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Consequences of Isospin Sum Rules for Photonuclear Reactions

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Isospin sum rules for photonuclear reactions are derived; they relate the strengths of the two isospin components of the giant resonance to the isoscalar, isovector, and isotensor radii of the nucleus. The connection between these radii and both the number and correlations of excess neutrons is discussed. A semiempirical formula for the fraction of the dipole strength in the $T+1$ giant resonance is derived and various experimental data are discussed in the light of the results obtained using this equation.

I. INTRODUCTION

Electric dipole isospin sum rules have already been discussed in connection with photonuclear physics by several authors.[1-5] In the following, we give a derivation of three sum rules, only two of which are linearly independent. These sum rules relate the strengths of the two isospin components of the electric dipole giant resonance to the isoscalar, isovector, and isotensor radii of the nucleus.

Section III contains a discussion of the properties of the three radii and their connection with the number of nucleons and their correlations. In Sec. IV a semiempirical formula is derived that gives the fraction of the dipole strength in the $T+1$ giant resonance taking into account center-of-mass correlations. Finally, Sec. V contains a short discussion of the connection between the present work and the existing experimental data.

II. DERIVATION OF THE SUM RULES

The electric dipole operator in the long-wavelength approximation can be expressed as

$$\vec{D}^z=\sum_{i=1}^{A}\vec{r}_i t_i^z, \tag{2.1}$$

where t_i^z has eigenvalues $+\frac{1}{2}$ (neutrons) and $-\frac{1}{2}$ (protons) and $\vec{r}_i$ is the center-of-mass coordinate of the ith nucleon. This operator will induce electric dipole transitions in the target nucleus from the ground state $|TT_z\rangle$ to the giant-resonance states characterized by $\langle TT_z|$ and $\langle T+1\,T_z|$. Since the ground states of stable nuclei have $T=T_z=\frac{1}{2}(N-Z)$ and the giant-resonance excitations are characterized by $\Delta T_z=0$, formal treatment of these transitions takes on a special simplicity. Here we consider the so-called bremsstrahlung-weighted cross section which is proportional to the sum of the squares of the transition matrix elements:

$$\sigma_{-1}\equiv\int\frac{\sigma}{E}dE=\frac{4\pi^2e^2}{3\hbar c}\sum_k\langle 0|\vec{D}^z|k\rangle\langle k|\vec{D}^z|0\rangle, \tag{2.2}$$

where 0 and k represent the ground state and excited states, respectively. Performing all indicated integrations and spin sums except those pertaining to isospin, we can rewrite Eq. (2.2) as

$$\sigma_{-1}=\sum_{T'}\sigma_{-1}(T'),$$

where

$$\sigma_{-1}(T')=\tfrac{4}{3}\pi^2\alpha\,|\langle T'T|\vec{D}^z|TT\rangle|^2 \tag{2.3}$$

is the total electric dipole strength integrated over energy and populating excited states having isospin T'.

Three sum rules can be derived relating the integrals $\sigma_{-1}(T)$ and $\sigma_{-1}(T+1)$ to the mean square isoscalar, isovector, and isotensor radii. These sum rules follow directly from the formula expressing the reduced matrix element of the tensor product of two vector operators as an expansion in products of reduced matrix elements of the individual vector operators, e.g., Eq. (15.15) of Fano and Racah,[6] or Eq. (7.1.1) of Edmonds.[7] For our problem it has the simple form

$$\langle T\|[\vec{D}\times\vec{D}]^{[\nu]}\|T\rangle=(2\nu+1)^{1/2}(-1)^{2T+\nu}\sum_{T'}\begin{Bmatrix}1&1&\nu\\T&T&T'\end{Bmatrix}\times\langle T\|\vec{D}\|T'\rangle\langle T'\|\vec{D}\|T\rangle, \tag{2.4}$$

where $\vec{D}$ is a vector in isospin space, $\vec{D}=\sum_i\vec{r}_i\vec{t}_i$. The index ν is limited by the usual angular momentum coupling rules to the values 0, 1, or 2 corresponding to scalar, vector, and tensor vector, and isotensor radii as

$$\langle R_S^2\rangle = \langle TT_z|\sum_{ij}(\vec{r}_i\cdot\vec{r}_j)(\vec{t}_i\cdot\vec{t}_j)|TT_z\rangle,$$

$$\langle R_V^2\rangle = \frac{\langle TT_z|\sum_i r_i^2 t_i^z|TT_z\rangle}{2T_z}, \tag{2.13}$$

$$\langle R_T^2\rangle = \frac{\langle TT_z|\sum_{ij}(\vec{r}_i\cdot\vec{r}_j)(3t_i^z t_j^z - \vec{t}_i\cdot\vec{t}_j)|TT_z\rangle}{3T_z^2 - T(T+1)}.$$

Since we are interested in the excitations of the nuclear ground state, we set $T_z = T$, and thus we finally obtain

$$\sigma_{-1}(T) - T\sigma_{-1}(T+1) = \tfrac{2}{3}\pi^2\alpha[2T\langle R_V^2\rangle + T(2T-1)\langle R_T^2\rangle], \tag{2.14a}$$

$$\sigma_{-1} = \sigma_{-1}(T) + \sigma_{-1}(T+1) = \tfrac{4}{9}\pi^2\alpha[\langle R_S^2\rangle + T(2T-1)\langle R_T^2\rangle], \tag{2.14b}$$

$$\sigma_{-1}(T) + (2T+3)\sigma_{-1}(T+1) = \tfrac{4}{3}\pi^2\alpha(\langle R_S^2\rangle - 2T\langle R_V^2\rangle). \tag{2.14c}$$

Equation (2.14a) is the sum rule of O'Connell.[1] The factor of 2 multiplying the right-hand side is missing in his treatment because his operator, τ^z, has eigenvalues ±1 and the summation in his definition of $\langle R_T^2\rangle$ is restricted.

The Eqs. (2.14) are not linearly independent in that the first two may be combined to obtain the third. Another way of writing them is

$$\sigma_{-1}(T) = \frac{4\pi^2\alpha}{3(T+1)}[\tfrac{1}{3}T\langle R_S^2\rangle + T\langle R_V^2\rangle + \tfrac{1}{6}T(2T-1)(2T+3)\langle R_T^2\rangle], \tag{2.15}$$

$$\sigma_{-1}(T+1) = \frac{4\pi^2\alpha}{3(T+1)}[\tfrac{1}{3}\langle R_S^2\rangle - T\langle R_V^2\rangle - \tfrac{1}{6}T(2T-1)\langle R_T^2\rangle].$$

In fact, this is a convenient representation for discussing the relative magnitudes of the T and $T+1$ transitions.

III. DIPOLE OPERATOR AND THE RADII

The Eqs. (2.13) define the isoscalar, isovector, and isotensor radii as being proportional to the reduced matrix elements of operators that transform as a scalar, a vector, and a tensor in isospin space. In addition, the ground-state expectation value of the square of the dipole operator, $\langle\vec{D}^2\rangle$, can be related to the weighted sum of $\langle R_S^2\rangle$ and $\langle R_T^2\rangle$. This may be seen simply from its definition, Eq. (2.1):

$$\sigma_{-1} = \tfrac{4}{3}\pi^2\alpha\langle\vec{D}^2\rangle = \tfrac{4}{3}\pi^2\alpha\langle TT_z|\sum_{ij}(\vec{r}_i\cdot\vec{r}_j)t_i^z t_j^z|TT_z\rangle. \tag{3.1}$$

Since

$$(\vec{r}_i\cdot\vec{r}_j)t_i^z t_j^z = \tfrac{1}{3}(\vec{r}_i\cdot\vec{r}_j)(\vec{t}_i\cdot\vec{t}_j) + (\vec{r}_i\cdot\vec{r}_j)[t_i^z t_j^z - \tfrac{1}{3}(\vec{t}_i\cdot\vec{t}_j)], \tag{3.2}$$

$$\sigma_{-1} = \tfrac{4}{3}\pi^2\alpha[\tfrac{1}{3}\langle R_S^2\rangle + \tfrac{1}{3}T(2T-1)\langle R_T^2\rangle], \tag{3.3}$$

which is Eq. (2.14b). In the following we discuss some properties of $\langle\vec{D}^2\rangle$ and the radii.

Of these four quantities the isovector radius, $\langle R_V^2\rangle$, is by far the simplest, since it involves only a one-body operator. One has

$$2T_z\langle R_V^2\rangle = \langle TT_z|\sum_{i=1}^{A} r_i^2 t_i^z|TT_z\rangle = \tfrac{1}{2}[N\langle R_n^2\rangle - Z\langle R_p^2\rangle], \tag{3.4}$$

where $\langle R_n^2\rangle$ and $\langle R_p^2\rangle$ are the mean square radii of the neutron and proton distributions, respectively. The proton-distribution radius can be measured by elastic electron scattering and μ-capture experiments,[8] while the neutron-distribution radius can be inferred from optical-model analysis of nucleon-nucleus scattering.[9] Thus the isovector radius can be considered to be a known quantity. In the approximation that $\langle R_n^2\rangle = \langle R_p^2\rangle$, the quantity $2T\langle R_V^2\rangle$, which appears in the sum rules of Eqs. (2.14), is simply $T\langle R_p^2\rangle$. In this sense it is proportional to the number of excess neutrons and the mean square nuclear-charge radius corrected for finite proton size.

The remaining three quantities, $\langle\vec{D}^2\rangle$, $\langle R_S^2\rangle$, and $\langle R_T^2\rangle$, are much more complicated. They involve the spatial correlations between pairs of nucleons, $\sum_{ij}(\vec{r}_i\cdot\vec{r}_j)$. Three separate factors affect this term: the center of mass, the Pauli principle, and the two-nucleon correlations resulting from the nucleon-nucleon potential itself (especially the repulsive core).

The center-of-mass correlations are small except in the lightest nuclei. Their effect on the total σ_{-1} may be seen by examining $\langle\vec{D}^2\rangle$, which may be written as

$$\langle\vec{D}^2\rangle = \langle 0|\sum_{ij}(\vec{r}_i\cdot\vec{r}_j)t_i^z t_j^z|0\rangle = \tfrac{1}{4}\langle 0|\sum_{i=1}^{A} r_i^2|0\rangle + \langle 0|\sum_{i\neq j}(\vec{r}_i\cdot\vec{r}_j)t_i^z t_j^z|0\rangle, \tag{3.5}$$

where $|0\rangle$ stands for the ground state. Defining the mean square matter radius to be

$$R_m^2 = \frac{1}{A}\langle 0|\sum_{i=1}^{A} r_i^2|0\rangle, \tag{3.6}$$

$$\langle\vec{D}^2\rangle = \tfrac{1}{4}AR_m^2 + \langle 0|\sum_{i\neq j}(\vec{r}_i\cdot\vec{r}_j)t_i^z t_j^z|0\rangle. \tag{3.7}$$

couplings. Making the substitutions:

$$\langle T\|\vec{D}\|T'\rangle\langle T'\|\vec{D}\|T\rangle=(-1)^{T+1-T'}|\langle T'\|\vec{D}\|T\rangle|^2, \tag{2.5}$$

we obtain

$$\langle T\|[\vec{D}\times\vec{D}]^{[\nu]}\|T\rangle=(2\nu+1)^{1/2}\sum_{T'}(-1)^{\nu+1-T-T'}\begin{Bmatrix}1 & 1 & \nu\\ T & T & T'\end{Bmatrix}|\langle T'\|\vec{D}\|T\rangle|^2. \tag{2.6}$$

Inserting explicit formulas for the 6-j symbols, we obtain the three relationships:

$$\langle T\|[\vec{D}\times\vec{D}]^{[0]}\|T\rangle=\frac{1}{[3(2T+1)]^{1/2}}[|\langle T-1\|\vec{D}\|T\rangle|^2+|\langle T\|\vec{D}\|T\rangle|^2+|\langle T+1\|\vec{D}\|T\rangle|^2],$$

$$\langle T\|[\vec{D}\times\vec{D}]^{[1]}\|T\rangle=\frac{1}{[T(2T+1)(2T+2)]^{1/2}}[(T+1)|\langle T-1\|\vec{D}\|T\rangle|^2+|\langle T\|\vec{D}\|T\rangle|^2-T|\langle T+1\|\vec{D}\|T\rangle|^2],$$

$$\langle T\|[\vec{D}\times\vec{D}]^{[2]}\|T\rangle=\frac{1}{[6(2T-1)(2T)(2T+1)(2T+2)(2T+3)]^{1/2}}$$
$$\times[2(T+1)(2T+3)|\langle T-1\|\vec{D}\|T\rangle|^2-2(2T-1)(2T+3)|\langle T\|\vec{D}\|T\rangle|^2+2T(2T-1)|\langle T+1\|\vec{D}\|T\rangle|^2]. \tag{2.7}$$

The bremsstrahlung-weighted cross sections, $\sigma_{-1}(T')$, may be expressed in terms of the reduced matrix elements, $\langle T'\|\vec{D}\|T\rangle$:

$$\sigma_{-1}(T-1)=0,$$
$$\sigma_{-1}(T)=\tfrac{4}{3}\pi^2\alpha|\langle T\|\vec{D}\|T\rangle|^2\frac{T}{(T+1)(2T+1)}, \tag{2.8}$$
$$\sigma_{-1}(T+1)=\tfrac{4}{3}\pi^2\alpha|\langle T+1\|\vec{D}\|T\rangle|^2\frac{1}{(T+1)(2T+3)}.$$

The quantity $\sigma_{-1}(T-1)$ vanishes because the vector-coupling coefficient multiplying $\langle T-1\|\vec{D}\|T\rangle$ is zero. We therefore eliminate $\langle T-1\|\vec{D}\|T\rangle$ from each of the three separate pairs of equations in Eq. (2.7), we replace $\langle T\|\vec{D}\|T\rangle$ and $\langle T+1\|\vec{D}\|T\rangle$ by $\sigma_{-1}(T)$ and $\sigma_{-1}(T+1)$ using the relations described in Eq. (2.8), and we substitute for the reduced matrix elements $\langle T\|[\vec{D}\times\vec{D}]^{[\nu]}\|T\rangle$ their respective components $\langle TT|[\vec{D}\times\vec{D}]^{[\nu]}_0|TT\rangle$:

$$\langle TT|[\vec{D}\times\vec{D}]^{[0]}_0|TT\rangle=\langle T\|[\vec{D}\times\vec{D}]^{[0]}\|T\rangle\frac{1}{(2T+1)^{1/2}},$$

$$\langle TT|[\vec{D}\times\vec{D}]^{[1]}_0|TT\rangle=\langle T\|[\vec{D}\times\vec{D}]^{[1]}\|T\rangle\frac{T}{[T(T+1)(2T+1)]^{1/2}}, \tag{2.9}$$

$$\langle TT|[\vec{D}\times\vec{D}]^{[2]}_0|TT\rangle=\langle T\|[\vec{D}\times\vec{D}]^{[2]}\|T\rangle\times\frac{2T(2T-1)}{[(2T-1)(2T)(2T+1)(2T+2)(2T+3)]^{1/2}}.$$

Finally, we obtain

$$\sigma_{-1}(T)-T\sigma_{-1}(T+1)=\tfrac{2}{3}\pi^2\alpha\{\sqrt{2}\langle TT|[\vec{D}\times\vec{D}]^{[1]}_0|TT\rangle-\sqrt{6}\langle TT|[\vec{D}\times\vec{D}]^{[2]}_0|TT\rangle\},$$

$$\sigma_{-1}(T)+\sigma_{-1}(T+1)=\tfrac{4}{9}\pi^2\alpha\{\sqrt{3}\langle TT|[\vec{D}\times\vec{D}]^{[0]}_0|TT\rangle-\sqrt{6}\langle TT|[\vec{D}\times\vec{D}]^{[2]}_0|TT\rangle\},$$

$$\sigma_{-1}(T)+(2T+3)\sigma_{-1}(T+1)=\tfrac{4}{3}\pi^2\alpha\{\sqrt{3}\langle TT|[\vec{D}\times\vec{D}]^{[0]}_0|TT\rangle-\sqrt{2}\langle TT|[\vec{D}\times\vec{D}]^{[1]}_0|TT\rangle\}. \tag{2.10}$$

The components $\langle TT|[\vec{D}\times\vec{D}]^{[\nu]}_0|TT\rangle$ may be evaluated through the use of Eq. (5.1.5) of Edmonds[7]:

$$[\vec{D}\times\vec{D}]^{[0]}_0=\frac{1}{\sqrt{3}}\sum_{ij}(\vec{r}_i\cdot\vec{r}_j)(\vec{t}_i\cdot\vec{t}_j),$$
$$[\vec{D}\times\vec{D}]^{[1]}_0=\frac{1}{\sqrt{2}}\sum_i r_i^2 t_i^z, \tag{2.11}$$
$$[\vec{D}\times\vec{D}]^{[2]}_0=\frac{1}{\sqrt{6}}\sum_{ij}(\vec{r}_i\cdot\vec{r}_j)(\vec{t}_i\cdot\vec{t}_j-3t_i^z t_j^z).$$

The phases appearing here stem from the use of the contrastandard convention given by Eq. (5.15) of Ref. 6 so that

$$t_i^+t_j^-+t_i^-t_j^+=\vec{t}_i\cdot\vec{t}_j+t_i^0t_j^0. \tag{2.12}$$

Next, we define the mean squared isoscalar, iso-

From the definition of the center of mass one has

$$\langle 0|\sum_{i=1}^{A}\vec{r}_i|0\rangle^2=0\,; \tag{3.8}$$

therefore, we can write

$$\langle 0|\sum_{ij}(\vec{r}_i\cdot\vec{r}_j)|0\rangle=\langle 0|\sum_{i=1}^{A}r_i^2|0\rangle+\langle 0|\sum_{i\neq j}(\vec{r}_i\cdot\vec{r}_j)|0\rangle=0\,, \tag{3.9}$$

so that

$$\langle 0|\sum_{i\neq j}(\vec{r}_i\cdot\vec{r}_j)|0\rangle=-AR_m^{\,2}\,. \tag{3.10}$$

Now for the 1s-shell nuclei, we can assume that the wave function is spatially symmetric with respect to all pairs of nucleons. Then the double sum over the $\frac{1}{2}A(A-1)$ pairs gives

$$\langle 0|\sum_{i\neq j}(\vec{r}_i\cdot\vec{r}_j)|0\rangle=A(A-1)\langle 0|(\vec{r}_1\cdot\vec{r}_2)|0\rangle=-AR_m^{\,2}\,. \tag{3.11}$$

Thus we have

$$\begin{aligned}\langle\vec{D}^2\rangle &=\tfrac{1}{4}AR_m^{\,2}-\frac{1}{A-1}R_m^{\,2}\langle 0|\sum_{i\neq j}t_i^z t_j^z|0\rangle\\ &=\tfrac{1}{4}R_m^{\,2}\left[A-\frac{Z(Z-1)+N(N-1)-2NZ}{A-1}\right]\\ &=\frac{NZ}{A-1}R_m^{\,2}\,,\end{aligned} \tag{3.12}$$

such that we obtain

$$\sigma_{-1}=\frac{4\pi^2\alpha}{3}\,\frac{NZ}{A-1}R_m^{\,2}\,. \tag{3.13}$$

This is the result of Foldy.[10] Therefore, the inclusion of center-of-mass correlations makes a correction of the order of $1/A$ with respect to the expression[11] in which it is neglected:

$$\sigma_{-1}=\frac{4\pi^2\alpha}{3}\,\frac{NZ}{A}R_m^{\,2}\,. \tag{3.14}$$

For nuclei beyond the 1s shell Fallieros and Goulard[5] have estimated in an oscillator model the magnitude of $\langle R_T^{\,2}\rangle$ resulting from center-of-mass correlations. We use their results in the next section to compute the relative strength of the $T+1$ giant resonance.

The contribution of Pauli correlations depends on the number of pairs of excess neutrons moving in orbits of opposite parity and differing by one unit of angular momentum j. Since the real nuclei have no such pairs, this effect is identically zero. We ignore here the contribution from two-nucleon force.

IV. NUMERICAL ESTIMATES

Since $\langle R_S^{\,2}\rangle$ is difficult to interpret and $\langle R_T^{\,2}\rangle$ is small, we turn to Eq. (2.14a) for an estimate of the magnitude of the $T+1$ giant resonance. Combining it with the identity

$$\sigma_{-1}=\sigma_{-1}(T)+\sigma_{-1}(T+1)\,, \tag{4.1}$$

we obtain

$$\frac{\sigma_{-1}(T+1)}{\sigma_{-1}}=\frac{1}{T+1}\left[1-\frac{4\pi^2\alpha}{3}\,\frac{T\langle R_V^{\,2}\rangle}{\sigma_{-1}}(1-\eta)\right], \tag{4.2}$$

where

$$\eta=-\frac{T(2T-1)\langle R_T^{\,2}\rangle}{2T\langle R_V^{\,2}\rangle}\,. \tag{4.3}$$

The ratio $\sigma_{-1}(T+1)/\sigma_{-1}$ is dominated by the geometrical factor $1/(T+1)$. The expression in the square brackets determines the amount by which the upper giant resonance is decreased by dynamical effects. The major part is proportional to the number of excess neutrons and their mean square radii, $T\langle R_V^{\,2}\rangle$. Of lesser importance is the correction term $(1-\eta)$, which measures the strength put back into the upper resonance as a result of correlations. On the basis of the center-of-mass correlations estimates of Fallieros and Goulard[5] we find that $\langle R_T^{\,2}\rangle$ is small and negative and that η is given by

$$\eta=\frac{(2T-1)A^{2/3}}{3NZ}\,. \tag{4.4}$$

This is a small correction never exceeding 5%.

In order to evaluate $\sigma_{-1}(T+1)/\sigma_{-1}$, we need an estimate for σ_{-1} to insert in the right-hand side of Eq. (4.2). To obtain this normalization factor we take $\sigma_{-1}(T+1)$ for ^{208}Pb to be zero; this is certainly very nearly true.[12] Then

$$\sigma_{-1}=\tfrac{4}{3}\pi^2\alpha T\langle R_V^{\,2}\rangle(1-\eta)\,. \tag{4.5}$$

Assuming that the mean square neutron- and proton-distribution radii are the same,

$$2T\langle R_V^{\,2}\rangle=T\langle R_p^{\,2}\rangle\,. \tag{4.6}$$

We approximate the proton radius by the charge radius of Hofstadter[13]:

$$\langle R_p^{\,2}\rangle^{1/2}=0.82A^{1/3}+0.58\text{ fm}\,. \tag{4.7}$$

For ^{208}Pb ($T=22$) Eq. (4.4) gives $\eta=0.049$ and we find

$$\sigma_{-1}=299\text{ mb}=0.244A^{4/3}\text{ mb}\,. \tag{4.8}$$

Here it has been assumed that σ_{-1} is proportional to $A^{4/3}$. This dependence is a consequence of the harmonic-oscillator model[14] and is consistent with the experiments. An $A^{5/3}$ dependence is also consistent with the experiments for $A>80$, but this difference is too small to affect any of the arguments made in the following. Inserting the numerical value given in Eq. (4.8) and combining all constants we finally obtain

$$\frac{\sigma_{-1}(T+1)}{\sigma_{-1}} = \frac{1}{T+1}\left[1 - \frac{1.97\,T\langle R_p{}^2\rangle}{A^{4/3}}(1-\eta)\right]. \qquad (4.9)$$

In Fig. 1 we plot the experimental values of σ_{-1} as a function of A where the upper limit of the integration is 30 MeV. The data for $A < 40$ are from the total-absorption experiment of Wyckoff *et al.*,[15] and data for the heavy elements are from the neutron-production experiments performed at Livermore.[16–19] Based on these data the constant of proportionality lies between the extreme values of 0.15 to 0.20. However, it is well known that there is electric dipole absorption at energies in excess of 30 MeV; the somewhat larger value $0.244A^{4/3}$ just obtained for ^{208}Pb presumably reflects this phenomenon.

Table I lists some numerical values obtained from Eq. (4.9) for various nuclei often discussed in the literature. These results show that the $T+1$ giant resonance is almost insignificant for nuclei having $A > 100$.

The results presented here reproduce the same trend with A as that obtained in the computation of Deague, Muirhead, and Spicer,[20] although our actual magnitudes are somewhat smaller. Approximate agreement also exists between our results for the even isotopes of nickel and those derived by Macfarlane[21] in a more detailed calculation.

Our estimates were obtained by assuming that the mean squared radius of the excess neutrons could be approximated by $\langle R_p{}^2\rangle$. Since the second term in Eq. (4.9) depends on the ratio of $\langle R_p{}^2\rangle$ for the nucleus under consideration to that for ^{208}Pb, the error involved cannot be very large. According to the three-fluid model of Mohan, Danos, and Biedenharn[22] the mean squared radius of the excess neutrons in ^{208}Pb is 13% smaller than $\langle R_p{}^2\rangle$. In that model our results would be in error by 1 or 2%.

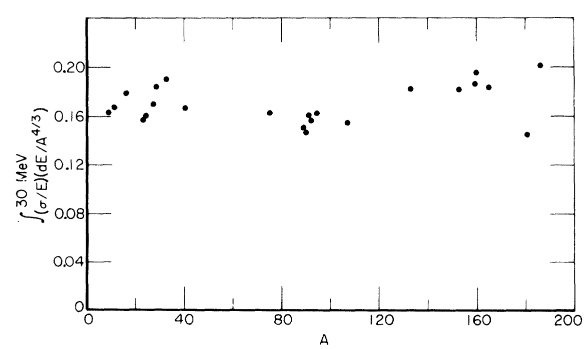

FIG. 1. The ratio $\int^{30\text{MeV}}(\sigma/E)(dE/A^{4/3})$ as a function of A. The data for the light elements come from the total-photon-absorption experiment of Ref. 15. Those for the heavy elements come from neutron-yield cross sections measured at Livermore, Refs. 16–19.

V. APPLICATIONS

The $T=\frac{1}{2}$ nuclei offer the greatest opportunity for observing the isospin splitting of the giant resonance. For these nuclei the geometric factor actually favors the $T+1$ resonance. In addition, the magnitude of the $T+1$ resonance is not affected by correlations. Thus, the two resonances have comparable strength. Various particle-hole calculations[23–25] describing the giant-resonance T splitting for $T=\frac{1}{2}$, p-shell nuclei show that the transitions which comprise the two giant resonances are somewhat interlaced. We should, therefore, not expect to observe a spectacular separation. This is borne out by the existing experiments[26–28] on ^{3}He, ^{11}B, ^{13}C, and ^{15}N.

The experiments[1] on ^{3}He and ^{13}C are consistent with the sum rule of Eq. (2.14a). A new measurement[29] of the three-body breakup of ^{3}He yields a value extrapolated to high energies of $\sigma_{-1} = 1.0$ mb. Calculating[30] the isovector and matter radii of ^{3}He from the electron scattering data, one can use Eqs. (2.14a) and (2.14b) to compute $\sigma_{-1}(\frac{1}{2}) = 1.34$ mb and $\sigma_{-1}(\frac{3}{2}) = 1.06$ mb. Since only three-body breakup can contribute to $\sigma_{-1}(\frac{3}{2})$, the result of Berman, Fultz, and Yergin is seen to exhaust the $T=\frac{3}{2}$ sum; i.e., there is no $T=\frac{1}{2}$ three-body breakup.

A new, high-resolution electron scattering experiment[31] shows clearly the isospin splitting of the ^{13}C giant resonance. The areas in the two energy regions of interest are again consistent with the sum-rule prediction.

TABLE I. Numerical evaluation of Eq. (4.9). The values given here for ^{3}He and ^{13}C were obtained using the actual radii (Ref. 30) instead of Eq. (4.7).

Nucleus	T	$\sigma_{-1}(T+1)/\sigma_{-1}$	$\sigma_{-1}(T+1)/\sigma_{-1}(T)$
^{3}He	$\frac{1}{2}$	0.44	0.79
^{13}C	$\frac{1}{2}$	0.55	1.22
^{26}Mg	1	0.39	0.64
^{48}Ti	2	0.24	0.32
^{52}Cr	2	0.25	0.33
^{64}Zn	2	0.26	0.35
^{58}Ni	1	0.44	0.79
^{60}Ni	2	0.25	0.33
^{62}Ni	3	0.16	0.19
^{64}Ni	4	0.11	0.12
^{74}Ge	5	0.08	0.09
^{90}Zr	5	0.10	0.11
^{108}Pd	8	0.07	0.08
^{112}Cd	8	0.05	0.05
^{120}Sn	10	0.03	0.03
^{208}Pb	22	0	0

A recent experiment[32] on inelastic electron scattering from the $T=1$ nucleus ^{26}Mg shows a spectacular splitting of the giant resonance into two bumps. This has been attributed to isospin splitting. Even though the idea is supported by the photoneutron spectra,[33] this phenomenon cannot be entirely an isospin effect; the higher-energy component contains most of the strength, whereas the upper limit for the ratio $\sigma_{-1}(T+1)/\sigma_{-1}$ predicted here is only 0.39. Indeed, the appreciable $(\gamma, 2n)$ cross section,[34] which coincides almost exactly with the upper resonance, can only proceed through $T=1$ states in ^{26}Mg up to an excitation energy of 28 MeV. Since the integrated cross section for the upper resonance is twice that of the lower, it is much more appealing to think that the splitting results mainly from a large intrinsic deformation of the nuclear ground state.

Various structures[35, 36] on the high-energy side of the giant resonances of the medium and heavy nuclei have been observed and discussed. The hydrodynamic model[37] predicts that the electric quadrupole giant resonance should lie at 1.6 times the energy of the electric dipole giant resonance and that its integrated absorption cross section should amount to 8% of that of the $E1$ giant resonance. Positive support for $E2$ absorption comes from the well-known fact that the high-energy protons resulting from photon absorption at these energies are peaked forward of 90°, suggesting an interference between $E1$ and $E2$ absorption. The $T+1$ giant resonance, on the other hand, is apparently[38, 39] ~$60(T+1)/A$ MeV above the main giant resonance, and an estimate of its strength is given in the present paper. Using these signatures we look at the data for some medium and heavy nuclei for evidence of the giant-resonance isospin splitting.

A recent work[40] on photon scattering, which reflects the total photon absorption cross section, identifies a structure 3–5 MeV above the giant resonance for a group of nuclei in the range $48 \leq A \leq 120$. The angular distribution of the scattered radiation excludes the possibility that it is electric quadrupole for $A<100$, and in every case the observed intensity is consistent with the value given in Table I. This is substantial evidence for its being the $T+1$ giant resonance.

The scattering experiment certainly lends credence to earlier conjectures concerning bumps observed in the neutron-production cross sections of nickel, zirconium, and tin. Min[41] has already pointed out that the large difference in the magnitudes of the (γ, n) cross sections of ^{58}Ni and ^{60}Ni is certainly an isospin effect. It is tempting to identify structures[42, 43] between 20 and 23 MeV in the neutron-production cross sections, as well as in the ^{59}Co(p, γ_0) cross section[44] with the $T+1$ giant resonance, but it is not possible to make any statement concerning the strength, since the (γ, p) cross section is surely very important.

The neutron-production cross sections[17] for the nuclei near $A=90$ contain a bump at 21 MeV which may be the $T+1$ giant resonance. The ^{89}Y(p, γ_0) cross section[45] also has a peak at this energy. This is the appropriate energy for the $T+1$ resonance and is too low to be the $E2$ giant resonance. Including a contribution for the (γ, p) process, Berman *et al.*[17] estimate that the $T+1$ component may represent as much as 16% of the total integrated cross section. Our result $10\times21/16\sim13\%$ is consistent with this.

In tin the energies of the $T+1$ giant resonance and the $E2$ giant resonance actually coincide near 25 MeV. Because the experimental integrated cross section[46] represented by the observed structure is ~12% of the giant-resonance integrated cross section, it must be predominantly an $E2$ phenomenon. The value of 3% for $\sigma_{-1}(T+1)/\sigma_{-1}$ predicted by Eq. (4.9) corresponds to only ~3% of the integrated absorption cross section.

Cook, Morrison, and Schamber[47] have studied the photodisintegration of ^{64}Zn and wish to identify the (γ, pn) cross section with the $T+1$ giant resonance. This cross section is 7.7 MeV above the main giant resonance and has a strength that is 20% of the latter. If the $T+1$ giant resonances for ^{48}Ti, ^{52}Cr, and ^{58}Ni and ^{60}Ni are really located near 22 MeV, it would be surprising for the $T+1$ giant resonance for ^{64}Zn to be at 25 MeV. Their value of 0.2 for $\sigma_{-1}(T+1)/\sigma_{-1}(T)$ is probably not inconsistent with the value 0.26 obtained here, since the unmeasured (γ, p) cross section must make an appreciable contribution.

In conclusion, we repeat our remark from the previous section: The $T+1$ giant resonance is theoretically almost insignificant for all nuclei having $A>100$. In addition, we would like to emphasize that by its very nature the giant-resonance isospin splitting must remain speculative, because there are so few instances in which the isospin of the giant-resonance states can be specified. Examples are the ^{3}He$(\gamma, d)p$ reaction,[48] the ^{10}B$(p, \gamma_0)^{11}$C reaction,[27] the ^{12}C$(p, \gamma_0)^{13}$N reaction,[28] and the ^{26}Mg$(\gamma, 2n)^{24}$Mg reaction[34] for $E_\gamma < 28$ MeV. In almost all other cases the isospin of the giant-resonance states is inferred.

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