Differences between the deformed-potential and folding-model descriptions of inelastic nuclear scattering

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(Received 24 March 1994)

The differences between the deformed-potential and folding-model descriptions of inelastic nuclear scattering, attention to which has been called recently by Beene, Horen, and Satchler [Phys. Rev. C **48**, 3128 (1993)], were pointed out already some time ago by contrasting the rules of equal deformation lengths and equal normalized multipole moments for the optical potential and the underlying nucleon distribution of the excited nucleus.

PACS number(s): 24.10.Ht, 24.50.+g, 21.10.Gv

Recently, Beene, Horen, and Satchler [1] (see also [2]) have compared the standard description of inelastic nuclear scattering, which uses deformed phenomenological optical potentials, with that of the folding model [3], in which an effective nucleon-nucleon interaction is folded with a deformed nucleon distribution of the nucleus excited in the scattering. They have shown that, with the exception of the unphysical case of a dipole deformation, these two descriptions are essentially different, and criticized the fact that this is usually overlooked in the analyses of inelastic-scattering data. They have also suggested that, because of this difference, the nuclear deformation lengths are extracted incorrectly from the inelastic-scattering data, in particular for multipolarities $l > 2$, when the deformed-potential procedure is followed.

The purpose of this Comment is to point out that similar conclusions (but note one difference below), using similar methods, were reached already some time ago [4]. It was shown in Ref. [4] that when the rule of equal normalized multipole moments of the optical potential and the deformed nucleon distribution [5], which follows from the folding model, is imposed on a phenomenological potential that is deformed according to the standard prescription [6]

$$U(r) \to U(r) - \sum_l \delta_l(U) \frac{dU(r)}{dr} Y_{l0}(\theta) \tag{1}$$

and on a similarly deformed underlying nucleon distribution, the relation

$$\delta_l(U) \, \langle r^{l-1} \rangle_U = \delta_l(\rho) \, \langle r^{l-1} \rangle_\rho \tag{2}$$

is obtained between the deformation lengths δ_l and radial moments $\langle r^{l-1} \rangle$ of the optical potential U and the nucleon distribution ρ. For the most commonly used Woods-Saxon shape with halfway radius R and diffuseness a, the radial moment $\langle r^{l-1} \rangle$ is to second order in a/R

$$\langle r^{l-1} \rangle_{\rm WS} = \frac{3R^{l-1}}{l+2} \left[1 + \frac{(l-1)(l+4)}{6} \left(\frac{\pi a}{R} \right)^2 + \cdots \right]. \tag{3}$$

Thus, Eq. (2) demands that the deformation length $\delta_l(U)$ of the optical potential be smaller than the deformation length $\delta_l(\rho)$ of the nucleon distribution, and that by a margin that grows rapidly with an increasing multipolarity $l \geq 2$, as $\langle r^{l-1} \rangle_U$ can be, especially for heavy ions, considerably greater than $\langle r^{l-1} \rangle_\rho$. It also follows immediately from Eq. (2) that an exception to this is the case of dipole deformation, $l = 1$, as the radial moment $\langle r^{l-1} \rangle$ with $l = 1$ always equals unity; however, for inelastic scattering to low-lying excited states, the $l = 1$ case is unphysical, as it corresponds to a spurious shift of the center of mass of the nucleus. The $l = 1$ exception to the above difference between the deformed-potential and nucleon-distribution deformation lengths has been emphasized in Refs. [1,2].

Notwithstanding Eq. (2), the empirical rule of equal deformation lengths

$$\delta_l(U) = \delta_l(\rho) \tag{4}$$

appears to be borne rather well by the analyses of light and heavy-ion inelastic-scattering data that employ deformed phenomenological potentials [7], including the analyses of inelastic scattering of heavy ions from deformed nuclei under the critical conditions of strong Coulomb-nuclear interference where the inelastic scattering is sensitive to possible differences between the deformation of the optical potential and the well-known deformation of the nuclear charge distribution [8]. Admittedly, these analyses deal with only relatively low multipolarities l, but according to Eq. (2), for heavy ions there should be an appreciable difference between the deformed-potential and nucleon-distribution deformation lengths already in the quadrupole case $l = 2$.

A conclusion somewhat different to that of Refs. [1,2] is then drawn that the rule of equal normalized multipole moments is not universal, in the sense of being valid for all optical potentials, because it is derived from nonspherical folded potentials, whose deformation results from the deformation of the underlying nucleon distribution but is of an essentially different kind than that which results from the standard prescription (1) used with phenomenological potentials [9]. The folding model, however, with

0556-2813/94/50(2)/1251(2)/$06.00

its unambiguous connection between the deformations of the optical potential and the underlying nucleon distribution, is to be preferred as a more physical description of inelastic scattering than that provided by deformed phenomenological potentials, where the rule of equal deformation lengths has only an empirical justification, presumably limited to low multipolarities l only.

[1] J. R. Beene, D. J. Horen, and G. R. Satchler, Phys. Rev. C **48**, 3128 (1993).
[2] D. J. Horen, J. R. Beene, and G. R. Satchler, Phys. Lett. B **316**, 463 (1993).
[3] G. R. Satchler and W. G. Love, Phys. Rep. **55**, 183 (1979).
[4] V. Hnizdo, J. Phys. G **13**, 27 (1987).
[5] G. R. Satchler, J. Math. Phys. **13**, 1118 (1972).
[6] For simplicity, the case of an axially symmetric deformation with respect to the body-fixed axis is assumed here.
[7] W. J. Thompson and J. S. Eck, Phys. Lett. **67B**, 151 (1977).
[8] J. Carter, R. G. Clarkson, V. Hnizdo, R. J. Keddy, D. W. Mingay, F. Osterfeld, and J. P. F. Sellschop, Nucl. Phys. **A273**, 517 (1976); J. Carter, R. G. Clarkson, V. Hnizdo, and J. P. F. Sellschop, *ibid.* **A297**, 520 (1978).
[9] A spherical folded potential can be, of course, deformed also according to the standard prescription (1). When the folded potentials used in the Coulomb-nuclear-interference test cases of Ref. [4] are deformed in this way, the inelastic-scattering data are fitted equally well as in the original nonspherical-folded-potential analysis of Ref. [4], using the same deformation lengths.