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Reduction of nuclear moment of inertia due to pairing interaction

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The BCS theoretical values of the moments of inertia of even-even nuclei are systematically smaller than the experimental ones by a factor of 10–40%. This long-standing discrepancy disappears in the particle-number-conserving treatment for the cranked shell model, in which the blocking effects are taken into account exactly. The calculated moments of inertia satisfactorily reproduce the experimental data covering a large number of rare-earth even-even nuclei, whose deformations and single-particle states are well characterized (Lund systematics). The pairing interaction strength G is unambiguously determined by the even-odd mass difference. The reduction of the moment of inertia due to the antialignment effect of pairing interaction is discussed and no systematic excessive reduction is found.

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I. INTRODUCTION

It is well known that the calculated moments of inertia in the cranked shell model (CSM) [1,2], neglecting the residual interaction, are near the rigid-body value, which is much larger than the observed ones [3]. Bohr, Mottelson, and Pines [4] suggested that the pairing interaction may be responsible for the observed reduction of the moments of inertia compared to that of a rigid rotor. Soon after, the BCS method and the concept of quasiparticle of superconductivity were used to treat the nuclear pairing correlation [5,6] and a significant reduction of the moments of inertia was successfully confirmed. However, the BCS theoretical moments of inertia of the ground bands in rare-earth and actinide even-even nuclei are systematically smaller than the experimental ones by a factor of 10–40%; i.e., a systematic excessive reduction of the nuclear moments of inertia was found [7–9]. Many efforts to reduce the discrepancy between theory and experiment have not been successful. The use of a more realistic mean field such as that of Woods and Saxon combined with a pairing strength depending on the level density near the Fermi surface [10] and the use of Nilsson's mean field with a pairing strength depending on the isospin and deformation [11,12] cannot reduce this discrepancy. General considerations show that the BCS theory is very suitable for a system of a large number of particles. However, the number of nucleons in a nucleus ($\sim 10^2$), particularly the number of valence nucleons (~ 10) which dominate the behavior of low-lying excited states, is very limited. To overcome the defect of particle number nonconservation in the BCS approximation, there have been developed various methods, including the generator coordinate method [13,14] and various types of particle number projection methods [15–21], and considerably improved agreement with experiment compared to the BCS approach was obtained. By using Bayman's wave function [15] generated from the BCS wave function, Rich [22] evaluated the moments of inertia of five rare-earth nuclei and obtained an improved agreement with experiment. This method allows the approximate cancellation of the major fluctuations in the number of particles. Later, by cancellation of the fluctuation in the number of particles, Frauendorf [23] demonstrated that only a particle-number-conserving description of pairing is able to provide a reliable estimate at which angular momentum the transition from the superfluid to the normal state takes place. Recently. Allah and Fellah [24] investigated the effects of nonconservation of the particle number in the BCS wave function on the moments of inertia and concluded that the problem of systematic excessive reduction of the calculated moments of inertia is due to the number-nonconservation effects of the BCS treatment. However, a completely different conclusion was drawn by the calculation of Hasegawa and Tazaki [25]. Therefore this long-standing puzzling problem still remains and presents a serious challenge to the mean-field (BCS) theory for nuclear pairing correlation.

It has been emphasized [26] that while the defect of particle number nonconservation of the BCS treatment for nuclear pairing may be partly remedied by various types of particle number projection, the most fatal weakness of the BCS approximation is that it cannot properly treat blocking effects, which are responsible for various even-odd differences in nuclear properties. Rowe [27] pointed out that while the blocking effects are straightforward, it is very difficult to treat them in the BCS formalism because they introduce different quasiparticle bases for different blocked levels. Usually, in the BCS calculation of the moments of inertia,the gap parameter Δ is set equal to the observed even-odd mass differ-

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ence. However, because of the blocking effect, the gap parameter for odd-A nuclei $\Delta(\nu_0)$ (ν_0 being the blocked Nilsson level) may be quite different from (smaller than) the Δ's of neighboring even-even nuclei [26]. In fact, $\Delta(\nu_0)$ depends sensitively on the location of the level ν_0 and the level distribution in the vicinity of Fermi surface. Therefore there exist some ambiguities regarding the relation between the gap parameter and the even-odd mass difference. This may be why the BCS calculations overestimate the reduction of the nuclear moment of inertia compared to that of a rigid rotor. In this paper the nuclear moments of inertia are calculated by means of a particle-number-conserving (PNC) treatment for the CSM, in which the blocking effects are taken into account exactly.

As usual, the CSM Hamiltonian of an axially symmetric nucleus in the rotating frame is expressed as

$$\begin{aligned} H_{\rm CSM} &= H_0 + H_P \\ &= H_{\rm SP} + H_C + H_P, \end{aligned} \tag{1}$$

where $H_{\rm SP}$ is the single-particle Hamiltonian (e.g., $H_{\rm SP} = H_{\rm Nil}$, the Nilsson Hamiltonian), $H_C = -\omega J_x$ is the Coriolis interaction with cranking frequency ω about the x axis perpendicular to the symmetry z axis, and H_P is the pairing interaction with strength G. Usually, it is thought that it would be very cumbersome and impractical to treat the eigenvalue problem of $H_{\rm CSM}$ using a PNC formalism. In fact, actual calculations show this is not the case. Considering the fact that the realistic pairing strength G is smaller than the average spacing of the Nilsson level b ($G/b < \frac{1}{2}$), the influence of the pairing interaction on nuclear properties is mainly concentrated in a very limited region around the Fermi surface. If we are interested in the yrast and low-lying excited states, the number of important many-particle configurations (MPC's) involved (say, with weight $> 10^{-3}$) is very limited; i.e., the effective MPC space is not too large. In Ref. [28] a MPC truncation scheme was suggested and the advantage of the MPC truncation over the conventional single-particle level (SPL) truncation has been discussed thoroughly [29]. Calculation shows that it is practical to obtain very accurate solutions to the low-lying eigenstates of $H_{\rm CSM}$ by diagonalizing $H_{\rm CSM}$ in a sufficiently large MPC space.

To reveal clearly the influence of pairing interactions on the moment of inertia, in this paper we shall adopt an improved PNC approach to the eigenvalue problem of $H_{\rm CSM}$; i.e., first, we diagonalize exactly the one-body part of $H_{\rm CSM}$, $H_0 = H_{\rm SP} + H_C$, to obtain the cranked Nilsson orbitals and then diagonalize $H_{\rm CSM}$ in a sufficiently large cranked many-particle configuration (CMPC) space to obtain accurate solutions of the low-lying excited eigenstates of $H_{\rm CSM}$. This will be described in Sec. II. In Sec. III the moments of inertia of the ground bands of well-deformed even-even rare-earth nuclei at low spin are calculated and compared with experimental ones. The mechanism of the reduction of the moments of inertia due to the antialignment effect of the pairing interaction is discussed in detail.

II. FORMALISM

A. Cranked Nilsson orbitals

First, considering the Coriolis interaction being a one-body operator, it is not difficult to diagonalize exactly the one-body part of $H_{\rm CSM}$, $H_0 = H_{\rm Nil} - \omega J_x = \sum_i h_0(i)$, the cranked Nilsson (CN) Hamiltonian. Usually, the eigenstates of $H_{\rm Nil}$ (Nilsson orbitals) [30] are characterized by π (parity) and Ω (eigenvalue of j_z, the z component of angular momentum) and are conventionally denoted by the asymptotic quantum numbers $[Nn_z\Lambda\Sigma]\Omega$ $(=\Lambda + \Sigma)$. Each Nilsson level is twofold degenerate ($\pm\Omega$). In the CN model, j_z is no longer conservative and the degeneracy is removed (signature splitting). However, the $R_x(\pi)$ invariance (rotation of π around the x axis) still remains. It can be easily shown that though $[j_z, R_x(\pi)] \neq 0$, we have $[j_z^2, R_x(\pi)] = 0$, and we can construct the simultaneous eigenstates of $R_x(\pi)$ and j_z^2. As in Ref. [30], let $|Nl\Lambda\Sigma\rangle$ be the simultaneous eigenstates of $h_{\rm osc}$ (spherical harmonic oscillation), $\mathbf{l}^2$, l_z, and s_z, and denote

$$\begin{aligned} |\xi\rangle &\equiv |N_\xi l_\xi \Lambda_\xi \Sigma_\xi\rangle, \quad \Omega_\xi = \Lambda_\xi + \Sigma_\xi > 0, \\ |-\xi\rangle &\equiv |N_\xi l_\xi - \Lambda_\xi - \Sigma_\xi\rangle, \end{aligned} \tag{2}$$

which are also the eigenstates of j_z with eigenvalues $\pm\Omega_\xi$. In terms of $|\pm\xi\rangle$ we can construct

$$\begin{aligned} |\xi\alpha\rangle &= \frac{1}{\sqrt{2}}[1 - e^{-i\pi\alpha}R_x(\pi)]|\xi\rangle \\ &= \frac{1}{\sqrt{2}}[|\xi\rangle \pm (-1)^{N_\xi}|-\xi\rangle], \quad \alpha = \pm\tfrac{1}{2}. \end{aligned} \tag{3}$$

It is easy to be verified that $|\xi\alpha\rangle$ is the eigenstate of $R_x(\pi)$ and j_z^2,

$$\begin{aligned} R_x(\pi)|\xi\alpha\rangle &= e^{-i\pi\alpha}|\xi\alpha\rangle, \\ j_z^2|\xi\alpha\rangle &= \Omega_\xi^2|\xi\alpha\rangle. \end{aligned} \tag{4}$$

Now we diagonalize $h_0 = h_{\rm Nil} - \omega j_x$ in the $|\xi\alpha\rangle$ space. The matrix elements of $h_0 = h_{\rm Nil} - \omega j_x$ can be calculated as

$$\langle\xi\alpha|h_{\rm Nil}|\xi'\alpha'\rangle = \langle\xi|h_{\rm Nil}|\xi'\rangle\delta_{\alpha\alpha'} \tag{5}$$

and

$$\langle\xi\alpha|j_x|\xi'\alpha'\rangle = \begin{cases} \langle\xi|j_x|\xi'\rangle\delta_{\alpha\alpha'} & \text{if } \Omega_\xi \neq \frac{1}{2} \text{ or } \Omega_{\xi'} \neq \frac{1}{2}, \\ (-1)^{N_\xi+1/2-\alpha}\langle\xi|j_x|-\xi'\rangle\delta_{\alpha\alpha'} & \text{if } \Omega_\xi = \Omega_{\xi'} = \frac{1}{2}. \end{cases} \tag{6}$$

The eigenstate of h_0 can be expressed as

$$|\mu\alpha\rangle = \sum_{\xi} C_{\mu\xi}(\alpha)|\xi\alpha\rangle \quad [C_{\mu\xi}(\alpha) \text{ real}], \tag{7}$$

which is characterized by the energy $\varepsilon_{\mu\alpha}$, parity π, and signature $\alpha = \pm\frac{1}{2}$. Hereafter, $|\mu\alpha\rangle$ is sometimes briefly denoted by $|\mu\rangle$.

Correspondingly, the cranked many-particle configuration of an n-particle system can be expressed as

$$|\mu_1\mu_2\cdots\mu_n\rangle = b^\dagger_{\mu_1}b^\dagger_{\mu_2}\cdots b^\dagger_{\mu_n}|0\rangle, \tag{8}$$

$\mu_1, \mu_2, \ldots, \mu_n$ being the occupied CN orbitals. Each configuration, simply labeled by $|i\rangle$, is characterized by E_i (configuration energy), parity, and signature. The angular momentum alignment of the CMPC $|i\rangle$ is

$$\langle i|J_x|i\rangle = \sum_{\mu(\text{occ})} \langle\mu|j_x|\mu\rangle = \sum_{\mu}\langle\mu|j_x|\mu\rangle P_{i\mu}, \tag{9}$$

$$P_{i\mu} = \begin{cases} 1 & \text{if } |\mu\rangle \text{ is occupied in } i|\rangle, \\ 0 & \text{otherwise,} \end{cases} \tag{10}$$

which is the sum of the contributions to angular momentum alignment from all the occupied orbitals. The kinematic moment of inertia is given by $J_i^{(1)} = \langle i|J_x|i\rangle/\omega$. Calculation shows that the calculated moments of inertia using the CN model (pairing interaction being neglected) are much larger than the experimental ones.

B. Influence of pairing interaction on the moment of inertia

Now we take the pairing interaction into account. The pairing interaction is usually expressed as

$$\begin{aligned} H_P &= -G\sum_{\xi\eta} a^\dagger_{\xi}a^\dagger_{\bar\xi}a_{\bar\eta}a_{\eta} \\ &= -G\sum_{\xi\eta}(-1)^{(\Omega_\xi-\Omega_\eta)}a^\dagger_{\xi}a^\dagger_{-\xi}a_{-\eta}a_{\eta}, \end{aligned} \tag{11}$$

where $\bar\xi$ $(\bar\eta)$ labels the time-reversed state of ξ (η). In the $|\xi\alpha\rangle$ representation [see Eq. (3)]

$$\beta^\dagger_{\xi\pm} = \frac{1}{\sqrt{2}}[a^\dagger_\xi \pm (-1)^{N_\xi}a^\dagger_{-\xi}] \quad \text{for } \alpha = \pm\tfrac{1}{2}, \tag{12}$$

H_P can be expressed as

TABLE I. The deformations ε_2 and ε_4 taken from the Lund systematics [31] are given in the second and third columns. The fourth and sixth columns give the measured proton and neutron even-odd mass differences [32] P_p and P_n, respectively, determined by Eq. (20). The corresponding pairing strengths G_p and G_n are listed in the fifth and seventh columns, respectively.

Nuclei	$\varepsilon_2, \varepsilon_4$	P_p (MeV)	G_p (keV)	P_n (MeV)	G_n (keV)
^{160}Dy	0.248,−0.016	0.807	315.8	0.872	285.3
^{162}Dy	0.261,−0.007	0.684	304.9	0.694	273.9
^{164}Dy	0.267,0.003	0.548	268.4	0.664	283.8
^{162}Er	0.245,−0.009	0.930	289.8	0.973	305.2
^{164}Er	0.258,0.001	0.835	298.1	0.913	304.2
^{166}Er	0.267,0.012	0.705	296.3	0.668	255.3
^{168}Er	0.273,0.023	0.604	282.6	0.628	238.8
^{170}Er	0.276,0.034	0.473	260.2	0.578	243.4
^{166}Yb	0.246,0.004	0.935	308.9	0.990	306.3
^{168}Yb	0.255,0.014	0.776	278.3	0.801	273.9
^{170}Yb	0.265,0.025	0.747	262.1	0.703	248.9
^{172}Yb	0.269,0.036	0.668	234.6	0.548	230.6
^{174}Yb	0.266,0.048	0.594	215.5	0.523	225.4
^{176}Yb	0.263,0.058	0.579	224.6	0.607	249.4
^{170}Hf	0.245,0.014	1.020	318.0	0.830	274.7
^{172}Hf	0.254,0.023	0.785	269.5	0.830	282.0
^{174}Hf	0.258,0.034	0.775	265.3	0.728	257.9
^{176}Hf	0.256,0.043	0.780	276.6	0.659	260.0
^{178}Hf	0.251,0.056	0.681	251.1	0.644	267.2
^{180}Hf	0.241,0.062	0.577	226.4	0.512	244.6
^{182}Hf	0.228,0.069	0.588	235.8	0.497	209.6
^{176}W	0.242,0.031	0.880	309.7	0.835	258.7
^{178}W	0.240,0.040	0.780	298.1	0.742	276.6
^{180}W	0.232,0.048	0.566	251.0	0.690	271.9
^{182}W	0.225,0.057	0.440	227.3	0.611	240.3
^{184}W	0.215,0.060	0.533	245.1	0.721	237.4
^{186}W	0.198,0.060	0.606	268.9	0.684	212.9

$$H_P = -G\sum_{\xi\eta}(-1)^{\Omega_\xi-\Omega_\eta}\beta^\dagger_{\xi+}\beta^\dagger_{\xi-}\beta_{\eta-}\beta_{\eta+}. \quad (13)$$

In the space spanned by the CN orbitals $|\mu\alpha\rangle$ [see Eq. (7)];

$$b^\dagger_{\mu\pm} = \sum_{\xi} C_{\mu\xi}(\pm)\beta^\dagger_{\mu\pm} \quad [C_{\mu\xi}(\alpha) \text{ real}], \quad (14)$$

we have

$$H_P = -G\sum_{\mu\mu'\nu\nu'} f^*_{\mu\mu'}f_{\nu\nu'}b^\dagger_{\mu+}b^\dagger_{\mu'-}b_{\nu-}b_{\nu'+}, \quad (15)$$

$$f^*_{\mu\mu'} = \sum_{\xi\xi'} e^{i\pi\Omega_\xi}C_{\mu\xi}(+)C_{\mu'\xi'}(-1),$$

$$f_{\nu\nu'} = \sum_{\eta\eta'} e^{-i\pi\Omega_\eta}C_{\nu\eta}(+)C_{\nu'\eta'}(-1).$$

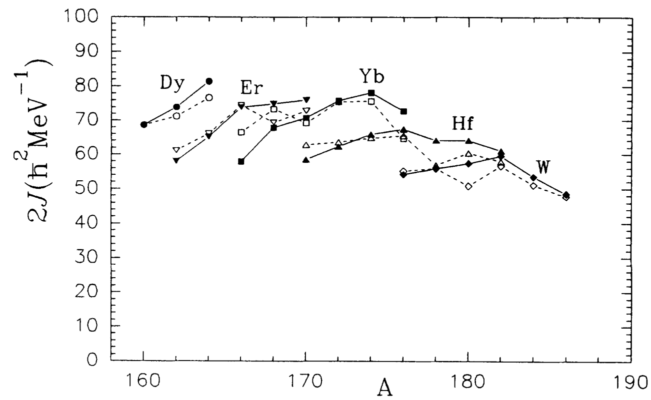

FIG. 1. Moments of inertia of the ground bands of even-even rare-earth nuclei (160< A < 186). The experimental results for each isotope chain are connected by solid lines, and the corresponding calculated ones are connected by dashed lines.

TABLE II. Comparison of the calculated and experimental moments of inertia of the ground bands of even-even rare-earth nuclei (160< A < 186) at low spin. Columns 2, 3, and 4 present the calculated results for $G_p = G_n = 0$. The corresponding calculated results with the pairing interaction being taken into account are given in columns 5 (proton), 6 (neutron), and 7 (total), respectively. Column 8 gives the experimental moments of inertia $J_{\text{expt}} = \hbar^2/2A$, where A is determined by fitting the observed three lowest levels [33] by the usual rotational spectrum formula $E(I) = AI(I+1) + BI^2(I+1)^2$.

	$2J_{\text{calc}}$ ($\hbar^2$ MeV^{-1})						
Rotational band	$G_p, G_n = 0$ Proton	Neutron	Total	$G_p, G_n \neq 0$ Proton	Neutron	Total	$2J_{\text{expt}}$ ($\hbar^2$ MeV^{-1})
^{160}Dy	61.26	126.32	187.58	28.98	39.70	68.68	68.6
^{162}Dy	59.46	101.14	160.60	29.66	41.56	71.22	74.0
^{164}Dy	59.42	95.38	154.80	30.80	45.88	76.68	81.4
^{162}Er	47.06	135.40	182.46	22.66	38.46	61.12	58.0
^{164}Er	44.16	106.74	150.90	23.82	42.32	66.14	65.2
^{166}Er	42.10	99.22	141.32	24.97	49.55	74.50	74.0
^{168}Er	40.62	77.34	117.96	24.24	45.18	69.42	75.0
^{170}Er	39.58	81.38	120.96	25.46	47.44	72.90	76.1
^{166}Yb	46.18	114.90	161.08	25.80	40.66	66.46	57.9
^{168}Yb	43.90	106.04	149.94	26.64	46.80	73.44	67.8
^{170}Yb	41.66	80.26	121.92	25.46	43.74	69.20	70.8
^{172}Yb	40.40	84.34	124.74	28.06	47.58	75.64	75.9
^{174}Yb	39.96	81.48	121.44	28.28	47.46	75.74	78.2
^{176}Yb	39.74	72.94	112.68	26.66	37.90	64.56	72.8
^{170}Hf	39.48	111.80	151.28	18.74	44.10	62.84	58.5
^{172}Hf	37.86	84.08	121.94	21.70	41.92	63.62	62.3
^{174}Hf	36.80	88.46	125.26	19.92	44.86	64.78	65.4
^{176}Hf	36.64	84.96	121.60	18.82	46.60	65.42	67.5
^{178}Hf	25.20	76.86	102.06	19.82	36.08	55.90	64.1
^{180}Hf	25.30	53.36	78.66	21.26	39.10	60.36	64.1
^{182}Hf	25.48	61.56	87.04	20.44	37.46	57.90	61.0
^{176}W	22.46	94.90	117.36	16.20	39.98	55.18	54.2
^{178}W	22.08	91.06	113.14	17.12	38.90	56.02	55.8
^{180}W	21.80	83.28	105.08	16.56	34.26	50.82	57.4
^{182}W	21.46	56.80	78.26	17.60	39.04	56.64	59.6
^{184}W	21.78	65.36	87.14	15.36	35.74	51.10	53.5
^{186}W	22.74	54.60	77.34	16.52	31.20	47.72	48.5

We can diagonalize H_{CSM} in the CMPC space [see Eq. (8)]. Considering the pairing interaction strength G being smaller than the average spacing of Nilsson levels, the number of important CMPC's mixed into the yrast and low-lying excited states is rather limited. Therefore we can diagonalize H_{CSM} in a sufficiently large CMPC space (i.e., all the CMPC's with energies $E_i - E_0 \le E_c$ are taken into account, E_0 being the energy of the lowest CMPC and E_c a sufficiently large cutoff energy) to obtain accurate solutions to the yrast and low-lying excited states.

Assume that one eigenstate of H_{CSM} is expressed as

$$|\psi\rangle = \sum_i C_i |i\rangle \quad (C_i \text{ real}); \tag{16}$$

the angular momentum alignment of $|\psi\rangle$ is

$$\langle\psi|J_x|\psi\rangle = \sum_i C_i^2 \langle i|J_x|i\rangle + 2\sum_{i<j} C_i C_j \langle i|J_x|j\rangle. \tag{17}$$

Considering J_x being a one-body operator, the matrix element $\langle i|J_x|j\rangle$ for $i \neq j$ is nonzero only when $|i\rangle$ and $|j\rangle$ differ by one particle occupation. Suppose that after a certain permutation of creation operators, $|i\rangle$ and $|j\rangle$ are brought into the forms

$$|i\rangle = (-1)^{M_{i\mu}} |\mu\cdots\rangle, \quad |j\rangle = (-1)^{M_{j\nu}} |\nu\cdots\rangle, \tag{18}$$

where the ellipsis stand for the same particle occupation and $(-1)^{M_{i\mu}} = \pm 1$, $(-1)^{M_{j\nu}} = \pm 1$ according to whether the permutation is even or odd. Therefore the kinematic moment of inertia of the state $|\psi\rangle$ is

$$\begin{aligned} J &= \frac{1}{\omega}\langle\psi|J_x|\psi\rangle = \sum_\mu J_{\mu\mu} + \sum_{\mu<\nu} J_{\mu\nu}, \\ J_{\mu\mu} &= \frac{1}{\omega}\langle\mu|j_x|\mu\rangle \sum_i C_i^2 P_{i\mu}, \\ J_{\mu\nu} &= \frac{2}{\omega}\langle\mu|j_x|\nu\rangle \sum_{i<j} (-1)^{M_{i\mu}+M_{j\nu}} C_i C_j \quad (\mu \neq \nu). \end{aligned} \tag{19}$$

If the pairing interaction is missing, only one CMPC appears in Eq. (16), and Eq. (19) is reduced to Eq. (9). In this case all the interference terms vanish, $J_{\mu\nu} = 0$, and the calculated moments of inertia are much larger than the experimental ones. However, when the pairing interaction is taken into account, because of its antialignment effect, the destructive interference ($J_{\mu\nu} < 0$) will significantly reduce the moment of inertia, usually by a factor of about $\frac{1}{2}$. The calculated moments of inertia of the ground bands of rare-earth even-even nuclei are given in the next section.

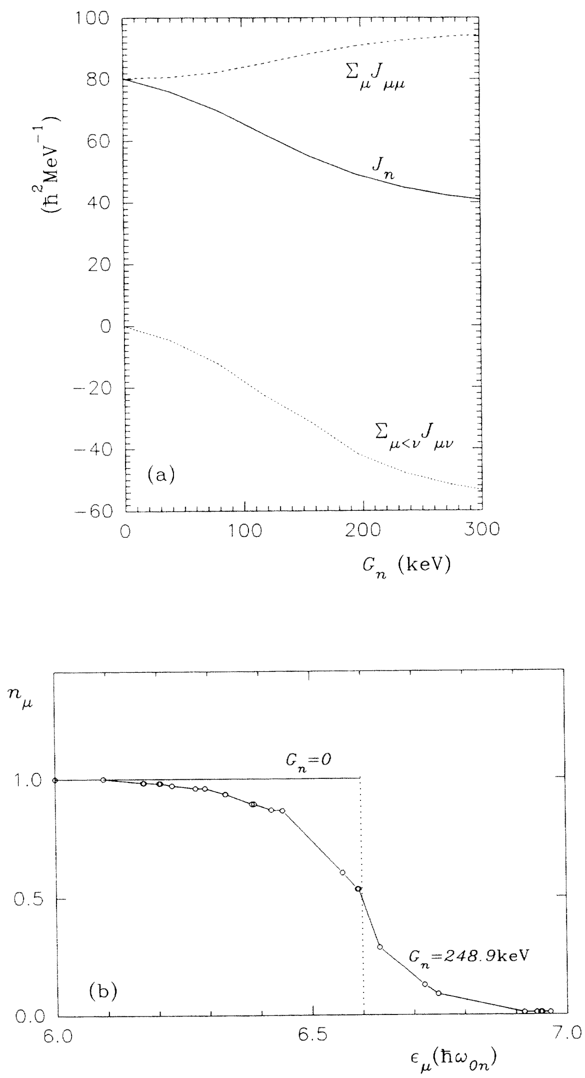

FIG. 2. (a) Variation with G of the moment of inertia (neutron part) of the ground band of ^{170}Yb at low spin. The diagonal part $\sum_\mu J_{\mu\mu}$ is shown by the dashed line, the off-diagonal part by the dotted line, and J_n by the solid line. (b) Occupation probability n_μ of each CN (neutron) orbital in the ground bands of ^{170}Yb at low spin.

III. CALCULATED RESULTS AND DISCUSSIONS

Using the PNC formalism presented in Sec. II, we have carried out numerical calculations for the moments of inertia of the ground bands of 27 well-deformed even-even nuclei in the rare-earth region (160< A < 186). The Nilsson parameters ($\kappa, \mu, \varepsilon_2, \varepsilon_4, \hbar\omega_0$) are taken from the Lund systematics [30,31], and no change to improve the calculated results is made. The pairing strengths G_n and G_p are unambiguously determined by the measured even-odd differences in binding energies [32],

$$\begin{aligned} P_n &= \tfrac{1}{2}[B(Z,N)+B(Z,N+2)]-B(Z,N+1) \\ &= E_g(Z,N+1)-\tfrac{1}{2}[E_g(Z,N)+E_g(Z,N+2)], \\ P_p &= \tfrac{1}{2}[B(Z,N)+B(Z+2,N)]-B(Z+1,N) \\ &= E_g(Z+1,N)-\tfrac{1}{2}[E_g(Z,N)+E_g(Z+2,N)] \\ & \qquad (Z,N,\ \text{even}), \end{aligned} \tag{20}$$

where E_g is the ground state energy of nucleus at $\omega = 0$. In the PNC calculation of the ground state energy (at $\omega = 0$) of an odd-A nucleus, the blocking effect is taken into account exactly. The values of ε_2, ε_4, P_p, G_p, P_n, and G_n are listed in Table I. In the PNC calculation the CMPC's with energies $E_i - E_0 \le E_C = 0.85\hbar\omega_0$ (truncation energy) are considered (e.g., for ^{170}Yb, $\hbar\omega_{0p} = 6.966$ MeV, $\hbar\omega_{0n} = 7.837$ MeV). Calculation shows that for the yrast band at low spin the energies of important CMPC's (weight $> 1\%$) are all below $0.5\hbar\omega_0$ and almost all the CMPC's with weight $> 10^{-3}$ are included in the calculation; so the calculated results are very accurate. It should be noted that in our calculation the one-body part of $H_{\rm CSM}$ (including the Coriolis interaction) has been treated exactly. For treating the pairing interaction, if different E_c (truncation energy) is adopted, renormalization of average pairing strength G should be made [26]. The calculated moments of inertia are given in Table II and Fig. 1. It is seen that the observed moments of inertia are reproduced very well (except for a few cases) and no systematic excessive reduction of moments of inertia compared to that of a rigid rotor is found.

It is seen that if the pairing interaction is missing ($G = 0$), the theoretical values of the moments of inertia in the CSM are much larger than the experimen-

TABLE III. Structure analysis of the contributions to the moments of inertia of three typical rare-earth nuclei ^{166}Er, ^{170}Yb, and ^{174}Hf. The proton part is shown in (a) and the neutron part in (b). No contribution comes from the closed shells $N = 0$, 1, 2, and 3. Contributions to the moments of inertia come mainly from the proton $N = 4, 5$ and neutron $N = 5, 6$ shells. Because of the antialignment effect of the pairing interaction, the moments of inertia are reduced by a factor of about $\frac{1}{2}$. $J_{\mu\mu}$ and $J_{\mu\nu}$ are in units of $\hbar^2$ MeV^{-1}.

(a)

		$G_p = 0$	$G_p \neq 0$		
		$2\sum_\mu J_{\mu\mu}$	$2\sum_\mu J_{\mu\mu}$	$2\sum_{\mu<\nu} J_{\mu\nu}$	Total
$^{166}_{68}$Er$_{98}$	$N = 4$	15.53	15.49	−4.37	11.12
	$N = 5$	26.57	31.65	−18.68	12.97
	$N = 6$	0.00	1.45	−0.57	0.88
	all shells	42.11	48.59	−23.62	24.97
$^{170}_{70}$Yb$_{100}$	$N = 4$	15.39	14.44	−3.36	11.08
	$N = 5$	26.27	27.55	−13.24	14.32
	$N = 6$	0.00	3.12	−3.06	0.06
	all shells	41.65	45.12	−19.67	25.46
$^{174}_{72}$Hf$_{102}$	$N = 4$	10.16	12.05	−2.81	9.24
	$N = 5$	26.66	25.62	−13.12	12.50
	$N = 6$	0.00	5.57	−7.38	−1.81
	all shells	36.82	43.23	−23.31	19.92

(b)

		$G_n = 0$	$G_n \neq 0$		
		$2\sum_\mu J_{\mu\mu}$	$2\sum_\mu J_{\mu\mu}$	$2\sum_{\mu<\nu} J_{\mu\nu}$	Total
$^{166}_{68}$Er$_{98}$	$N = 4$	0.00	0.05	−0.01	0.04
	$N = 5$	34.74	36.47	−12.38	24.09
	$N = 6$	64.49	62.62	−37.20	25.42
	all shells	99.23	99.15	−49.59	49.55
$^{170}_{70}$Yb$_{100}$	$N = 4$	0.00	0.03	0.00	0.03
	$N = 5$	35.29	36.92	−13.32	23.60
	$N = 6$	44.97	56.16	−36.04	20.12
	all shells	80.26	93.10	−49.36	43.74
$^{174}_{72}$Hf$_{102}$	$N = 4$	0.00	0.04	0.00	0.04
	$N = 5$	41.28	37.30	−14.09	23.21
	$N = 6$	47.21	52.78	−31.16	21.62
	all shells	88.50	90.11	−45.25	44.86

tal ones, as expected by general consideration [3]. To illustrate how the moments of inertia are significantly reduced (see Table II) due to the pairing correlation, let us make a more detailed analysis of the calculated results for three typical nuclei ${}^{166}_{68}$Er, ${}^{170}_{70}$Yb, and ${}^{174}_{72}$Hf. Contributions to the moments of inertia may be divided into two parts [see Eq. (19)]. If the pairing interaction is absent ($G = 0$), only one CMPC (say, i_0) configuration

TABLE IV. The off-diagonal part $\sum_{\mu<\nu} J_{\mu\nu}$ of the contributions to the moments of inertia of the ground bands of ^{166}Er, ^{170}Yb, and ^{174}Hf at low spin. (a) is for the protons and (b) for the neutrons.

(a)

$2J_{\mu\nu}$ ($\hbar^2$ MeV^{-1})

μ,ν	^{166}Er $\alpha=\frac{1}{2}$	^{166}Er $\alpha=-\frac{1}{2}$	^{170}Yb $\alpha=\frac{1}{2}$	^{170}Yb $\alpha=-\frac{1}{2}$	^{174}Hf $\alpha=\frac{1}{2}$	^{174}Hf $\alpha=-\frac{1}{2}$
[422]1/2,[413]5/2	−0.11	−0.11				
[420]1/2,[411]1/2	−0.20	−0.18	−0.11	−0.10	−0.05	−0.05
[420]1/2,[411]3/2	−0.17	−0.17	−0.05	−0.05	−0.04	−0.04
[413]5/2,[411]3/2	−0.03	−0.03				
[413]5/2,[404]7/2	−0.90	−0.90	−0.72	−0.72	−0.49	−0.49
[411]3/2,[402]5/2	−0.64	−0.64	−0.50	−0.50	−0.45	−0.45
[411]1/2,[402]3/2	−0.09	−0.10	−0.25	−0.29	−0.20	−0.24
[411]1/2,[400]1/2	−0.07					
[404]7/2,[402]5/2			−0.03	−0.03	−0.13	−0.13
[550]1/2,[541]3/2	−0.76	−0.57				
[541]3/2,[532]5/2	−0.53	−0.54	−0.31	−0.31	−0.34	−0.34
[532]5/2,[523]7/2	−2.71	−2.71	−0.92	−0.92	−0.35	−0.35
[532]7/2,[514]9/2	−5.04	−5.04	−4.30	−4.30	−3.63	−3.63
[514]9/2,[505]11/2	−0.17	−0.17	−0.30	−0.30	−0.85	−0.85
[541]1/2,[532]3/2	−0.12	−0.13	−0.40	−0.48	−0.76	−0.98
[541]1/2,[530]1/2	−0.09	−0.11	−0.04	−0.08	−0.09	−0.11
[532]3/2,[523]5/2			−0.27	−0.27	−0.16	−0.16
[530]1/2,[521]3/2					−0.27	−0.26
[660]1/2,[651]3/2	−0.17	−0.36	−0.91	−1.61	−0.79	−1.75
[660]1/2,[642]5/2					−0.02	−0.10
[651]3/2,[642]5/2		−0.06	−0.27	−0.27	−1.34	−1.28
[642]5/2,[633]7/2					−1.03	−1.04
Total	−23.63		−19.67		−23.31	

(b)

$2J_{\mu\nu}$ ($\hbar^2$ MeV^{-1})

μ,ν	^{166}Er $\alpha=\frac{1}{2}$	^{166}Er $\alpha=-\frac{1}{2}$	^{170}Yb $\alpha=\frac{1}{2}$	^{170}Yb $\alpha=-\frac{1}{2}$	^{174}Hf $\alpha=\frac{1}{2}$	^{174}Hf $\alpha=-\frac{1}{2}$
[514]9/2,[505]11/2	−0.24	−0.24	−0.16	−0.16	−0.09	−0.09
[530]1/2,[521]3/2	−0.38	−0.38	−0.29	−0.34	−0.18	−0.18
[530]1/2,[521]1/2	−0.32	−0.25	−0.33	−0.24	−0.23	−0.14
[532]3/2,[523]5/2	−0.68	−0.67	−0.42	−0.42	−0.27	−0.28
[532]3/2,[521]1/2	−0.31	−0.37	−0.29	−0.37	−0.18	−0.26
[521]3/2,[523]5/2	−0.15	−0.15	−0.09	−0.09	−0.05	−0.05
[521]3/2,[512]5/2	−1.95	−1.95	−2.07	−2.07	−1.88	−1.88
[523]5/2,[514]7/2	−1.71	−1.71	−1.72	−1.72	−1.78	−1.78
[521]1/2,[510]1/2	−0.18	−0.15	−0.45	−0.38	−0.71	−0.61
[521]1/2,[512]3/2	−0.13	−0.16	−0.31	−0.38	−0.48	−0.58
[512]5/2,[514]7/2			−0.16	−0.16	−0.50	−0.50
[512]5/2,[503]7/2	−0.10	−0.10	−0.25	−0.25	−0.53	−0.53
[514]7/2,[505]9/2			−0.07	−0.07	−0.16	−0.16
[660]1/2,[651]3/2	−0.47	−1.03	−0.69	−1.24	−0.34	−1.22
[651]3/2,[642]5/2	−2.37	−2.36	−1.67	−1.64	−1.11	−1.10
[642]5/2,[633]7/2	−13.30	−13.30	−7.94	−7.94	−4.23	−4.23
[633]7/2,[624]9/2	−2.07	−2.07	−7.13	−7.13	−8.88	−8.88
[624]9/2,[615]11/2	−0.08	−0.08	−0.29	−0.29	−0.54	−0.54
Total	−49.59		−49.36		−45.25	

is present ($C_i = \delta_{ii_0}$), the off-diagonal part vanishes. In general, the diagonal part may be expressed as

$$\sum_\mu J_{\mu\mu} = \frac{1}{\omega}\sum_\mu \langle\mu|j_x|\mu\rangle \sum_i C_i^2 P_{i\mu}$$
$$= \frac{1}{\omega}\sum_\mu \langle\mu|j_x|\mu\rangle n_\mu, \tag{21}$$

where $n_\mu = \sum_i C_i^2 P_{i\mu}$ is the particle occupation probability of the CN orbital $|\mu\alpha\rangle$ [α being omitted in Eqs. (19) and (21) for brevity]. Figure 2(a) shows that the diagonal part $\sum_\mu J_{\mu\mu}$ of neutrons in ^{170}Yb (dashed line) changes slowly with the pairing strength G_n, which can be understood from the slight change in the particle occupation due to the pairing interaction. The change in the occupation probability of CN neutron orbitals $|\mu\alpha\rangle$ due to the pairing interaction for the yrast band of ^{170}Yb at low spin is shown in Fig. 2(b).

Contrary to the diagonal part, the off-diagonal part $\sum_{\mu<\nu} J_{\mu\nu}$ [dotted line in Fig. 2(a)] drops rapidly with the pairing strength G_n, which results in a significant reduction of the moments of inertia in realistic nuclei. The reduction of the moments of inertia originates mainly from the destructive interference ($\sum_{\mu<\nu} J_{\mu\nu} < 0$ [see Fig. 2(a) and Tables III and IV] due to the antialignment effect of the pairing interaction. The interference between various CMPC's [see Eq. (17)] can be viewed from the transitions of particles between various CN orbitals [see Eq.(19)]. The off-diagonal part $\sum_{\mu<\nu} J_{\mu\nu}$ depends sensitively on the properties and level distribution of the CN orbitals near the Fermi surface. Each $J_{\mu\nu}$ ($\mu \neq \nu$) depends on the energetic location of the CN orbitals ε_μ and ε_ν and the magnitude of the matrix element $\langle\mu|j_x|\nu\rangle$, which is especially large for both μ and ν belonging to the high-j intruder orbitals (for rare-earth nuclei, the $h_{11/2}$ protons and $i_{13/2}$ neutron orbitals). Needless to say, if μ or ν is far away from the Fermi surface, $J_{\mu\nu}$ would be negligibly small. Therefore only when both μ and ν are in the vicinity of Fermi surface would $J_{\mu\nu}$ be of importance. The non-negligible $J_{\mu\nu}$'s (weight $> 10^{-3}$) for the ground bands of $^{166}_{68}$Er,

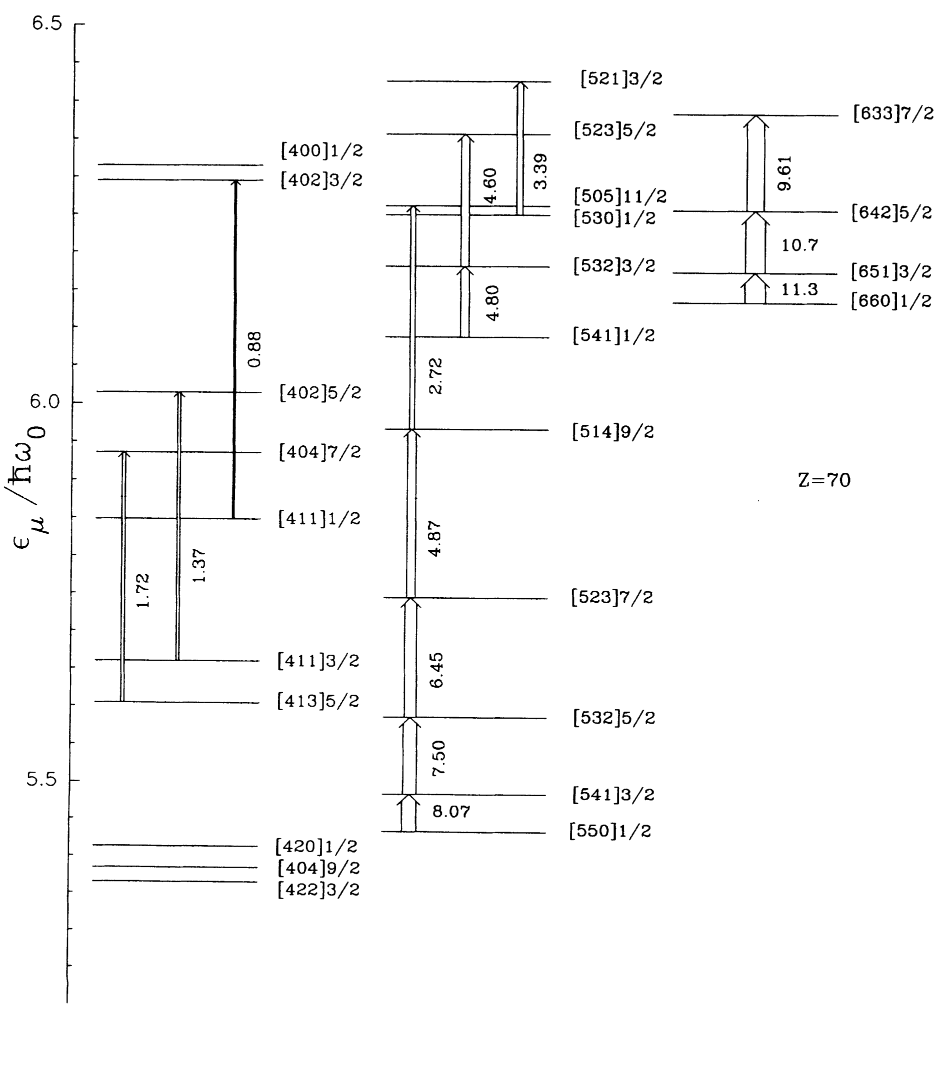

FIG. 3. Proton Nilsson orbitals near the Fermi surface for ^{170}Yb ($\varepsilon_2 = 0.265$, $\varepsilon_4 = 0.025$). The width of each arrow indicates the magnitude of the matrix element $|\langle|\mu|j_x|\nu\rangle|^2$. In this figure are not shown the transitions with very small matrix elements $|\langle|\mu|j_x|\nu\rangle|^2$ and the transitions for μ or ν being far away from the Fermi surface.

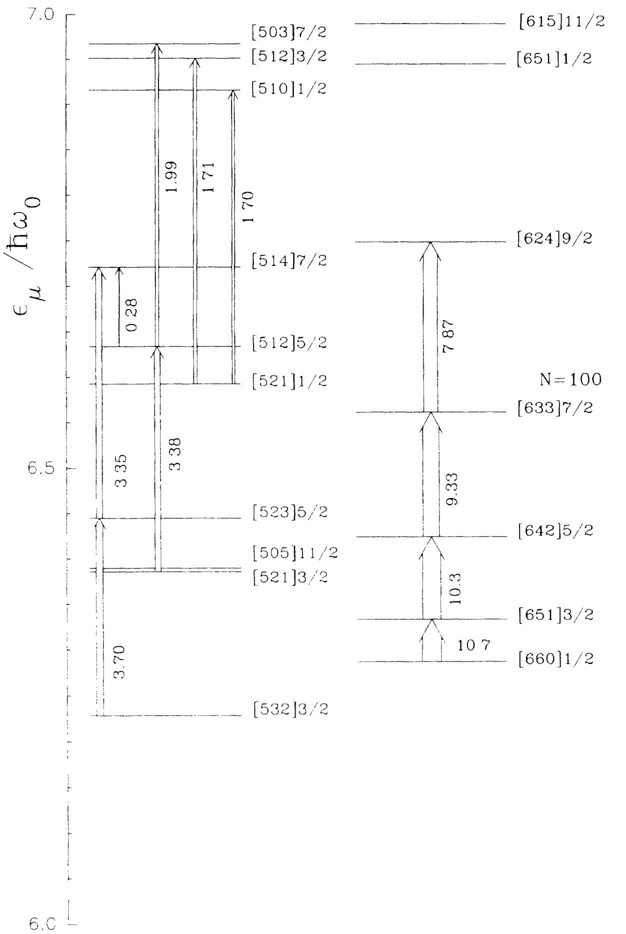

FIG. 4. Same as Fig. 3, but for the neutron orbitals.

$^{170}_{70}$Yb, and $^{172}_{72}$Hf are listed in Tables IV(a) (proton) and IV(b) (neutron). It is seen that the transitions between adjacent high-j intruder orbitals in the vicinity of the Fermi surface play a decisive role in the contribution to the moments of inertia (e.g., proton [523]5/2-[514]7/2, neutron [642]5/2-[633]7/2, [633]7/2-[624]9/2). The particle transitions which contribute greatly to the moment of inertia are shown by arrows in Figs. 3 (proton) and 4 (neutron). The width of each arrow indicates the magnitude of $|\langle\mu|j_x|\nu\rangle|^2$.

It should be noted that the contribution to the moments of inertia from a closed harmonic oscillator major shell is zero. Therefore no contribution comes from $N \leq 3$ proton shells and $N \leq 4$ neutron shells, which are closed for the ground bands of rare-earth nuclei at low spin. Similarly, the contributions from the $N \geq 6$ proton shells and the $N \geq 7$ neutron shells are very small, even when the pairing interaction is taken into account, because these shells are completely vacant in the ground configurations of rare-earth nuclei. Therefore almost all the contributions to the moments of inertia of rare-earth nuclei come from the $N = 4,5$ proton and $N = 5,6$ neutron shells [see Tables III(a) and III(b)]. The contribution of neutrons is approximately twice as large as that of protons, i.e., $J_p/J \sim \frac{1}{3}$ and $J_n/J \sim \frac{2}{3}$ (see Table II), which is easily understood, because the valence neutrons occupy higher major shells ($N = 5,6$) than the valence protons ($N = 4,5$ shells) and so are more strongly influenced by the Coriolis interaction.

To summarize, the moments of inertia of the ground bands of a large number of even-even rare-earth nuclei at low spin have been calculated using the PNC treatment, in which blocking effects are taken into account exactly. The Nilsson parameters are taken from the Lund systematics, and the pairing strength is determined unambiguously from the observed odd-even mass differences. No change of the parameters has been made to improve the calculated results. Because of the antialignment effect of the pairing interaction, the moments of inertia are reduced by a factor of about $\frac{1}{2}$. The experimental data are reproduced very well, and no systematic excessive reduction of the moments of inertia is found.

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