoted by τ_{3i}. The g_s, g_ω, g_ρ, and $e^2/4\pi=1/137$ are the coupling constants for σ,ω, and ρ mesons, and the photon, respectively. M is the mass of a nucleon, and m_σ, m_ω, and m_ρ are the masses of σ, ω, and ρ mesons, respectively. $\Omega^{\mu\nu}$, $\vec{B}^{\mu\nu}$, and $F^{\mu\nu}$ are the field tensors for the V^μ, $\vec{\rho}^\mu$ and the photon fields, respectively [11,13].

We obtain field equations for mesons and nucleons from the relativistic Lagrangian. These equations are solved by expanding the upper and the lower components of the Dirac spinors and the meson fields in a deformed harmonic oscillator basis [11] with initial deformation parameter β_0. The set of coupled equations are then solved numerically by a self-consistent iteration method. The quadrupole deformation parameter β is evaluated from the resulting quadrupole moment using the formula

$$Q=Q_n+Q_p=\sqrt{9/5\pi}\,AR^2\beta . \quad (2.3)$$

Note that the self-consistently determined β are in general different for protons and for neutrons. We also calculate the hexadecapole moments for protons and for neutrons by

$$Q_4^{n,p}=\langle r^4Y_{40}(\theta)\rangle_{n,p} , \quad (2.4)$$

where $R=1.2A^{1/3}$. The total hexadecapole moment is then given as the sum of Q_4^n and Q_4^p. The total energy of the system is

$$E_{\text{total}}=E_{\text{part}}+E_\sigma+E_\omega+E_\rho+E_C+E_{\text{pair}}+E_{\text{c.m.}} , \quad (2.5)$$

where E_{part} is the sum of the single-particle energies of nucleons and E_σ, E_ω, E_ρ, E_C, and E_{pair} are the contributions of the meson fields, the Coulomb field, and the pairing energy, respectively. For open shell nuclei the effect of pairing interaction has been added in the BCS formalism with a constant pairing gap. The value given in Ref. [19] is assumed for the gap parameter. $E_{\text{c.m.}}=-\frac{3}{4}41A^{-1/3}$ is the center-of-mass energy correction in the nonrelativistic approximation.

In numerical calculations the wave functions are expanded in a deformed harmonic oscillator basis with maximum oscillator shells $N_{\max}=12$ for both fermions and bosons. In our calculations, we used the nonlinear (NL1) parameter set [15] ($M=938.0$, $m_\sigma=492.25$, $m_\omega=795.359$, and $m_\rho=763.0$ MeV, $g_\sigma=10.138$, $g_\omega=13.285$, $g_\rho=4.9755$, $g_2=-12.172$ fm^{-1}, $g_3=-36.265$).

III. RESULTS AND DISCUSSIONS

We solve the mean field equations self-consistently by starting from a prolate, an oblate, and a spherical configuration. The self-consistently obtained final shape is not necessarily the same as the initial shape. For a given nucleus the solution with the maximum binding energy corresponds to the ground state configuration and the other solutions are excited intrinsic states. We present the binding energies and quadrupole and hexadecapole moments for Pt, Hg, and Pb nuclei. Single-particle energy spectra are also shown for some specific nuclei.

In the RMF calculations the maximum binding energy for ^{176}Pt is 1400.5 MeV at the prolate β value of 0.286. The oblate solution for ^{176}Pt lies about 3 MeV (BE=1397.3 MeV, $\beta=-0.193$) above the prolate ground state. Similarly the ground state binding energies for ^{178}Pt and ^{180}Pt are found to be 1419.1 ($\beta=0.338$) and 1436.6 MeV ($\beta=0.347$), while the oblate binding energies for these two nuclei are 1415.9 and 1434.4 MeV (see Table I), respectively. There are no spherical solutions for ^{176,178,180}Pt isotopes.

For ^{180,184}Hg nuclei, the ground states are the prolate solutions, whereas the prolate and the oblate solutions are nearly degenerate for the ground state of ^{182}Hg. The maximum binding energies for ^{180}Hg, ^{182}Hg, and ^{184}Hg are 1422.6, 1443.3, and 1460.5 MeV at the deformation parameters $\beta=0.334, 0.339(-0.202)$, and 0.327, respectively. One can see from Table I that the oblate solutions for all of these nuclei lie very close to the prolate configurations, and they are almost degenerate. The spherical solutions for ^{180}Hg and ^{182}Hg have less intrinsic binding energies than the deformed states. There is no spherical solution for ^{184}Hg nucleus. The spherical solutions are found at the binding energies 1420.2 and 1437.5 MeV for ^{180}Hg and ^{182}Hg, respectively.

In the case of Pb isotopes, there is a shape transition from prolate to oblate at mass number $A=186$ and from oblate to spherical between $A=196$ and $A=200$. The deformation parameter for the ground state of Pb isotopes is shown in Fig. 1 for $A=208-178$. For ^{186}Pb, the prolate minimum ($\beta=0.329$) has an intrinsic binding energy of 1465.4 MeV, and there is an oblate solution ($\beta=-0.205$) with a binding energy 1464.0 MeV. The binding energies, β values, and the hexadecapole moments for Pb isotopes are listed in Table I. The spherical solution for ^{186}Pb is much above the deformed intrinsic states. From Fig. 1, it is clear that the ground states are oblate in shape for $^{188-196}$Pb isotopes.

In Table I, we compare the results of RMF theory with the experimental binding energies. We also compare our results for the deformation parameters with the theoretical prediction of Möller *et al.* [19]. In contradiction with the results in Ref. [19], our calculations predict that ^{180}Hg and ^{184}Hg are prolate. Not only the sign, but also the absolute magnitude of the quadrupole deformation parameter differs very much in the relativistic and nonrelativistic calculations. Similar disagreement in shapes between RMF theory and Möller *et al.* has been noticed also in previous calculations [20]. The $Z\approx 80$ region is well known for the shape coexistence, and the shape transition in both the experimental [22] and theoretical studies [23]. In this mass region, the energy minima of the prolate and the oblate solutions lie very close to each other. This can be seen also in Table I. In our previous paper [24], we have shown that our calculations of the quadrupole deformation parameter of Hg isotopes agree

well with the experimental data, and with nonrelativistic microscopic [i.e., Hartree-Fock (HF) and HF+BCS] calculations. Also it has been noticed that the RMF calculations well reproduce the experimental data of the quadrupole deformation parameter and other bulk properties throughout the periodic table [11]. For Pt isotopes the prolate solutions are the ground states in our RMF calculations in agreement with the predictions of Möller *et al.*

TABLE I. The results of RMF theory for the binding energies (BE) of prolate, oblate, and spherical solutions are compared with the experimental values [21] and the quadrupole deformation parameter (β) with the theoretical predictions of Möller *et al.* [19] for Pt, Hg, and Pb nuclei. The hexadecapole moment (Q_4) is in fm^4 and the binding energy is in MeV.

Nucleus	A	BE(RMF)	BE(Expt.)	β	Q_4	Möller *et al.*
Pt	176	1400.5	1388.4	0.286	6140.5	0.163
		1397.3		−0.193	4198.8	
	178	1419.1		0.338	8301.7	0.185
		1415.9		−0.221	5175.3	
	180	1436.6		0.347	6943.1	0.246
		1434.4		−0.236	5427.8	
Hg	180	1422.6	1410.4	0.334	8913.0	−0.129
		1421.8		−0.327	7948.6	
		1420.2		0.0	−8.6	
	182	1443.3	1430.7	0.339	7082.5	−0.137
		1443.3		−0.202	3942.9	
		1437.5		0.001	−8.7	
	184	1460.5	1448.7	0.327	4782.5	−0.137
		1458.9		−0.213	4047.4	
Pb	178	1381.7	1369.8	0.274	9358.6	0.018
		1382.7		−0.169	2933.8	
		1385.8		0.0	−5.6	
	180	1402.8	1390.6	0.282	8753.3	0.011
		1403.1		−0.178	3198.7	
		1406.9		0.0	−7.1	
	182	1420.7	1411.6	0.220	8442.9	0.003
		1422.9		−0.201	3998.0	
		1424.3		0.0	−5.7	
	184	1443.2	1432.0	0.303	6285.7	0.003
		1442.8		−0.201	4115.6	
		1441.4		0.018	1049.8	
	186	1465.4	1452.5	0.329	6120.5	0.003
		1464.0		−0.205	3870.1	
		1461.3		0.004	−2.9	
	188	1482.5	1470.9	0.309	3292.0	0.003
		1483.8		−0.205	3270.5	
		1478.4		0.0	−12.0	
	190	1500.0	1498.7	0.292	562.9	0.003
		1501.8		−0.196	1939.1	
		1497.3		0.0	−12.8	
	192	1516.2	1508.1	0.273	−1852.3	0.011
		1518.1		−0.184	238.7	
		1514.7		0.0	−15.2	
	194	1532.6	1525.9	0.1	−369.7	0.011
		1534.9		−0.173	−1164.6	
	196	1547.6	1543.3	0.054	−888.2	0.011
		1549.4		−0.172	−2883.7	
	198	1565.2	1560.1	0.036	−781.5	0.011
		1565.1		−0.166	−2925.9	
	200	1580.7	1576.4	0.002	−5.8	0.003
		1579.4		−0.157	−2051.5	
	202	1597.5	1592.2	0.001	−2.3	0.003
		1595.4		−0.145	−868.4	
	204	1613.2	1607.5	0.002	−38.7	−0.003
		1611.0		−0.017	−574.9	
	206	1628.2	1622.3	0.001	−38.1	−0.003
		1626.9		−0.005	−86.3	
	208	1641.6	1636.4	0.0	−1.0	0.003

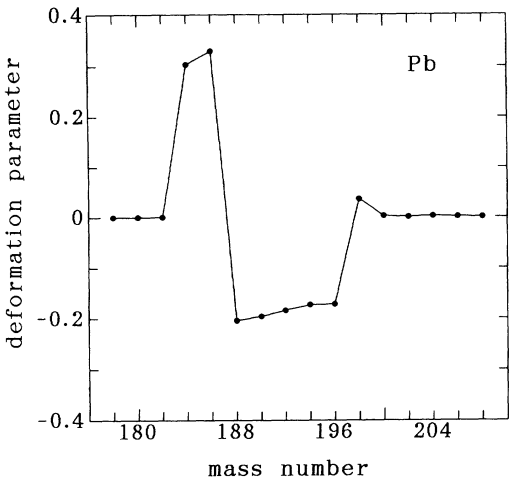

FIG. 1. The quadrupole deformation parameter (β) in the ground state of Pb isotopes as a function of the mass number.

From the analysis of quadrupole moments (Table I), one can see that the Pb isotopes are considerably deformed in their ground configurations in the region $N=102$ to $N=116$. There is a change in sign of the hexadecapole moment from positive to negative value in both the prolate and oblate solutions.

In Fig. 2 we plot the density distribution of protons for ^{186}Pb and ^{208}Pb along the $r_\perp$ and z directions, where z is the axially symmetric axis. In ^{186}Pb the density distribution is fairly different along the $r_\perp$ and z directions, whereas for ^{208}Pb, it is almost similar along these two axes. The unequal distribution of nucleons along the $r_\perp$ and z directions, and the considerably large values of β and the hexadecapole moment indicate a large deformed shape of ^{186}Pb nucleus. On the other hand, the similar density distribution along the $r_\perp$ and z directions, zero β value, and a very small hexadecapole moment for ^{208}Pb, confirm the spherical shape.

The single-particle spectra of protons and neutrons for the ground state solutions of ^{186}Pb and ^{208}Pb are shown in Fig. 3. Each level for ^{208}Pb is an eigenstate of the orbital as well as the total angular momenta, and has the degeneracy with respect to their z components, while each level for ^{186}Pb is not an eigenstate of the angular momentum and has only twofold degeneracy. It is clear from these figures that the shell gaps of protons and neutrons disappear because of the large deformation of the ground state of ^{186}Pb. On the other hand, in the case of ^{208}Pb, the shell gaps are distinctly visible. We analyzed the single-particle spectra, density distributions, and multipole moments of all the other Pb isotopes considered here. From our analysis, we find that the nucleus becomes more and more deformed with decrease of the neutron number (Fig. 1). ^{186}Pb is the most deformed nucleus in the series. If one further reduces the neutron number, the nucleus becomes again spherical in the ground state at $A=182$ agreeing with the experimental observation [3]. This trend continues even beyond the mass number $A=182$; i.e., ^{178,180}Pb are predicted to be spherical in their ground states. It is natural to expect from our studies on the shape of nuclei that the reduced width for the α decay of ^{186}Pb is smaller than that of ^{184}Pb, because the shape changes sign in the parent and in the daughter nuclei in the former. This contradicts, however, with the experimental data. Further studies including the possibility of the shape fluctuation in these nuclei is needed to resolve this puzzle. The disappearance of the shell gaps due to the large deformed configurations of $^{184-198}$Pb as well as the unequal density distribution along the $r_\perp$ and z directions and the considerably large magnitude of multipole moments are clear evidences that $Z=82$ is not a magic number for neutron number between $N=102$ and $N=116$.

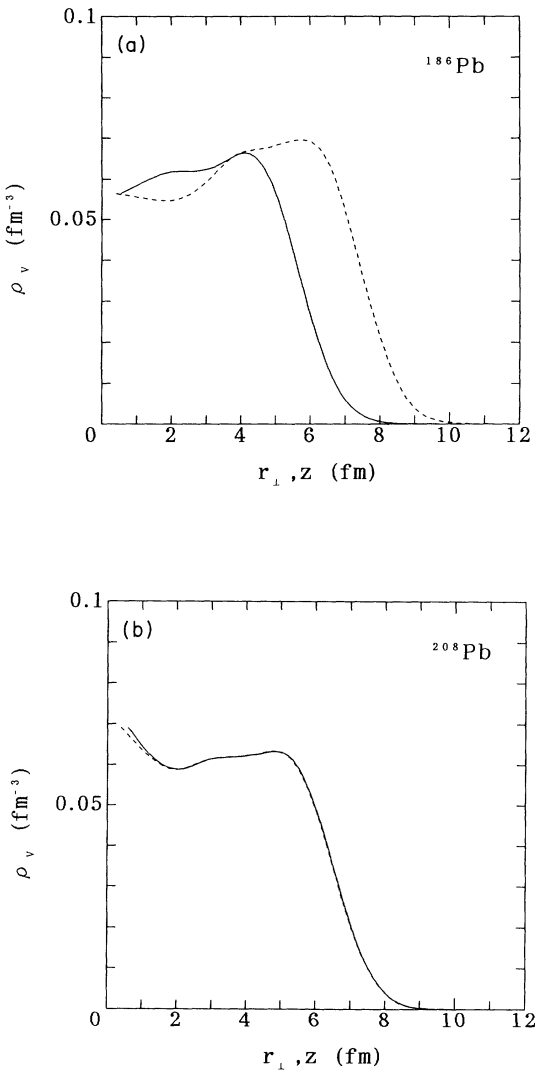

FIG. 2. The density distributions along the $r_\perp$ (the solid line) and the z directions (the dashed line) for ^{186}Pb and ^{208}Pb. The value of z is 0.419 fm for the solid line and $r_\perp$ is 0.552 fm for the dashed line. (a) is for ^{186}Pb and (b) for ^{208}Pb.

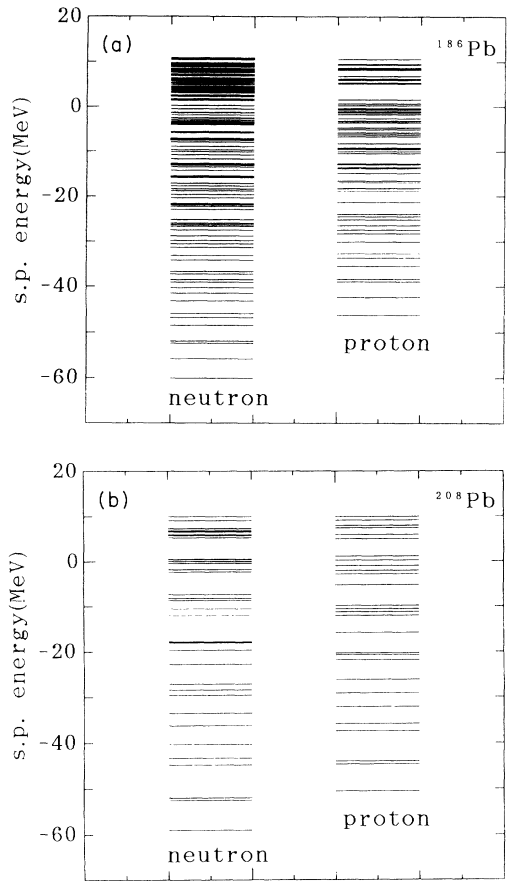

FIG. 3. Single-particle energy spectra of protons and neutrons for (a) ^{186}Pb and (b) ^{208}Pb.

IV. SUMMARY AND CONCLUSIONS

We investigated the lowest energy prolate, oblate, and spherical solutions of very neutron-deficient Pt, Hg, and Pb isotopes by using the RMF theory. We found that all the 't isotopes considered here are prolate in shape, whe :as ^{180,184}Hg are prolate and the prolate and oblate sha] s are nearly degenerate for ^{182}Hg. There is a shape trar .tion from prolate to oblate and oblate to spherical in t : ground states of Pb isotopes. Also, it was found that he hexadecapole moment changed its sign from positive :o negative values both in the prolate and the oblate solu ons for Pb isotopes. There are no spherical solutions for Pt isotopes, whereas excited spherical solutions wer found for some of the Hg and Pb isotopes.

T : absence of shell gaps in the ground state con ;urations for some of the neutron-deficient Pb isotop were clearly shown in our calculations. This disappearance of shell gap was caused by the large deformations of these nuclei. Also, the reappearance of the shell gaps and hence the gain of magicity was observed for $A < 184$. The multipole moments, density distributions, and the single-particle spectra indicate that neutron-deficient Pb isotopes do not behave as good magic nuclei between $N = 102$ and $N = 116$. The difference in binding energies between the prolate and oblate configurations was found to be very small for many of the nuclei considered here. This small difference in binding energy is an indication of shape coexistence, whereby the prolate and oblate minima lie very close to each other. Alternatively these nuclei might have a large shape fluctuation. It is an interesting future problem to determine the actual shape of nuclei in this region either experimentally or by a theory including axially nonsymmetric, i.e., gamma mode of vibration.

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