Double Positron-Electron Pair Creation by Photons: An Experimental Limit at $E_\gamma = 6.13$ MeV*

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A search has been made for double positron-electron pair creation by photons of 6.13 MeV. γ rays from the reaction $^{19}F(p,\alpha)^{16}O$ were incident on Ge(Li) detectors of either 15- or 4-cm^3 volume and the pulse-height spectra were examined for the presence of a four-escape peak. Such a peak was, in fact, observed using the 15-cm^3 detector, but is accounted for by secondary effects, namely, the production of a second pair by bremsstrahlung from the first pair and the direct production of a second pair by the members of the first pair. These secondary effects are evaluated in the appendixes. An upper limit on the cross section for the genuine production of two pairs (σ_2) in one photon-nucleus encounter at 6.13 MeV relative to the formation of a single pair (σ_1), is represented by the experimentally determined ratio $\sigma_2/\sigma_1 = -(2\pm5)\times10^{-5}$; this is consistent with the theoretical expectation of order $(\alpha/\pi)^2$.

I. INTRODUCTION

Quantum electrodynamics is commonly regarded as a closed book, at least so far as the first few orders of expansion in powers of α/π are concerned. The two great exemplars are the Lamb shift and the electron-muon anomalous g factor; the importance of the second term in the α/π expansion is well established. The third term, of order $(\alpha/\pi)^2$ relative to the first, is clearly demanded by the g-2 data and is becoming a matter of significant concern on whose comparison with experiment, assuming the previous terms to be correct, matters such as hadronic contributions and/or a finite length in the fundamental theory begin to turn. But while such matters of great weight hang on the correctness of the theoretical higher-order corrections, and although we have no significant reason for doubting the validity of the theory at these levels, it remains true that direct experimental verification of the theoretical correctness of higher-order processes is extremely sparse.

In this paper we direct attention to the possibility of measuring one such higher-order process, namely, double positron-electron pair creation by photons; that is to say, the simultaneous production of two positron-electron pairs in one photon-nucleus encounter, rather than the usual single pair. Single pair creation involves three photon vertices and so the cross section is of order[1] $r_0^2(\alpha/\pi)$ where $r_0 = e^2/mc^2$. To produce two pairs in the single act another virtual photon line must be introduced and so this process is of higher order than the ordinary single pair creation by $(\alpha/\pi)^2$; i.e., the cross section is presumably of order[2] $r_0^2(\alpha/\pi)^3$. We may note in passing that another higher-order process, namely, double Compton scattering in which one photon impinges on an electron and emerges as two, is lower in cross section than the single process only by the single order α/π, since in that case the extra photon line is real and is attached only at one end. Double pair creation may therefore be represented as a test of quantum electrodynamics that is of higher order than double Compton scattering; the analog in pair creation to double Compton scattering would more nearly be radiative pair creation in which the final state contains a pair and a photon, a process lower in cross section than ordinary pair creation by α/π.

No general calculation of double pair creation has yet been made but an estimate is available in the extreme relativistic limit[3]:

$$\sigma_2 = \tfrac{1}{9} Z^2 r_0^2 \alpha(\alpha/\pi)^2 L \ln^2 k\,,$$

where σ_2 is the total cross section for the process $\gamma + Z \to Z + 2e^+ + 2e^-$, k is the incident photon energy in natural units (used throughout this paper unless otherwise stated or implied) and

$$L = \tfrac{175}{4}\zeta(3) - \tfrac{19}{2} = 43.09\cdots\,.$$

[$\zeta(3) = \sum_1^\infty n^{-3} = 1.202\cdots$.] We compare this cross section with that for single pair creation, σ_1, which, for consistency, we also take in the extreme relativistic limit, similarly dropping all constant terms:

$$\sigma_1 = \tfrac{28}{9} Z^2 r_0^2 \alpha \ln k .$$

We may therefore anticipate, very roughly:

$$\sigma_2/\sigma_1 \approx (L/28)(\alpha/\pi)^2 \ln k , \qquad (1)$$

which shows the expected factor $(\alpha/\pi)^2$.

Our present attempt to measure σ_2 has been made using γ rays of 6.13 MeV for which expression (1) predicts $\sigma_2/\sigma_1 \approx 2\times 10^{-5}$, but with unknown reliability. This attempt has been unsuccessful: We find $\sigma_2/\sigma_1 = -(2\pm 5)\times 10^{-5}$, a result that does not conflict with the crude expectation of expression (1), but is clearly less than Heitler's expectation of $\sigma_2/\sigma_1 \approx \alpha/\pi \approx 2\times 10^{-3}$. It would be of great interest to know the accurate theoretical value for σ_2 at the low γ-ray energies at which measurements may be possible; this is evidently a matter of some complexity.

Our interest in double pair creation was first stimulated by a curious effect that we observed when an experiment[4] using the reaction $^{15}N(p,\gamma)^{16}O$ was being carried out to measure the γ-ray branching of the $J^\pi = 1^-$ state of ^{16}O at 13.09 MeV to the $J^\pi = 0^+$ state at 6.05 MeV. The method used three NaI(Tl) crystals, one of 5×6 in. to detect the 7.04-MeV cascade transition, and the others of 3×3 in. on opposite sides of the target to detect the positron-annihilation radiation from the decay of the 6.05-MeV state. Figure 1 displays the resulting coincidence spectrum.

The bombardment of ^{15}N with protons also results in the strong competing reaction $^{15}N(p,\alpha)^{12}C$ which produces 4.44-MeV γ rays from the first excited state of ^{12}C. We expected to see the characteristic 4.44-MeV lines as a random-coincidence background, and we were therefore surprised to observe that the 4.44 (2) peak in Fig. 1 is much too intense to be accounted for as a random effect, that there is another peak at the 4.44 (3) position and the suggestion of a weak 4.44 (4) peak. It was apparent that these extra peaks must be due to real coincidence effects in which the 4.44-MeV γ rays produced two positron-electron pairs in the 5×6-in. crystal. This would result in time-coincident but spatially uncorrelated 0.511-MeV quanta emerging from the 5×6-in. crystal; one annihilation quantum from one positron was being detected in one 3×3-in. crystal, while another annihilation quantum from the other positron was being detected in the other 3×3-in. crystal. The question was then whether the double pair production was in the single quantum act discussed earlier or whether it was the result of some cascade process in which ordinary single pair creation was followed by production of a second pair by the electron or positron of the first pair, either directly in the course of their slowing down in the crystal or indirectly by their producing bremsstrahlung which gave rise to the second pair.

We have more recently carried out another experiment to display this effect. γ rays of 6.13 and 6.92 MeV, made through the reaction $^{19}F(p,\alpha)$-^{16}O, irradiated a conventional three-crystal pair spectrometer. Pulse-height windows were placed around the 0.511-MeV peaks from both side crystals so as to give the normal three-crystal display shown in the upper curve of Fig. 2. In order to detect the simultaneous emission of four annihilation quanta from the center crystal, two in each side crystal, another coincidence condition was set up with pulse-height windows centered at 1.02 MeV on both outputs and of a width appropriate to a full-energy-loss peak at that energy. The lower curve in Fig. 2 was recorded under these conditions during the same run as the upper curve. Clear peaks at 6.13 − 2.04 MeV and 6.92 − 2.04 MeV occur. When the center crystal was pulled back so that only half of its length lay between the two side crystals the ratio of intensities (four-escape to two-escape) was about 3 times smaller than in Fig. 2. This indicated that the two positrons responsible for the four-escape peaks were not being produced at a common point in the crystal but at different sites, a situation that would be consistent with the formation of the second pair by bremsstrahlung radiation produced by one member of the first pair.

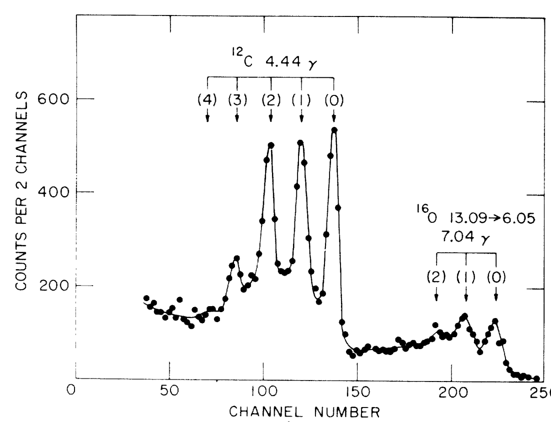

FIG. 1. Spectrum of γ rays from $^{15}N+p$ in triple coincidence with annihilation γ rays from the target region as obtained in the work of Ref. 4. The 4.44 (0) and 4.44 (1) peaks are due to random coincidences, whereas the enhanced 4.44 (2) peak, the 4.44 (3) peak, and a possible 4.44 (4) peak are attributed to the production of two pairs, the second following a bremsstrahlung process involving one of the electrons of the first pair.

The cascade mechanisms for double pair production are evaluated in the appendixes and can indeed account for the results displayed in Figs. 1 and 2.

It became evident from the experiments described above that a sensitive search for double pair production arising from single photon-nucleus encounters should make use of a detecting system that minimizes the formation and detection of pairs in secondary processes such as bremsstrahlung. A more ideal detector for this purpose would be one consisting of a thin slab of low-Z material. Following the formation of a positron-electron pair by an incident γ ray, the production of secondary radiation in such a detector would be small by virtue of the low Z, and any secondary radiation would have a high probability of escaping before producing another pair. A thin detector would also enhance the escape of the annihilation quanta associated with the positron. Thus, the predominant yield arising from single pair creation would lie in the two-escape peak, while in double pair creation most of the yield would be in the four-escape peak. Considerations of peak-to-background ratio and statistical accuracy suggested the use of a high-resolution γ-ray detector having a small volume, a planar configuration, excellent gain stability, and high counting-rate capability without loss of pulse-height resolution. While a detector fabricated from Si would be more favorable from the point of view of minimizing bremsstrahlung production, the detectors available to us were of the Ge(Li) type.

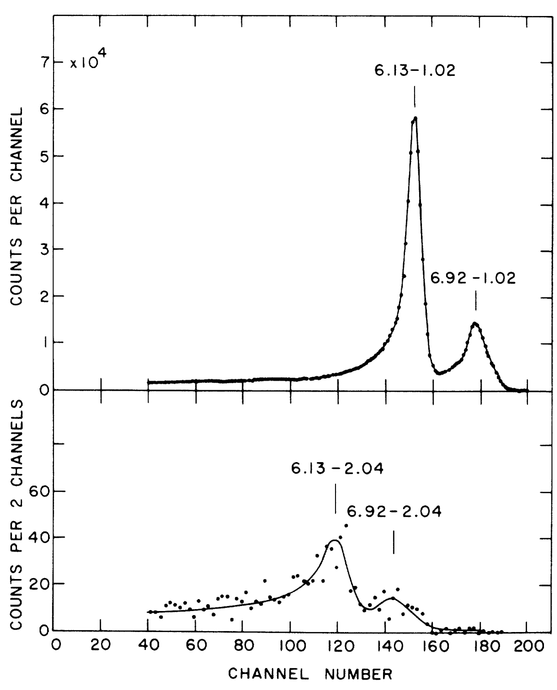

FIG. 2. Three-crystal pair-spectrometer measurement of the 6.13- and 6.92-MeV γ rays from the reaction $^{19}F(p,\alpha)^{16}O$. The upper curve is the ordinary spectrum taken with each pulse-height window on the side crystals encompassing the 0.51-MeV peak. The lower curve was obtained (during the same run) with a window on each side-crystal output centered at 1.02 MeV so as to detect the simultaneous emission of four annihilation quanta from the center crystal.

II. EXPERIMENTAL METHODS AND RESULTS

The source requirement for experiments on double pair creation is to produce a high-energy γ-ray line that is not Doppler broadened and which can be developed with sufficient intensity and without disturbing background radiations. These conditions are met by the 6.13-MeV γ rays from the $J^{\pi}=3^-$ state of ^{16}O ($\tau_m = 24$ psec) that may be made in abundance through the reaction $^{19}F(p,\alpha)^{16}O$. In the bombardment of a thick CaF_2 target with 0.7-MeV protons the 6.13-MeV γ rays dominate the spectrum; the only other high-energy γ ray is of 6.92 MeV, but its intensity is only 8% as great as that of the 6.13-MeV γ ray; this line is Doppler broadened by about 200 keV ($\tau_m = 7$ fsec) and so will just contribute to the background. Most of our work was carried out using targets of CaF_2 ~3 mg/cm^2 thick evaporated on Au backings that were cooled with an air jet. Beam currents of up to 5 μA at $E_p = 0.7$ MeV were obtained from the 3.5-MeV Van de Graaff accelerator.

The Ge(Li) detectors were selected on the basis of volume, configuration, and pulse-height resolution. Initial work was done with a 15-cm^3 coaxial detector. This was placed several cm from the target, and a $\frac{1}{8}$-in.-thick brass absorber was placed in front of the detector to keep positron-electron pairs from the 6.05-MeV state of ^{16}O from entering the detector. The output of the detector preamplifier was fed to a Tennelec model No. TC203BLR amplifier and thence to a 16 384-channel pulse-height analyzer utilizing an 8192-channel analog-to-digital converter. A gain setting was used that placed the 6130-keV full-energy-loss peak at about channel 7000 such that the dispersion was 0.88 keV per channel. In all of the runs a Th^{228} source was located near the detector so as to provide 2614.5 (0) and 2614.5 (2) energy calibration lines in each spectrum.

Tests were first made to determine that counting rate above which there was a noticeable worsening of pulse-height resolution. Having established the optimum rate for achieving good statistical accuracy without loss of pulse-height resolution, the spectrum was stored in separate runs of 8- to 16-h duration. Most of the runs preserved good pulse-height resolution, but there were slight differences in the peak positions from one run to another, indicating a slow gain drift. The data of each run were therefore adjusted in the region expected for the 6130 (4) peak so as to result in the same position of this peak, to the nearest half channel. All runs were then added.

The final data from the 15-cm^3 detector included runs totaling 88 h. In the summed spectrum the 6130 (2) line contained 2.64×10^7 counts [full width at half maximum (FWHM) 6.5 channels or 5.8 keV, net height 4.06×10^6 counts per channel]. Its amplitude was 5.5 times the net amplitude of the 6130 (1) peak, and it was a factor of 40 above the Compton background level at the expected position of the 6130 (4) line. Figure 3(a) shows the region of the pulse-height spectrum containing the possible 6130 (4) peak where the Compton background level is ~100 000 counts per channel. The expected position of the peak was obtained from a polyfit to the peak positions of the lines at 2614.5 (2), 2614.5 (0), 6130.0 (2), 6130.0 (1), and 6130.0 (0).

In order to locate and put in evidence a possible peak in the data of Fig. 3(a), the following procedure was adopted. The data were systematically fitted to a polynomial (quadratic in channel number) plus a Gaussian whose width was that thought probable for the 6130 (4) peak, if it existed, and whose peak height, N counts per channel, was determined by a χ^2 minimization routine. This Gaussian test peak was moved along, channel by channel, so that for each channel of its centering an N value with an associated error was determined. The results are shown in Fig. 3(b), where a strong "N" signal is seen as the test peak crosses the position expected for the 6130 (4) peak. The errors in the points of Fig. 3(b) vary little from channel to channel and have a magnitude of about ±200 counts (standard deviation) as indicated on the figure. The significance of the N values as representing a genuine peak is shown in Fig. 3(c), where the probability that such N values could have arisen by chance is given, according to the usual χ^2 test, as a function of the centering channel number. We see that the peak at channel 4554 is unequivocally established as real, while the probability away from the peak fluctuates around the value 0.5, as is proper.

We have thus demonstrated the 6130 (4) peak at precisely its expected position. We may now go further and extract its width and area from the "N peak" of Fig. 3(b), which, as is there shown, is well represented by a Gaussian of FWHM = 9.0 ± 0.5 channels.

It is easy to show that if a Gaussian $e^{-x^2/2\sigma_1^2}$ is hunted for by the above-described χ^2 minimiza-

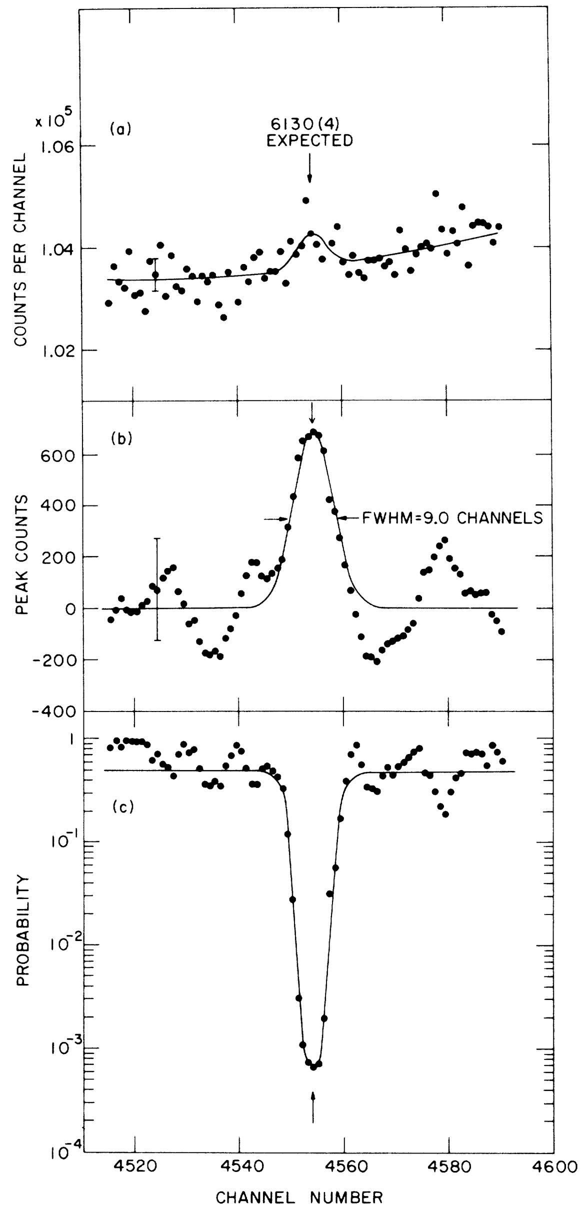

FIG. 3. Curve (a) shows the region of the expected double pair creation peak 6130 (4) seen with the 15-cm^3 Ge(Li) detector. The arrow shows the expected position of this peak. The full curve is the computed quadratic background plus Gaussian peak generated by the moving-Gaussian-test-peak procedure described in the text. As described in the text, this 6130 (4) peak is chiefly due to cascade production mechanisms, not to genuine double pair creation. Curve (b) shows the peak height (N counts per channel of the text) of the moving Gaussian test peak that, centered on successive channel numbers together with a quadratic background, minimizes χ^2 for each of them. Curve (c) shows, as a function of channel number, the probability that the associated N values of curve (b) might have arisen as statistical fluctuations. The arrow indicates the expected position of the 6130 (4) peak.

tion procedure using a Gaussian $N(\Delta)e^{-(x+\Delta)^2/2\sigma_2^2}$, then the resultant values of $N(\Delta)$ that give the χ^2 minimization as a function of the displacement Δ between the centers of the Gaussians form a third Gaussian:

$$N(\Delta)=\frac{\sqrt{2}\,\sigma_1}{\Sigma}\,e^{-\Delta^2/2\Sigma^2},$$

where $\Sigma^2=\sigma_1^2+\sigma_2^2$, so that the total number of counts in the Gaussian being sought is

$$\sqrt{2\pi}\,\sigma_1=\sqrt{\pi}\,N(0)\Sigma$$

and may be obtained from $N(\Delta)$, i.e., Fig. 3(b), without reference to σ_1. In this way we may say that the total number of counts in the 6130 (4) peak is $(4.7\pm1.4)\times10^3$. To complete the demonstration it is interesting to extract σ_1 from the FWHM of 9.0 ± 0.5 channels of Fig. 3(b). The FWHM expected for the 6130 (4) peak, based on the observed widths of the five calibration peaks listed above, is 6.1 channels; we find $\sigma_1=5.5\pm0.8$ channels.

Although this is thought to be the first time that a four-escape peak has been identified in the γ-ray spectrum from a Ge(Li) detector, the line is accounted for by secondary effects, as will be shown later, and not to the genuine double-pair-creation phenomenon being searched for. Since the calculations outlined in the appendixes and to be discussed in the next section indicated that the relative importance of secondary effects should decrease with decreasing detector thickness, the effort was shifted to detectors of smaller volume and of planar configuration.

Best results were obtained with a 4-cm^3 detector (an ORTEC detector 5 mm thick and 8 cm^2 in area) in runs totaling 63 h. In this case the ratio of the 6130 (2) peak to the Compton background at the 6130 (4) position was 32, not quite as favorable as the ratio using the 15-cm^3 detector, but the calculations (see appendixes) indicated that the 6130 (4) peak, resulting from secondary effects, should be a factor of about 3 lower, relative to the 6130 (2) peak, than for the 15-cm^3 detector. Also, because of the smaller volume of this detector, the ratio of the 6130 (2)/ 6130 (1) peak amplitudes was 18, compared with 5.5 for the 15-cm^3 detector. All of the procedures for taking and analyzing the data with the 4-cm^3 detector were similar to those described above for the work with the 15-cm^3 detector. In the summed data the 6130 (2) peak had a net amplitude of 1.72×10^6 counts per channel, and its net area was 1.19×10^7 counts. The computer fit to the region of the 6130 (4) line, carried out using the procedure described for the 15-cm^3 detector, gave a net peak amplitude of 170 ± 140 counts per channel, or an area of $(1.14\pm0.94)\times10^3$ counts for the peak.

III. ANALYSIS

In the case of the 15-cm^3 detector we observe a definite 6130 (4) double-pair line with an intensity of $(4.7\pm1.4)\times10^3$ counts, to be compared with the 6130 (2) single-pair line of 2.64×10^7 counts. The question is whether this double-pair signal is due to genuine double pair creation or whether it is due to cascade processes, presumably chiefly (real) bremsstrahlung from the initial pair, giving further pair creation, but also direct pair production by either electron of the initial pair. These two processes are calculated, following simplifying assumptions, in Appendix A. The estimate of the former is probably good to 50% but that of the latter has an error of unknown magnitude because the basic process has never been properly treated theoretically in the low-energy regime. In Appendix A the effects of the finite range of the electrons is ignored, but in practice our detectors are not large in relation to these ranges. This means that some of the $e^\pm$ and $2e^\pm$ events will fall out of their respective 6130 (2) and 6130 (4) peaks because not all the pair electrons will stop in the detectors. These effects are treated in Appendix B (for the initial pair and genuine double-pair cases) and in Appendix C (for the cascade processes). Finally, we must calculate the chance that, for single pair creation, both annihilation quanta escape from the detector and, for double pair creation, that all four annihilation quanta escape, since we are comparing the 6130 (2) and 6130 (4) peaks. This is done in Appendix D.

When all these factors are computed we predict that, owing to the two cascade processes, the 2.64×10^7 counts of the 6130 (2) peak should be accompanied by 4.2×10^3 counts in the 6130 (4) peak which we must compare with the $(4.7\pm1.4)\times10^3$ counts that we have found there.

It is then clear that the cascade processes are capable of explaining the experimental double-pair effect in the 15-cm^3 detector and that our estimates of those processes are not too bad (in this case the bremsstrahlung-mediated cascade process is computed to be about 3.5 times more important than the direct-production cascade process).

In the 4-cm^3 detector the bremsstrahlung effect is much reduced (see Appendix A) and the $2e^\pm$ signal is enhanced relative to the $e^\pm$ signal because of the smaller probability for the escape of the $2e^\pm$ electrons owing to their lower average energy (see Appendix B). Owing to the two cascade processes, we expect, according to the procedures detailed in the appendixes, the 1.19×10^7 6130 (2) counts to be accompanied by 1.47×10^3 counts in the 6130 (4) peak, to be compared with the $(1.14\pm0.94)\times10^3$ counts observed. The difference of $-(0.33\pm0.94)\times10^3$ counts we may associate with a genuine double-pair-creation effect and, after allowance for electron escape (Appendix B) and annihilation-photon escape (Appendix D) we find

$$\sigma_2/\sigma_1 = -(2\pm5)\times10^{-5}.$$

IV. DISCUSSION

Our value for σ_2/σ_1 at $E_\gamma = 6.13$ MeV is consistent with the crude theoretical expectation of 2×10^{-5} given in the Introduction, but goes no further than establishing that the effect is not of gross order greater than $(\alpha/\pi)^2$.

We note that our figure for σ_2/σ_1 is much less than the factor $\alpha/\pi\approx2\times10^{-3}$ suggested by Heitler.[2] We must therefore review the suggestion of Hooper and King[5] that they have observed double pair creation by the γ rays of the cosmic radiation. They apparently found by means of photographic plates that $\sigma_2/\sigma_1\approx$ (2 cases)/(1400 cases), which is consistent with α/π and so, under the influence of Heitler's expectation for the σ_2/σ_1 ratio, claimed the establishment of the phenomenon; this now must appear most unlikely. (The photon energies involved in the two apparent cases were about 220 and 2000 MeV. For these energies we expect, according to Eq. (1), $\sigma_2/\sigma_1\approx10(\alpha/\pi)^2\approx6\times10^{-5}$ so that, assuming the correctness of Eq. (1), the two apparent cases must be due either to some other phenomenon or to a most unlikely fluctuation.)

It would appear that our present technique cannot confidently be pushed much further, since we are already at the point of major reliance on the calculated cascade-production probabilities of which the direct cascade production is very poorly known theoretically. [In the 4-cm^3 detector the calculated direct cascade contribution to the 6130 (4) peak already exceeded the calculated bremsstrahlung contribution by a factor of 1.4.] As is apparent from Appendixes A and C the direct cascade production dominates rapidly as we press to thinner detectors so that the total calculated cascade effect must rapidly become rather unreliable. Also, as is seen from Appendix B, the containment problem becomes very severe for thinner detectors; the estimate of the containment probability becomes very unreliable when that probability becomes small, particularly for the genuine double-pair-creation process that we are trying to find and where we have little guidance as to the energy distributions. It would seem that with Ge detectors we are unlikely to be able to establish a genuine double-pair-creation cross section with reliability if it is indeed of the expected order (see Appendix A). With Si detectors the situation is improved, since the limiting direct cascade production goes as Z, a factor of 2.3 lower for Si than for Ge. For confident work, however, it would appear to be necessary actually to measure the direct production of pairs by electrons over the relevant energy range.

As mentioned in Appendix A, if other techniques employing low-Z moderators were to be used, limiting sensitivities in σ_2/σ_1 of a few times 10^{-6} should be attainable, and this should enable the genuine effect to be put in evidence. Alternatively, a visual technique such as a streamer chamber would permit a separation of genuine from cascade events.

ACKNOWLEDGMENTS

We are indebted to H. Kraner for fabricating the 15-cm^3 Ge(Li) detector used in this work, and to R. Chrien for the loan of the ORTEC 4-cm^3 detector.

APPENDIX A: ESTIMATES OF CASCADE PAIR PRODUCTION VIA BREMSSTRAHLUNG AND BY DIRECT PRODUCTION

Throughout the appendixes energies are measured in units of mc^2.

Bremsstrahlung

γ rays of energy $k_0>4$ are incident normally on a slab, thickness t, of a material of atomic weight A, atomic number Z, and density ρ. Electron pairs are produced; some of these generate further pairs by making bremsstrahlung which then enjoys a second pair-producing interaction. We want to calculate the over-all probability $P_{\pi b}(k_0, t)$ that an initial pair gives rise to a subsequent pair in this way. We carry out the calculation semi-analytically under a number of simplifying assumptions:

(i) The Born-approximation formulas for bremsstrahlung and pair production are used.
(ii) There is at most one interaction for the bremsstrahlung.
(iii) All processes propagate in the beam direction.
(iv) The electron range is ignored in relation to t.

It is difficult to estimate the errors that these approximations entrain, but they probably do not exceed 50%; the last approximation is removed in Appendix C. We carry out the calculation with γ rays of $k_0\approx12$ in mind and make approximations that optimize for this case; the results should have fair validity for somewhat lower energies also.

The first step is to consider the bremsstrahlung from an electron of (kinetic) energy T. Call $\sigma_k\,dk$ the cross section for emission of a photon in the energy range k to $k+dk$. The Born-approximation formula[1] in the range of importance for our present problem admits of the adequately accurate parametrization:

$$k\sigma_k \approx \bar{\phi}T^{0.29}(10.15 - 9.5f), \qquad \text{(A1)}$$

where $\bar{\phi} = Z^2 r_0^2 \alpha$, $\alpha = e^2/\hbar c$, $r_0 = e^2/mc^2$, $f = k/T$, as is shown in Fig. 4.

The electrons involved in the possible cascade process are all relativistic with a rate of energy loss in the relativistic minimum, so that we may write:

$$dT/dx = -G\rho N_0 \phi_0 Z/A, \qquad \text{(A2)}$$

$$\phi_0 = \tfrac{8}{3}\pi r_0^2,$$

N_0 = Avogadro's number,

and where G is a numerical constant of value about 15.

Pair production is described by the Racah formula. Figure 5 shows this formula (evaluated by Maximon[6]) over the range of interest to us here where it is well approximated by

$$\sigma_\pi \approx \bar{\phi}\,0.264(k - 2.74). \qquad \text{(A3)}$$

It is trivial to make the appropriate analytical convolution of these three expressions (A1), (A2), and (A3) to find the probability $B_{\pi b}(T_1)$ per unit distance that a pair will be produced immediately subsequent to the stopping of an electron of initial energy T_1 (i.e., before the bremsstrahlung is sensibly weakened by absorption).

$B_{\pi b}(T_1)$ must now be integrated over the spectrum $I(T_1)$ of electron-positron energy distribution in the initial-pair-production process. For this spectrum we use the accurate and convenient parametrization due to Hough.[7]

Convolution of $B_{\pi b}(T_1)$ with $I(T_1)$ now yields the probability $P_{\pi b}(k_0)$ per unit distance, immediately subsequent to an initial pair production, that another will follow via the cascade process; further integration through the thickness t of the detector slab, with allowance for the absorption both of the incident radiation and of the bremsstrahlung, yields the total secondary-pair-production probability by this process, $P_{\pi b}(k_0, t)$.

It is interesting to enquire into the limit set by this cascade process to the sensitivity of the present type of search for double pair creation. The detector slab must be of sufficient thickness to catch the pair electrons with reasonable efficiency. We define this as a thickness t_0 equal to the range of the most energetic electron (of energy $k_0 - 2 \approx k_0$):

$$t_0 = k_0 A/G N_0 \phi_0 \rho Z.$$

We then find, inserting numerical values for Ge and the γ rays of the present investigation, that $P_{\pi b}(k_0, t_0)$ is a little under 10^{-4}. It may, of course, be possible to *demonstrate* double pair creation by using slabs of thickness considerably less than t_0, but the problem of estimating the absolute cross section then becomes more difficult as is discussed in Appendix B.

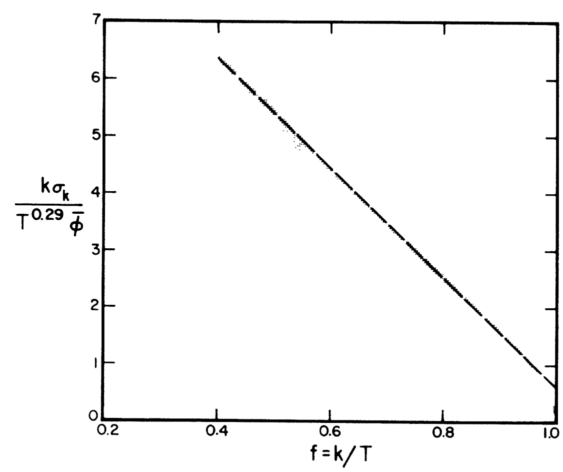

FIG. 4. The analytical approximation of Eq. (A1) to the Born-approximation bremsstrahlung formula is shown by the dashed line. The stippled area contains the exact expression over the range $T = 4$ to 10.

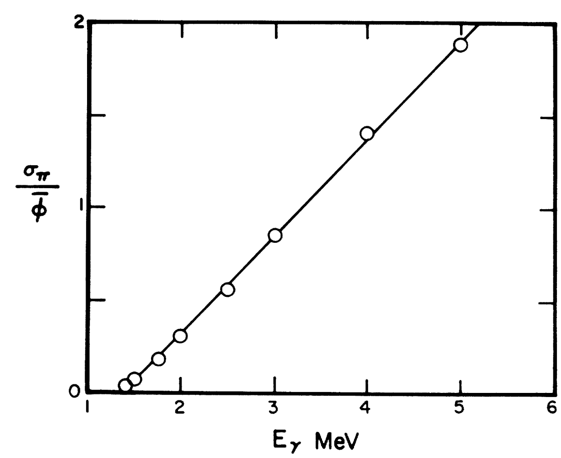

FIG. 5. The Maximon evaluation of the Racah formula for the total pair-production cross section σ_π is shown as circles. The straight line is the approximation given by Eq. (A3).

Direct Production

In addition to the cascade process via bremsstrahlung that we have just evaluated we must consider direct production of a second pair by one or other of the electrons of the first pair. This process is independent of the thickness t of the slab, and so will become the more important relative to the bremsstrahlung process as the slab gets thinner.

Unfortunately the theoretical cross section for this process is available only in the relativistic limit[8]:

$$\sigma_{\pi d} = \frac{28}{27\pi}\,\bar{\phi}\,\alpha \ln^3(T-1)\,.$$

Use of this formula will entrain considerable uncertainty at the electron energies of interest here. Using the notations and other procedures discussed above, the probability of direct pair production by the electrons of the initial pair is

$$P_{\pi d}(k_0) = \frac{7}{9\pi^2}\,\frac{Z\alpha^2}{G}\int_2^{k_0-2} F(T_1)I(T_1)\,dT_1\,,$$

where

$$F(\eta+1) = \eta(\ln^3\eta - 3\ln^2\eta + 6\ln\eta - 6) + 6\,.$$

It is now of interest to compare the direct and bremsstrahlung production processes:

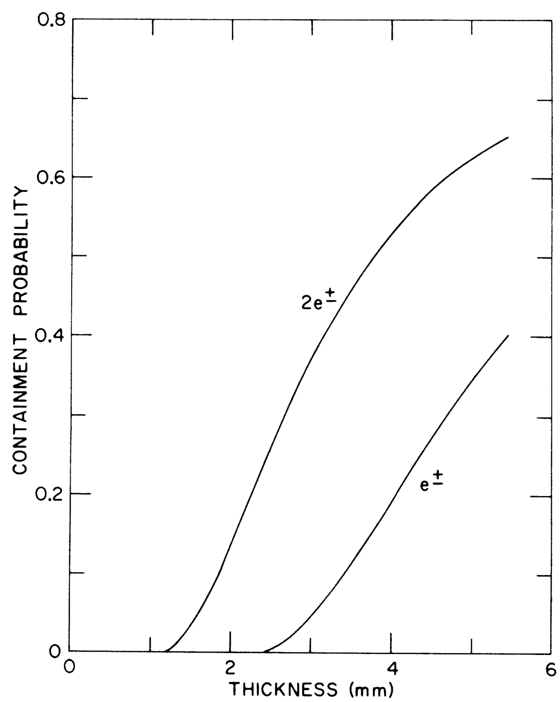

FIG. 6. Containment probabilities for $e^\pm$ and $2e^\pm$ production, respectively, for photons of $k_0=12$ incident normally on Ge detectors of thickness t mm.

$$R(k_0, t) = P_{\pi d}(k_0)/P_{\pi b}(k_0, t)\,.$$

It is also of interest to enquire into the value of $R(k_0, t)$ at the limiting slab thickness t_0 defined earlier. Inserting numerical values we find for Ge and 6.13-MeV γ rays

$$R(k_0, t_0) \approx 0.7\,.$$

The use of Ge is therefore not likely to be seriously inhibited by the lack of an accurate theoretical estimate of $\sigma_{\pi d}$. However, if other techniques, employing much lower-Z materials, were to be used, ignorance of $\sigma_{\pi d}$ could become the limiting factor. If lithium were to be used to moderate the pair electrons produced in a thin high-Z radiator we should have, within the considerable uncertainties of the present estimates, a limiting sensitivity of $P_{\pi d} \approx 6\times10^{-6}$ from the direct process for $E_\gamma = 6$ MeV. The corresponding number for the bremsstrahlung cascade process would be $P_{\pi b} \approx 8\times10^{-7}$.

APPENDIX B: EFFICIENCY OF THIN DETECTORS

Single Pair Production

If the pair-creation act takes place at a distance T_e from the back face of the detector, in terms of

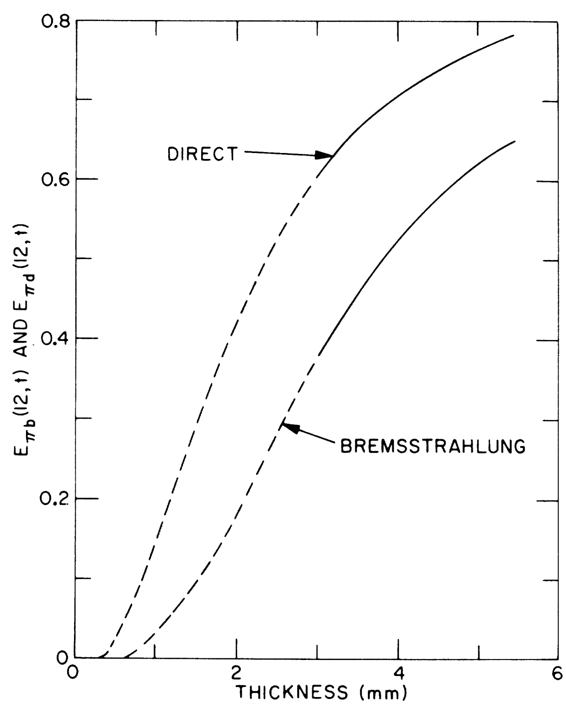

FIG. 7. The factors $E_{\pi b}(12, t)$ and $E_{\pi d}(12, t)$ by which the results of Appendix A for the cascade production of a second pair due to bremsstrahlung and direct production, respectively, must be multiplied on account of electron escape from a Ge detector of thickness t mm. The incident photon energy is $k_0=12$.

energy loss by a relativistic electron, the probability of containment $C_1(k_0, T_e)$ of the pair within the detector for normal incidence of the γ-ray beam of energy k_0 is, within the very crude approximation of neglecting electron scattering,

$$\begin{aligned} C_1(k_0, T_e) &= 1 \quad \text{for } k_0 - 2 < T_e \\ &= 0 \quad \text{for } \tfrac{1}{2}(k_0 - 2) > T_e \\ &= 2\int_{k_0 - T_e - 2}^{\frac{1}{2}k_0 - 1} I(T)\,dT \\ &\qquad\qquad \text{for } \tfrac{1}{2}(k - 2) < T_e < k_0 - 2\,. \end{aligned}$$

Integration of $C_1(k_0, T_e)$ through the detector gives the results that are shown in Fig. 6 for the γ rays of present interest, $k_0 = 12.0$, where t_e has been translated into mm of detector thickness for Ge.

Double Pair Production

We are hampered in our attempt to estimate the containment probability for double pair production by ignorance of the energy distribution among the four electrons. Suppose that the total energy k_0 is divided, W to one electron pair and $k_0 - W$ to the other, according to $P(W)$. We suppose, *faute de mieux,* that the energy distribution within each pair is given by the Hough parametrization; we also suppose, in keeping both with the principle of considering all events to propagate in the forward direction and also with our ignorance, that $P(W)$ is given simply by phase space in one dimension. The results are shown in Fig. 6 for $k_0 = 12.0$.

We must emphasize the crudeness of these containment calculations, due to the neglect of electron scattering. The actual containment will be somewhat better than estimated here. This same stricture applies to the loss estimates given in the following appendix.

APPENDIX C: BREMSSTRAHLUNG AND DIRECT-PRODUCTION EFFECTS IN THIN DETECTORS

In Appendix A we estimated the chance, $P_{\pi b}(k_0, t)$, that an initial act of pair production by a photon of energy k_0 incident normally upon a detector of thickness t should give rise, via bremsstrahlung from the electron or positron, to a further act of pair creation within the same detector, thus simulating a genuine act of double pair creation. We also estimated the chance, $P_{\pi d}(k_0)$, that the electrons of the initial pair should give rise to a further pair by the direct (bremsstrahlung-free) process in the course of their slowing down.

However, in order for such cascade processes to register in the detector as double-pair processes, all four electrons must be contained within the detector, and so we must estimate this further chance of containment just as, in Appendix B, we did for the cases of single and (genuine) double pair creation. We therefore seek to estimate the factors $E_{\pi b}(k_0, t)$ and $E_{\pi d}(k_0, t)$ by which $P_{\pi b}(k_0, t)$ and $P_{\pi d}(k_0)$, respectively, as estimated in Appendix A, must be multiplied on account of this possibility of escape.

As is clear from Appendix B, the use of detectors (of Ge) of $t \lesssim 3$ mm is unlikely to be profitable, and so in the present study we allow ourselves approximations that, although poor for very thin detectors, are reasonable for those of practicable thickness. These approximations are: For the bremsstrahlung case, that the bremsstrahlung arises at the point of origin of the initial pair (but that its subsequent interaction occurs with equal probability anywhere downstream of that point), and that we neglect the possibility of escape of the electron which underwent bremsstrahlung itself; for the direct production, that all three electrons concerned in the act of direct production are contained in the detector.

The results, which may also be used to correct, for electron loss, the results for thicker detectors given in Appendix A are given in Fig. 7.

APPENDIX D: ESCAPE OF ANNIHILATION QUANTA

What we observe are peaks at energies $k_0 - 2$ and $k_0 - 4$ corresponding to single and double pair creation. Before we can compare the number of events of both kinds we must correct for the fact that all (two or four) annihilation quanta involved must escape from the detector. This must be calculated, since the ratio of "one-escape" to "two-escape" peaks involves also the "peak-to-total" ratio of the annihilation quantum that interacts, and this cannot be determined experimentally, since the annihilation quanta originate inside the detector.

For a thin parallel-sided detector of thickness t and lateral dimension much greater than t the problem is simple and we have a double-escape probability (escape of both annihilation quanta from a single positron annihilation) of

$$\begin{aligned} \epsilon &= \int_1^\infty \frac{e^{-\mu t x}}{x^2}\,dx \\ &= E_2(\mu t)\,, \end{aligned}$$

which is given in standard tabulations.[9] ϵ does not depend on the depth within the detector at which absorption and annihilation take place. μ refers to photons of $k = 1$ ($\mu = 0.44\ \text{cm}^{-1}$ for Ge). Correction for finite lateral dimension is easily effected numerically. The problem is much more complicated for cylindrical detectors; we have handled it numerically in our present case.

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[1]W. Heitler, *The Quantum Theory of Radiation* (Oxford U. P., Oxford, England, 1954), 3rd ed.

[2]Reference 1 (p. 228) is in error in asserting that the cross section for double pair creation is less than that for single pair creation by only the factor α/π rather than by $(\alpha/\pi)^2$.

[3]V. G. Serbo, Zh. Eksperim. i Teor. Fiz – Pis'ma Redakt. 12, 50 (1970) [transl.: JETP Letters 12, 39 (1970)]. The value of the constant L in this expression is corrected from that in the original publication. We are grateful to Dr. M. Ram and to Dr. J. J. Sakurai for communications on this point.

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[6]L. C. Maximon, J. Res. Natl. Bur. Std. 72B, 79 (1968).

[7]P. V. C. Hough, Phys. Rev. 73, 266 (1948).

[8]The *ER* formula (see Ref. 1) has $T+1$ rather than $T-1$, but we use the latter to reproduce the threshold at $T=2$. The detailed calculations of G. Racah [Nuovo Cimento 14, 93 (1937)] are not useful here because they are not valid below $T \cong 50$; in our energy regime they show a cross section that increases as T decreases.

[9]E. g., *Handbook of Mathematical Functions*, edited by M. Abramowitz and I. A. Stegun, National Bureau of Standards Applied Mathematics Series, No. 55 (U. S. GPO, Washington, 1964), pp. 228 and 245ff.

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Theoretical Aspects of Quadrupole Perturbations of Time-Integrated Angular Correlations*

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Computations have been made in order to determine the optimum geometries for time-integrated angular-correlation experiments perturbed by static axially symmetric electric field gradients. Several new properties of the interaction are discussed, including planes of negative attenuation (enhancement), cases for which no interaction is allowed to occur, and new aspects of the concept of hard-core angular correlations.

I. INTRODUCTION

Although Alder, Schonberg, Heer, and Novey[1] published the theory of static quadrupole perturbations of angular correlations many years ago, not much attention has been given to this source of information about nuclear shapes. Experiments have been hindered because the techniques of preparing the appropriate single crystals and implanting them with radioactive impurity nuclei were not sufficiently developed. In recent years these experiments have become much more viable, since now single crystals of almost any material are commercially available, and the techniques of implantation using isotope separators or in-beam recoil after Coulomb excitation and other reactions are well known.[2] Further interest in performing static measurements of nuclear quadrupole moments has been stimulated by the recent controversial measurements of quadrupole moments using the dynamic-reorientation effect.[3] Comparison of results from the two methods could lead to new insights concerning the static or dynamic nature of nuclear deformations during Coulomb excitation.

It is the purpose of this paper to report on theoretical aspects of the static, axially symmetric quadrupole interaction which bring to light some hitherto unknown properties of the interaction and which correct several errors and ambiguities that exist in the literature. As has previously been reported,[4-6] extensive computational studies have been carried out in order to determine the optimum geometries for time-integrated angular-correlation experiments. Although the interpretation of time-differential experiments is less ambiguous, the majority of nuclear states must be studied by the integrated technique because of their short lifetimes. Therefore we concentrate here on time-integrated angular correlations. Experimental data that test some of the ideas and geometries proposed below will be presented in a forthcoming paper.[7]

II. BASIC THEORY

A. Internal Field Gradients

The internal electric field gradient (EFG) acting upon an impurity nucleus in a crystalline lattice

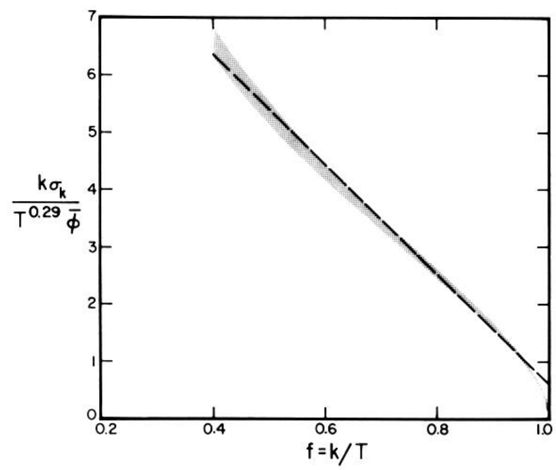

FIG. 4. The analytical approximation of Eq. (A1) to the Born-approximation bremsstrahlung formula is shown by the dashed line. The stippled area contains the exact expression over the range $T = 4$ to 10.