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Microscopic investigation of nuclear structure with dynamic Bose-Fermi symmetry

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We have described a seniority conserving mapping procedure from the shell model space to the boson-quasifermion space for odd nuclei within the framework of the Otsuka-Arima-Iachello mapping which is suitable in regions where seniority is a good quantum number. We show that, when a number of levels are allowed to the odd fermion in the interacting boson-fermion model (IBFM), one can use the dynamic symmetry, if present, to unambiguously find out the parameters in the IBFM Hamiltonian. As a simple example of the technique, we have discussed the case of ^{59}Ni where an approximate $U_{B+F}(5) \times SU_F(2)$ is known to exist.

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I. INTRODUCTION

The interacting boson model (IBM) and the interacting boson-fermion model (IBFM) have enjoyed considerable success in describing the collective behavior of nuclei since their inception about twenty years ago. Though the IBM is considered a purely phenomenological model, the IBFM is treated as a semimicroscopic extension of it on the basis of its derivation from the shell model.

The usual boson-fermion interaction in the IBFM consists of three terms, the monopole-monopole term, the quadrupole-quadrupole term, and the exchange term. The microscopic justification of such an interaction can be traced from the mapping of a shell model Hamiltonian, consisting of monopole-pairing, quadrupole-pairing, and quadrupole-quadrupole interactions, on to a boson-quasifermion basis. However, in regions where the shell model Hamiltonian is not well represented by these interactions mentioned above, the usual boson-fermion interaction should not be expected to provide a good description of the nucleus and one has to consider more terms. Such a case arises in singly closed shell nuclei when the number of valence nucleons is not large. Besides, in the dynamical symmetry limit of the IBFM the simple boson-fermion interaction is clearly insufficient to explain the presence of the symmetry. Few microscopic investigations have been conducted in this direction [1].

II. THEORY

The various mapping procedures from the shell model space to the boson-quasifermion space are discussed in a review article by Klein and Marshalek [2]. One approach [3] is to apply the mapping technique of Otsuka, Arima, and Iachello (OAI) [4] to the single-fermion operator, giving an approximate boson-quasifermion mapping. The maps of generators of the shell model algebra are obtained by coupling the products of fermion maps. In another approach [5,6] Van Egmond and Allaart have used the OAI mapping to map the fermion Hamiltonian on the boson-quasifermion space directly and estimate the values of the different parameters.

In the present work we have performed OAI mapping of the shell model Hamiltonian. This is ideally suited for our purpose as we have chosen the vibrational limit of the IBFM where this mapping works well. The collective subspace of the full shell model space is separated and mapped onto the boson-quasifermion space. This is essentially a Marumori mapping [7] and the mapped Hamiltonian contains an infinite number of terms. The truncation of this infinite Hamiltonian is achieved in three steps. For comparison with the IBFM, all three- or higher-body terms are dropped. Since in the IBFM the number of quasifermions for the low-lying states of odd nuclei is 1, all two-body fermion terms are dropped. Finally, since in the IBM the collective behavior is explained in terms of the s ($J = 0$) and d ($J = 2$) bosons only, all terms involving collective degrees of freedom other than s and d are neglected. The different parameters in the IBFM Hamiltonian are now estimated by imposing the condition that the matrix elements of the shell model Hamiltonian between the different collective states are equal to the matrix elements of the mapped Hamiltonian between the corresponding states in the boson-quasifermion space.

We assume that the boson states represent fermion states built up of coherent pairs with angular momentum $J = 0$ and $J = 2$. We also assume that the structures of the collective pairs do not vary from the even-even core to the odd-A nucleus. The collective fermion states

$$|J = 0, \nu = 0\rangle, \tag{1a}$$

$$|J = 2, \nu = 2\rangle, \tag{1b}$$

where ν is the generalized seniority, can be found in a number of ways, like broken pair calculation, number-projected BCS method, etc. Once this part has been carried out one can map these states on the

$$\frac{1}{n!}(s^\dagger)^n|0), \tag{2a}$$

$$\frac{1}{(n-1)!}(s^\dagger)^{n-1}d^\dagger|0) \tag{2b}$$

boson states, respectively. In our notation $|\ \rangle$ $[|\ )]$ denotes a fermion [boson-quasifermion] state. For the odd-A nuclei the basis states can be obtained by coupling the odd fermion to the collective states defined earlier. However, this cannot simply be mapped on the boson-quasifermion basis. The reasons for this can be understood from the following argument. A fermionic single-particle operator $c_j^\dagger$ operating on a fermionic state with $\nu = 2$ gives a combination of states with $\nu = 1$ and 3. So the states obtained by coupling a single-fermion operator $c_j^\dagger$ to the collective states (1a) and (1b) in the fermion space are not orthogonal. However, the quasifermion operator $a_j^\dagger$ operating on the state (2b) will only raise the seniority to $\nu = 3$ and so is orthogonal to the state obtained by coupling $a_j^\dagger$ to (2a). One may, following Ref. [5], map the state $[c_j^\dagger \times |1a\rangle]$ on the state $[a_j^\dagger \times |2a)]$ after suitable normalization, take the part of the wave function $[c_j^\dagger \times |1b\rangle]$ orthogonal to it, and map it on the state $[a_j^\dagger \times |2b\rangle]$ after normalization. However, it seems more natural to map the state with a particular seniority in the fermion space on the state with the same seniority in the boson-quasifermion space. We would like to map the fermion states

$$|c_j\rangle = N(c_j)[c_j^\dagger \times |1a\rangle] \tag{3a}$$

and

$$|c_j d; J\rangle = N(c_j d; J)P[c_j^\dagger \times |1b\rangle] \tag{3b}$$

on the corresponding boson-quasifermion states. Here P is the operator which projects out the maximum seniority part and N is the appropriate normalization factor. The states (3a) and (3b) are now orthogonal. However, the states $|c_j d; J\rangle$ and $|c_{j'} d, J\rangle$ are still nonorthogonal for $j \neq j'$ although the corresponding states in the boson-fermion space are orthogonal. If the overlap between the states is small, this can be neglected [5] but for large overlap one has to orthogonalize them. The simple mapping from fermion to boson-quasifermion system is lost as a result.

However, there is a particular situation where the simple mapping may still be restored. This occurs in the dynamical symmetry limit when the boson core obeys U(5) symmetry. In this case, the wave function for a particular state with seniority 3, written in the basis $[a_j^\dagger \times |2b)]$, does not depend on the parameters of the IBFM Hamiltonian. We therefore impose the condition on the orthogonalization of (3b) that the wave function for a particular state, calculated with the shell model Hamiltonian in the orthogonalized basis, should exactly correspond to the IBFM wave function. Imposing this condition on the lowest-lying states one can orthogonalize the states (3b) uniquely and map them on the boson-quasifermion basis.

After the collective subspace of the shell model has been identified and mapped on the boson-quasifermion basis, one can calculate the values of the different parameters in the IBFM Hamiltonian. The different restrictions on the values of the parameters, for a dynamical symmetry to exist, can then be tested and microscopic justification of the existence of the symmetry can be sought. Alternately, one can calculate the different parameters in a dynamically symmetric Hamiltonian and compare them with the phenomenological results.

III. EXAMPLE

As an illustration of the technique outline above we discuss the case of a particular symmetry. To keep the calculation as simple as possible, we choose a symmetry where the number of fermion levels is 2. Such a dynamical symmetry is given by the $U_{B+F}(5)$ limit of $\mathrm{U}_B(6) \times \mathrm{U}_F(10)$ with the odd fermion occupying $j = \frac{3}{2}$ and $\frac{5}{2}$ orbits. The Ni isotopes obey the above Bose-Fermi dynamic symmetry [8] and are suitable for calculation as they have only a few valence neutrons outside the closed ^{56}Ni core. Once again, to keep the calculation as simple as possible, we take the case of ^{59}Ni which has only three nucleons. The group chain and the quantum numbers are given by

$$\begin{array}{ccccccccc}
\mathrm{U}_B(6)\times\mathrm{U}_F(10) & \supset & \mathrm{U}_B(5)\times\mathrm{U}_F(5)\times\mathrm{SU}_F(2) & \supset & \mathrm{U}_{B+F}(5)\times\mathrm{SU}_F(2) & \supset & \mathrm{O}_{B+F}(5)\times\mathrm{SU}_F(2) & \supset & \mathrm{O}_{B+F}(3)\times\mathrm{SU}_F(2) \supset \mathrm{SU}_{B+F}(2). \\
N \quad (1) & & n_d \quad n_F \quad S & & [N_1, N_2] & & (\tau_1, \tau_2) & & L \qquad\qquad J
\end{array} \tag{4}$$

The dynamically symmetric Hamiltonian for excitation spectra is

$$H = AC_1[\mathrm{U}_B(5)] + A'C_2[\mathrm{U}(5)] + BC_2[\mathrm{U}_{B+F}(5)] + DC_2[\mathrm{O}_{B+F}(5)]$$
$$+EC_2[\mathrm{O}_{B+F}(3)] + FC_2[\mathrm{SU}_{B+F}(2)], \tag{5}$$

where C_n[G] is the nth-order Casimir operator for the group G. All the other Casimir operators of the group chain can be neglected for the excitation energy calculation.

The Hamiltonian in terms of quantum numbers is

$$H = An_d + A'n_d(n_d+4) + B[N_1(N_1+4) + N_2(N_2+2)] + D[\tau_1(\tau_1+3) + \tau_2(\tau_2+1)] + EL(L+1)] + FJ(J+1). \tag{6}$$

The wave function may be written in the form

$$|N,\{1\},n_d,n_F,[N_1,N_2],\{\tau_1,\tau_2\},\alpha LJM) = \sum_{\substack{\{B\}\\\{F\}}} \left\langle \begin{matrix} n_d,n_F \\ \nu_B,\nu_F \end{matrix} \middle| \begin{matrix} [N_1,N_2] \\ \{\tau_1,\tau_2\} \end{matrix} \right\rangle \left\langle \begin{matrix} \nu_B & \nu_F \\ \alpha_B L_B & \alpha_F L_F \end{matrix} \middle| \begin{matrix} \{\tau_1,\tau_2\} \\ \alpha L \end{matrix} \right\rangle$$
$$\times \sum_j (-1)^{L_B+J+1/2}[(2L+1)(2j+1)]^{1/2} \begin{Bmatrix} L_B & L_F & L \\ \frac{1}{2} & J & j \end{Bmatrix}$$
$$\times [|N,n_d,\nu_B,\alpha_B L_B) \times a_j^\dagger]_{JM}, \tag{7}$$

where the wave function is written in terms of the usual isoscalar factors and $6j$ coefficients. The values of the necessary isoscalar factors can be found in Ref. [9].

In our calculation, we have considered the valence shell model space to consist of only two orbitals, $j = \frac{3}{2}$ and $\frac{5}{2}$. We have chosen the pairing plus surface tensor [10] interaction as the residual interaction as it fits the lower mass region of these isotopes rather well within this valence space and has certain realistic features [11]. Earlier studies have shown the applicability of this interaction in the $1f2p$ shell [11,12]. The interaction has the added advantage that one can make the interaction coincide with monopole pairing and study the mapping procedure in this limit. The interaction is given by

$$V = -A_\tau q_{12} + B_\tau \left(\frac{r}{R_0}\right)^2 \delta(r_1 - R_0)\,\delta(r_2 - R_0)\,S_{12}, \tag{8}$$

where

$$S_{12} = \left[\frac{1}{r^2}(\vec{\sigma}_1\cdot\vec{r})(\vec{\sigma}_2\cdot\vec{r}) - \tfrac{1}{3}(\vec{\sigma}_1\cdot\vec{\sigma}_2)\right].$$

The form of the matrix elements can be found in Ref. [11]. The two parameters used in the present calculation are $A = 0.225$ keV and $B = 0.335$ keV and are obtained from a best fit calculation in this region. The single-particle energy values are taken from the energy level data of ^{57}Ni [13]. The experimental energy values are from Ref. [14].

Our next task is separating the collective subspace. Since the basis is not large, to calculate (1a) and (1b) we diagonalize the shell model Hamiltonian within the basis of all the states with the required generalized seniority and angular momentum for the particular nucleus and identify the lowest $\nu = 0$, $J = 0$ state as (1a) and the lowest $\nu = 2$, $J = 2$ state as (1b). The basis states (3a) and (3b) are found by coupling a single fermion to these states. However, for orthogonalization of (3b), we have to neglect that part of the Hamiltonian which mixes states with different seniority. This is not too drastic an assumption as for singly closed nuclei generalized seniority mixing is known to be small. For each state with seniority 3, we orthogonalize the states so that the wave function mixing in the collective space for a particular level corresponds to that given by (7). These orthogonalized states together with the states (3a) are mapped on the basis states in the boson-quasifermion space.

The boson-quasifermion Hamiltonian, after the truncation procedure discussed earlier, can be written as

$$H = H_B + H_F + V_{BF}, \tag{9}$$

where H_B is the boson part, H_F is the fermion part of the Hamiltonian, and V_{BF} is the boson-fermion interaction term. In this work we consider only the boson core with U(5) symmetry and only those states which can be obtained by coupling one quasifermion to a bosonic state with $n_d = 0$ and 1. So the boson Hamiltonian is simply written as

$$H_B = H_0 + \epsilon_d[d^\dagger\tilde{d}]_0, \tag{9a}$$

where $\tilde{d}_m = (-1)^m d_{-m}$. The fermionic part is written as

$$H_F = \sum \epsilon_j [a_j^\dagger \tilde{a}_j]_0, \tag{9b}$$

where $\tilde{a}_{jm} = (-1)^{j-m}a_{j-m}$. The boson-fermion part is written as

$$V_{BF} = \sum_{jj'} A_{jj'}(s^\dagger\tilde{d} + d^\dagger s)_2(a_j^\dagger\tilde{a}_{j'})_2 + \sum_{Ljj'} B^L_{jj'}(d^\dagger\tilde{d})_L(a_j^\dagger\tilde{a}_{j'})_L. \tag{9c}$$

The parameters in (9a) can be calculated through the equations

$$H_0 = \langle 1a|H_{\rm SM}|1a\rangle \tag{10a}$$

and

$$\epsilon_d = \langle 1b|H_{\rm SM}|1b\rangle - H_0, \tag{10b}$$

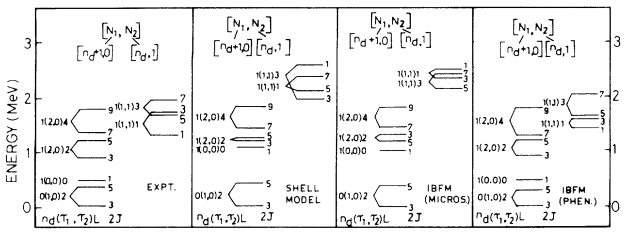

FIG. 1. Experimental, shell model, microscopic IBFM and phenomenological IBFM low-lying energy level schemes of ^{59}Ni. The quantum numbers are indicated in the figure.

where $H_{\rm SM}$ is the shell model Hamiltonian. The parameters have been calculated from the boson core as we have assumed that the structure of the collective states does not vary from the even core to the odd nucleus.

All the other parameters are estimated from the odd-A nucleus. Since we are interested in comparison with parameters for the excitation spectrum, we find only the value of $\epsilon = (\epsilon_{5/2} - \epsilon_{3/2})$:

$$\epsilon = \langle c_{5/2}|H_{\rm SM}|c_{5/2}\rangle - \langle c_{3/2}|H_{\rm SM}|c_{3/2}\rangle. \tag{10c}$$

Here $|c_j\rangle$ is given by (3a).

The parameters $A_{jj'}$ and $B^L_{jj'}$ are calculated from the relations

$$\langle c_j d; j'|H_{\rm SM}|c_{j'}\rangle = -(2j'+1)^{-1/2}N^{1/2}A_{jj'}, \tag{10d}$$

$$\langle c_j d; J|H_{\rm SM}|c_{j'}d; J\rangle$$

$$= \sum_L (-1)^{L+J-j}(2L+1)\begin{Bmatrix} 2 & J & j \\ j' & L & 2 \end{Bmatrix} B^L_{jj'} - [\langle c_j|H_{\rm SM}|c_{j'}\rangle + \epsilon_d]\,\delta_{jj'}, \tag{10e}$$

where N is the number of bosons.

The first part of (9c) mix states with different seniority and are absent in the dynamical symmetry limit. The contribution of these terms, calculated through the method described above, is found to be small. So we have assumed $A_{jj'} = 0$.

For (9) to be Hermitian, we must have

$$A_{jj'} = (-1)^{j-j'}A_{j'j} \tag{11a}$$

and

$$B^L_{jj'} = (-1)^{j-j'}B^L_{j'j}. \tag{11b}$$

Equation (11b) is satisfied in the OAI mapping but (11a) does not necessarily follow from it. Van Egmond and Allaart have checked the values of $A_{jj'}$ to test the validity of (11a) and have concluded that the relation is approximately satisfied. In our case, since we have assumed the seniority mixing part of the Hamiltonian to be absent, the requirement of Hermiticity for the mapped Hamiltonian is automatically satisfied. After the calculation of the parameters in (9), the spectra in the boson-quasifermion space can now be calculated. The experimental energy scheme is compared with the shell model calculation and the IBFM level scheme obtained from the calculation described above in the first three energy level schemes in Fig. 1.

TABLE I. The phenomenological best fit parameters and the parameters obtained from microscopic calculation for Eq. (6).

Parameter	Phenomenological (keV)	Microscopic (keV)
$A+5A'$	1962	3036
B	−186	−277
D	41	27
E	−21	−23
F	56	37

Ideally the lowest few states should be sufficient to calculate the best fit parameters of (6). However, since the chosen microscopic interaction does not describe the experimental level scheme very well, we perform a best fit calculation of the energy levels obtained in the IBFM calculation and compare them with the phenomenological best fit parameters (Table I). In the present example, A and A' cannot be found separately from the experimental energy level schemes of ^{59}Ni alone and the values of $A+5A'$ are tabulated. The fourth energy level spectrum shows the results of the phenomenological fitting.

IV. CONCLUSION

The above mapping procedure from the shell model space to the boson-quasifermion space for odd nuclei is suitable in regions where seniority is a good quantum number. However, these are the regions where mapping from the shell model space to the boson-quasifermion space shows reasonably good results. When a number of levels are allowed to the odd fermion in the IBFM, one can take advantage of the existence of a dynamic symmetry to find the parameters in the IBFM Hamiltonian unambiguously. An obvious extension of the present method can be made to include the case of supersymmetry.

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