Mott scattering as a probe of long range QCD effects

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We investigate the possibility of using the Mott scattering between identical nuclei to assess the existence of long range QCD effects, e.g., a color van der Waals interaction, as suggested recently. We show that the inclusion of atomic effects is very important and should be considered in order to extract limits on the strength of the color van der Waals force. We compare our calculations with the analysis of a recent heavy ion experiment.

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I. INTRODUCTION

Since the Coulomb force between heavy ions is very well known, Rutherford scattering can be used as a probe of long range small interactions between the nuclei. For example, it has been used in Ref. [1] to assess the effects of relativity, nuclear polarizability, and screening in sub-Coulomb elastic heavy ion scattering by looking at deviations from the Rutherford cross section. More exotic experiments have also used this technique, e.g., Rutherford backscattering was used by Brügger *et al.* [2] to search for supermassive matter with strange nuggets. Another possibility is the use of the scattering of identical nuclei, say Pb+Pb. Due to the interference between the scattering amplitudes for forward and backward angles, the effects of small corrections are seen as shifts in the Mott oscillations. This method has been used successfully by Vetterli *et al.* [3, 4] to test the effect of vacuum polarization on sub-Coulomb scattering. With this method the effect could be identified on a 7% level.

The use of sub-Coulomb scattering as a probe for a hypothetical color van der Waals (CVDW) force between hadrons has been proposed in Ref. [5]. This suggestion has been investigated experimentally at the heavy ion facility of GANIL, France [6]. One of the experimental difficulties is to produce a completely stripped ion beam for the experiment. As we show in this article, the presence of the atomic electrons makes it very difficult to deduce constraints on the strength of a CVDW force from the experimental data. Our arguments are based on a simplified calculation, which includes the main physical effects. We have compared our calculations with those of previous authors [1, 4–7]. A good agreement was obtained, especially with the calculations of Ref. [4]. Our results, however, are in disagreement with those of Ref. [6] for the shifts in the Mott oscillations caused by the nuclear dipole and quadrupole polarizabilities.

The data of Ref. [6] were collected for the system ^{208}Pb+^{208}Pb at $E_{\rm lab}$ = 873 MeV and 1130 MeV. The main interaction between the nuclei at these energies is the Coulomb force. The other interactions may be taken as small perturbations:

$$V(r) = \frac{Z_1 Z_2 e^2}{r} + \Delta V(r) \ . \tag{1}$$

Among the small effects contributing to $\Delta V(r)$ we consider (a) relativistic corrections, (b) nuclear dipole polarizability, (c) nuclear quadrupole polarizability, (d) vacuum polarization, (e) color van der Waals interaction, (f) tail of the nuclear interaction, (g) atomic screening, (h) bremsstrahlung, (i) emission of δ electrons, and (j) the formation of a quasimolecule.

II. COLOR VAN DER WAALS INTERACTION

The idea discussed in Ref. [5] is that a color van der Waals interaction of the form given by [8] (we use $\hbar = c = 1$)

$$V_{\rm CVDW} = -(\alpha_6/r_0)\ (r_0/r)^6,$$

or

$$V_{\rm CVDW} = -(\alpha_7/r_0)\ (r_0/r)^7 \ , \tag{2}$$

results in sizable shifts in the Mott oscillations. This is because the estimates of the color van der Waals force due to multigluon exchange [8] and from constraints imposed by experiments yield large values for the strength constants, $\alpha_6 \simeq 1$ and $\alpha_7 \simeq 100$ (with $r_0 = 1$ fm), respectively. Especially in the latter case, the shifts in the Mott oscillations are appreciable.

The van der Waals (VDW) force is a higher order correction to the exchange of massless particles. In the case of photons, it was shown by London [9] using perturbation theory that this force is proportional to $1/r^6$. Later, Casimir and Polder [10] showed that when r is large compared to the electromagnetic wavelength associated with an atomic transition frequency ($r \gg a_B/e^2 = 137 a_B$, where a_B is the Bohr radius) retardation effects are important, and the CVDW interaction is proportional to $1/r^7$. More recently, Feinberg and Sucher [11], using a relativistic covariant calculation, have shown that the conclusions above remain valid and that the proportionality constant for a VDW between particles 1 and 2 is given by

$$\alpha_{12} = \frac{23}{4\pi}\left(\alpha_1^e\alpha_2^e + \alpha_1^m\alpha_2^m\right) - \frac{7}{4\pi}\left(\alpha_1^e\alpha_2^m + \alpha_1^m\alpha_2^e\right), \tag{3}$$

where the α^e and α^m are electric and magnetic polarizabilities defined in terms of tensor amplitudes $M^{\mu\nu} = M^{\mu\nu}(p'_A k'_\gamma; p_A, k_\gamma)$ associated with the Compton scattering $\gamma + A \longrightarrow \gamma + A$.

The quantum chromodynamics theory of strong interactions predicts that hadrons are color neutral and that the color charge is carried by gluons which mediate the strong interactions. Therefore, one-gluon exchange between hadrons is forbidden. However, a two-gluon exchange is not ruled out. Based on phenomenological potentials for quark-quark interaction and on dispersion relations, the existence of a CVDW force has been suggested by some authors [8]. Their study has shown that if such a force exists limits on its strength can be obtained from several existing experiments. Until now, the best constraints have been obtained from the analysis hadronic atoms. The above-mentioned limits for the α_6 and α_7 in Eq. (2) were obtained by studying the energy shifts in pionic atoms. The existence of such a long range QCD interaction has also been strongly criticized theoretically [12]. However, a test of the theoretical predictions based on a different experimental method is worth doing [6].

If successful, these experiments could test corrections due to interactions mediated by very light particles. For example, one could try to study the contribution of weakly interacting particles, such as axions, to the long range force between nuclei. In this case, the force would be of the form [13]

$$V_a \sim A_1\ A_2\ g_a^2 \frac{m_a^2}{4m_n^2}\ \frac{\exp(-m_a r)}{r}, \tag{4}$$

where m_n is the nucleon mass. With $m_a \sim 1$ eV this force is of very long range. However, the coupling constant g_a is very small, $g_a \sim 10^{-8}$, as compared to the Coulomb constant $Z_1 Z_2 \alpha$. The corrections to the Coulomb force between heavy ions due to this interaction are therefore very small compared to the other effects considered here.

III. MOTT SCATTERING

Assuming the validity of a semiclassical approach (for heavy ion scattering this is often the case), the Mott symmetrized cross section including the potentials $\Delta V(r)$ is given by

$$\frac{d\sigma}{d\Omega} = \frac{d\sigma_{\rm cl}}{d\Omega}(\theta) + \frac{d\sigma_{\rm cl}}{d\Omega}(\pi-\theta) + 2\left[\frac{d\sigma_{\rm cl}}{d\Omega}(\theta)\frac{d\sigma_{cl}}{d\Omega}(\pi-\theta)\right]^{1/2} \cos\left\{2\left[\delta(\theta) - \delta(\pi-\theta)\right]\right\}. \tag{5}$$

The phase shifts δ in the above equation can be calculated with the WKBJ approximation. At these energies and for heavy systems it has been shown [4] that this approximation works extremely well. Use of the Coulomb phase shift in Eq. (5) yields the well-known Mott oscillations for the scattering of identical nuclei. The potentials $\Delta V(r)$ in Eq. (1) introduce corrections to $\delta(\theta)$ which induce shifts in the Mott oscillations.

In Eq. (5), θ is the scattering angle. The classical cross sections $d\sigma_{\rm cl}(\theta)/d\Omega$ are Rutherford cross sections including corrections arising from the same set of potentials considered, i.e., $\Delta V(r)$. But the corrections to the magnitude of $d\sigma_{\rm cl}/d\Omega$ are irrelevant. They damp the Mott oscillations, but do not shift them. The shift is essentially caused by the corrections $\Delta\delta(\theta) - \Delta\delta(\pi-\theta)$ in the last term of Eq. (5). The measurement of the shifts in the Mott oscillations is thought to be easier to accomplish than the measurement of the magnitude of the cross sections [6].

The potentials (a)–(d) listed before are treated in the same way as in Refs. [5, 6]. Our results are presented in Table I. We find small differences from our calculation and those of Refs. [5, 6]. However, our calculations agree well with those of Refs. [4, 14]. We now proceed to study the effects of other mechanisms.

IV. BREMSSTRAHLUNG

The emission of radiation in form of bremsstrahlung is well-known theoretically. In Ref. [15] a semiclassical calculation of bremsstrahlung radiation for all multipolarities was performed. There is no dipole radiation for a symmetrical system and we concentrate on the quadrupole radiation. Higher order multipolarities are strongly suppressed. For a collision with scattering angle θ the total energy emitted in form of quadrupole

TABLE I. The angular shifts (in degrees) in the Mott scattering oscillations for Pb+Pb scattering at three different laboratory energies and $\theta_{\rm lab} = 30°$, due to relativistic corrections (Δ_R), nuclear dipole polarization (Δ_{DP}), nuclear quadrupole polarization (Δ_{QP}), vacuum polarization (Δ_{VP}), atomic screening (Δ_{AS}), quasimolecule (Δ_{QM}), tail of the nuclear interaction (Δ_N), bremsstrahlung ($\Delta_{\rm brems}$), production of δ electrons ($\Delta_{\delta-e^-}$), and color van der Waals force ($\Delta^{(6)(7)}_{\rm CVDW}$) with a $1/r^6(1/r^7)$ dependence.

$E_{\rm lab}$ [MeV]	873	975	1130
Δ_R	0.0345	0.039	0.0435
$\Delta_{\rm DP}$	0.021	0.0285	0.0435
$\Delta_{\rm QP}$	0.003	0.006	0.0105
$\Delta_{\rm VP}$	−0.036	−0.038	−0.042
$\Delta_{\rm AS}$	0.018	0.0165	0.0135
$\Delta_{\rm QM}$	0.0255	0.024	0.021
Δ_N	$< 10^7$	$< 10^{-5}$	0.0015
$\Delta_{\rm brems}$	9.2×10^{-5}	7.2×10^{-5}	0.0015
$\Delta_{\delta-e^-}$	$< 10^{-4}$	$< 10^{-5}$	$< 10^{-5}$
$\Delta^{(6)}_{\rm CVDW}$	0.0028	0.0034	0.007
$\Delta^{(7)}_{\rm CVDW}$	0.0075	0.015	0.0345

bremsstrahlung is given by

$$\Delta E_{\rm brems}(\theta) = \frac{4}{\pi}\, E_0\, \alpha\, v^4\, A_r^4\, f_2^2 \int_0^\infty \nu\, h_2(\nu,\, \theta)\, d\nu\ , \tag{6}$$

where $\alpha = 1/137$, v is the relative velocity, $A_r = A_1A_2/(A_1 + A_2)$ is the reduced mass number, $f_2 = A_r\left(Z_1/A_1^2 + Z_2/A_2^2\right)$, $E_0 = v/a$, and $a = Z_1Z_2e^2/A_rm$. The function h_2 is given by

$$h_2(\nu,\, \theta) = \frac{2}{5}\, e^{-\pi\nu}\, \nu \left\{ \left[\frac{1}{\epsilon^4} \left\{ \nu^2(\epsilon^2-1)^3 + \left(\frac{1}{3}\epsilon^4 - \epsilon^2 + 1 \right) \right\} \right] K_{i\nu}^2 \right.$$
$$\left. + 2\frac{\epsilon^2-1}{\epsilon^3}\left(4 - 3\epsilon^2\right)\nu K_{i\nu}K'_{i\nu} + \frac{\epsilon^2-1}{\epsilon^2}\left[1 + \nu^2(\epsilon^2-1)\right]K'^2_{i\nu} \right\}, \tag{7}$$

where

$$K_{i\nu}(\epsilon\nu) = \frac{1}{2}\int_{-\infty}^{\infty} \exp\left\{ -\nu\epsilon\, \cosh t \right\} \cos\nu t\ dt\ , \tag{8}$$

and K' means the derivative of $K_{i\nu}$ with regard to its argument $\epsilon\nu$.

At $\theta_{\rm c.m.} = 0°$ the bremsstrahlung radiation vanishes, since this corresponds to very large impact parameters for which no appreciable acceleration occurs. The radiation is maximum (1.07 MeV for Pb+Pb at 873 MeV) for $\theta_{\rm c.m.}$ around 90°, and is not negligible at 180° (see Fig. 1). This is because at $\theta_{\rm c.m.} = 180°$ the movement of the charges is similar to that of an imperfect dipole antenna, which also emits quadrupole radiation.

V. DELTA ELECTRONS

To obtain the energy loss by emission of δ electrons we performed a coupled channels calculation using the two-center Dirac program described in Ref. [16]. The coupled channels equations for the transition amplitudes from state $|i\rangle$ to state $|j\rangle$ are given by

$$\frac{da_{ij}}{dt}(t) = -\sum_{k\neq j} a_{ik}\left\langle \phi_j \left| \frac{\partial}{\partial t} + i\mathcal{H}_{\rm TCD} \right| \phi_k \right\rangle e^{-i(E_i - E_f)t}\ , \tag{9}$$

with the initial condition that $a_{ij}(t = -\infty) = \delta_{ij}$ and an initial configuration of occupied states. Integrating these equations along the heavy ion trajectory one gets the transition probabilities $|a_{ij}(t = \infty)|^2$. $\mathcal{H}_{\rm TCD}$ is the two-center Dirac Hamiltonian. The calculation is enormously simplified with the use of the monopole approximation for the two-center potential. The validity of this approximation is discussed in more detail in Ref. [17].

Summing over all possible transitions from an initial state $|i\rangle$, we obtain the differential probability dP_i/dE_{e^-} for the emission of a δ electron with final energy E_{e^-}. To obtain the energy loss we multiply it by $E_{e^-} + E_i$ and integrate over E_{e^-}, where E_i is the binding energy of the emitted electron. After that we sum over all occupied orbitals to obtain the energy spectrum dP_{e^-}/dE_{e^-}. In Fig. 2 we show the energy spectrum of the emitted electrons as a function of the electron kinetic energy for a collision between a fully charged Pb atom with a 23^+ Pb atom at 873 MeV/nucleon in the laboratory and at an impact parameter of 20 fm. The numerical calculation is shown as filled circles. Also shown is a fit (dashed line) based on a simplified model developed in Ref. [18], which gives

$$\frac{dP_{e^-}}{dE_{e^-}} = P_0\, \frac{\kappa^2}{E^2}\, e^{-2E\hat{t}}\ , \tag{10}$$

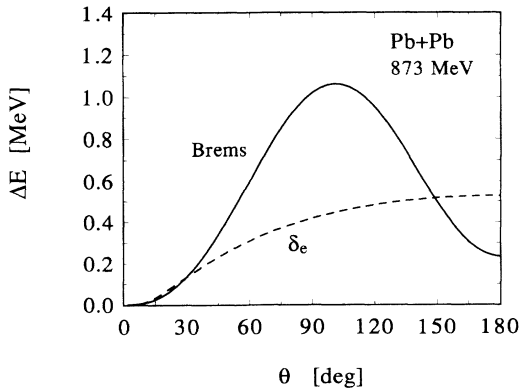

FIG. 1. Energy emitted in form of bremsstrahlung (solid curve) and of δ electrons (dashed curve) in a collision of Pb+Pb at $E_{\rm lab} = 873$ MeV, as a function of the center-of-mass scattering angle.

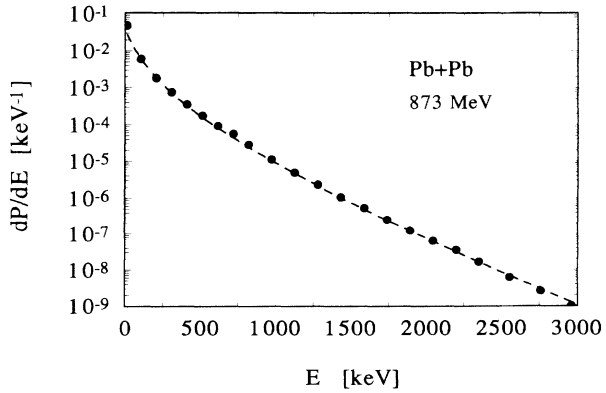

FIG. 2. A coupled-channels calculation (circles) of the energy spectrum of δ electrons emitted in a collision of Pb+Pb at 873 MeV, as a function of the kinetic energy of the electron. A fit using the scaling model given by Eq. (10) is also shown.

where P_0 is a normalization constant, and

$$\hat{t} = \frac{a}{v}\left(\epsilon + 1.6 + \frac{0.449}{\epsilon}\right), \qquad \kappa = 1 + \frac{0.174}{\epsilon},$$

and

$$E = E_{e^-} + \langle E_i \rangle , \tag{11}$$

where $\langle E_i \rangle$ is the average binding energy of the emitted electron. A detailed study presented in Ref. [19] has shown that this scaling model works very well in general. It is also very useful for a quick evaluation of the energy spectrum of δ electrons. The fit in Fig. 2 was obtained with $P_0 = 0.46$ MeV and $\langle E_i \rangle = 100$ keV for Pb+Pb at 873 MeV/nucleon. In Fig. 1 the energy loss by emission of δ electrons for this system is shown (dashed line). The energy loss increases steadily with the scattering angle and is maximum for $\theta_{\text{c.m.}} = 180°$ (0.52 MeV for Pb+Pb at 873 MeV). This is because for scattering at backward angles the ions come closer, leading to a greater probability for the ejection of δ electrons. We observe that the total energy loss by means of emission of bremsstrahlung and of δ electrons is not small. However, this has no great influence on the shifts of the Mott oscillations, as we show next.

VI. DISSIPATIVE EFFECTS

The small effects in sub-Coulomb scattering considered in previous publications [1, 3, 5–7] are of conservative character. The emission of bremsstrahlung and of δ electrons are not. They induce an attenuation of the elastic cross sections. For the same reason as stated before, more important is the shift caused by phase differences for forward and backscattering. We can calculate this phase shift by using the formula

$$\delta = -\int \Delta E(\theta, t)\, dt , \tag{12}$$

where $\Delta E(\theta, t)$ is the energy loss (positively defined) by emission of as a function of time t along a Rutherford trajectory with scattering angle θ. Inserting the result in Eq. (4) we get the values presented in Table I (Δ_{brems} and $\Delta_{\delta - e^-}$) for the shifts in the Mott oscillations at $\theta_{\text{lab}} = 30^0$. As compared to relativistic corrections, vacuum polarization, etc., these corrections are very small. The effects of Coulomb excitation are less relevant than those of bremsstrahlung and emission of δ electrons. Pb has its first state at 2.6 MeV. The excitation probability for any scattering angle is much smaller than 1 (typically of order of 0.01, or less). Thus the "probability averaged" energy loss, in a single collision, by means of Coulomb excitation is much less than 1 MeV.

It should be noted that for dissipative processes the scattered particles are not really phase shifted, as implied by our use of Eq. (12). Since the wavelength associated with the particle is different in the initial and final channels a time independent phase shift does not exist. Thus, the reason for using Eq. (12) is for qualitative purposes only, to infer the relative importance of attenuation processes. A good energy resolution is required experimentally to obtain a pure elastic process and avoid such difficulties.

VII. ATOMIC SCREENING

To account for screening of the electrons remaining in the nuclei the bare Coulomb potential has to be multiplied by a screening function, i.e.,

$$\frac{Z_1 Z_2 e^2}{r} \longrightarrow \frac{Z_1 Z_2 e^2}{r}\, \phi(r) . \tag{13}$$

With inclusion of screening the bare Coulomb potential between the nuclei is modified to

$$V_S(r) = \int d^3r_1\, d^3r_2\, \rho_1(\mathbf{r}_1)\, \rho_2(\mathbf{r}_2)\, \frac{1}{|\mathbf{r} + \mathbf{r}_1 - \mathbf{r}_2|} , \tag{14}$$

where $\rho_i(r)$ contains contributions from the electron cloud. We assume a spherically symmetric charge distribution and use the approximation

$$\rho_i(r_i) = \frac{Z_i e}{4\pi r_i^2}\, \delta(\mathbf{r}_i) - n_i e \frac{\nu^3}{8\pi}\, e^{-\nu r_i}, \qquad i = 1, 2 . \tag{15}$$

The first term corresponds to naked, pointlike nuclei. The second term is a correction due to the charge distribution of n_i electrons remaining in the atom. For the parameter ν we use the Bohr value $\nu = Z^{1/3}/a_B$, where a_B is the Bohr radius. The second term in Eq. (15) introduces a correction ΔV to the unscreened Coulomb potential between the two nuclei. The phase shifts depend on the bulk properties of the atomic screening and the approximation given by Eq. (15) should be as good for our purposes as the use of a more realistic Hartree-Fock calculation of the electronic orbitals [4]. Using the approximation (15) in Eq. (14) the integrals can be evaluated analytically, as shown in Ref. [4]. The shifts in the Mott oscillations caused by the atomic screening are shown in Table I (Δ_S).

VIII. TAIL OF THE NUCLEAR POTENTIAL

For the tail of the nuclear potential we used the Christensen-Winther parametrization [20]

$$V_N(r) = -50[\text{MeV}]\, \frac{R_1 R_2}{R_1 + R_2}\, \exp\left\{-\frac{r - r_1 + R_2}{a_v}\right\} , \tag{16}$$

which was shown to describe very well the heavy ion elastic scattering data at low energies. We used the nuclear radii given by $R_i = (1.233 A_i^{1/3} - 0.98 A_i^{-1/3})$ fm and the diffuseness $a_v = 0.63$ fm.

IX. QUASIMOLECULE EFFECT

As the nuclei come close together the atomic electrons feel the influence of the Coulomb field of both nuclei. This reduces the energy of the system since the electronic levels become more bound. The effective potential describing this effect is given by

$$V_{\text{QM}}(r, Z_1, Z_2) = \sum_i E_i^{Z_1 + Z_2}(r) - \left(\sum_j E_j^{Z_1} + E_k^{Z_2}\right) , \tag{17}$$

which is a function of the internuclear distance $r(t)$. $E_i^{Z_1+Z_2}(r)$ is the energy of the ith electron of the compound system $Z_1 + Z_2$ separated by the distance r. $E_j^{Z_1}$ and $E_k^{Z_2}$ are the energies of the electrons in the individual atoms. The energy of the compound, or quasimolecule, system was calculated by solving the two-center Dirac equation for the electrons, neglecting the electron-electron interaction, and in the monopole approximation [17]. A similar approach was used in Ref. [21] to calculate the effect of formation of a quasimolecule on Rutherford scattering of heavy ions.

In Fig. 3 we show the calculated potential (17) for the Pb+Pb quasimolecule with 141 electrons (59 in the projectile and 82 in the target). We see that the potential is quite large at small separations between the nuclei, but, more important for the shift in the Mott oscillations is the form of the potential. It is roughly proportional to $R^{-0.24}$. This weak dependence on r causes sizable shifts of the Mott oscillations, as we shall see next.

We remark that our calculations are based on a static procedure to calculate the quasimolecule potential. The process is considered to be adiabatic. This procedure is valid for inner electronic shells ($n < 4$) since the electronic velocities are much larger than the relative velocity between the nuclei. The inner shells are the ones that most contribute to the quasimolecule potential in Eq. (17). Another effect not considered here is the electron-electron interaction. However, the increase of the electronic binding due to the increase of the nuclear charge greatly exceeds the binding corrections due to electron-electron interactions, as obtained in, e.g., Hatree-Fock calculations [17]. As shown in Ref. [21] the effects of electron-electron interaction and the failure of the adiabaticity argument occur for scattering at very forward angles, or order of $\theta_{\rm c.m.} = 5°$ or less. We are considering the scattering at $\theta_{\rm c.m.} = 60°$, where such effects should be irrelevant.

X. SHIFTS IN THE MOTT OSCILLATIONS

The results of our calculations for the respective shifts in the Mott scattering of Pb+Pb at $\theta_{\rm lab} = 30°$ are shown in Table I. We observe from these results that the most relevant effects are the relativistic corrections, dipole polarizability, vacuum polarization, atomic screening, formation of a quasimolecule and the color van der Waals force with a r^{-7} dependence.

The results presented in Ref. [6] for the shifts in the Mott oscillations due to a CVDW force are in complete disagreement with ours. The value used in Ref. [6] was $\alpha_7 = 20$ for which we obtain $\Delta_7 = 0.0015°$, $0.0029°$, and $0.0074°$ for $E_{\rm lab}$= 873 MeV, 975 MeV, and 1130 MeV, respectively. These values are much smaller than those present in Ref. [6] (their Fig. 3b). The same disagreement was also found [14] by using the theoretical approach of Ref. [4].

If, instead of using the color van der Waals interaction between the nuclei as A_1A_2 times the elementary interaction, we use a folding potential over the nuclear matter densities with the same elementary interaction, we get Δ_7 shifts which are about a factor 2 larger than the ones presented above. However, a great discrepancy with the results of [6] remains.

In Fig. 4 we show all the shifts added (dashed curve), except for the effect of formation of a quasimolecule and of a CVDW force. We observe that the shifts are not sufficient to explain the data. When the shifts caused by the formation of a quasimolecule is added we obtain the solid curve. The agreement with the experimental data is quite reasonable, although a slight overestimate is obtained. This can be explained in terms of the uncertainties in the several interactions which contribute to the whole effect. Assuming a reasonable uncertainty of 10% in all corrections treated here the experimental data would lie within the limits of the theoretical predictions.

The inclusion of a CVDW force overestimates the data by a large amount for a $1/r^7$ interaction, with $\alpha_7 = 100$. As seen from Table I, a $1/r^6$ interaction gives a very small shift in the oscillations, unless a nonrealistic value of the constant α_6 is taken.

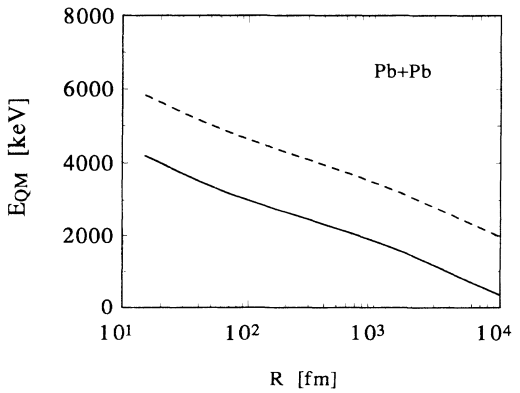

FIG. 3. The total binding energy of 141 electrons in a Pb+Pb quasimolecule (dashed line) as a function of the distance between the nuclear centers. Extracting from it the binding energies of individual atoms (i.e., at $r = \infty$) yields the quasimolecule potential (solid line).

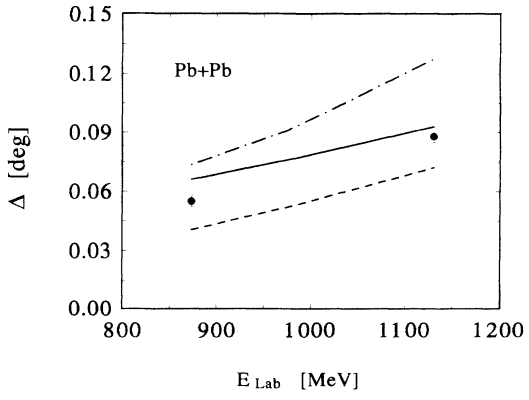

FIG. 4. The shifts in the Mott oscillations for the elastic scattering of Pb+Pb. The data are from Ref. [6]. The dashed curve is a calculation which includes all effects displayed in Table I, except for the formation of a quasimolecule and a CVDW force. When one includes the first effect the solid curve is obtained. The dashed-dotted curve contains both effects.

XI. CONCLUSIONS

We have shown that the shifts in the Mott oscillations due to bremsstrahlung, emission of δ electrons, tail of the nuclear potential, and of quadrupole polarizability are not important. But the modification of the electronic cloud due to the influence of both nuclear charges is shown to be an important effect. In view of this, we think that the existence of a hypothetical color van der Waals interaction does not seem to be the main cause for the shifts observed in the experiment of Ref. [6].

We feel, however, that such questions could be better assessed if an experiment with completely stripped nuclei could be performed. This is because the effects of atomic screening and of the formation of a quasimolecule are very large. If such experiments could be performed, than a more definite statement about a possible long range force could be done. Such an experiment could perhaps be done with cooled beams intersecting at a small angle [22]. Such beams are currently obtainable at the GSI/Germany facility. With sufficient experimental data the radial dependence of the potential corrections could be obtained by the inverse scattering technique [23].

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