[15]We are greatly indebted to J. Hüfner who kindly provided us with a copy of his muon-cascade program, which served as the basis for all of our cascade calculations.

[16]The capture schedule for ^{6}Li was obtained using capture broadening of 150 eV and 2×10^{-8} eV for the $1s$ and $3d$ states, respectively. For ^{12}C, the corresponding values were 3250 eV and 1.5×10^{-5}eV, respectively. [See R. J. Harris *et al.*, Phys. Rev. Letters 20, 505 (1968).] The results given in Table II are insensitive to uncertainties in these assumed values.

[17]H. Hilscher *et al.* (Ref. 9) apparently assumed that mesons are captured from only the $2p$ or $1s$ state, whereas Bistirlich *et al.* (Ref. 10) apparently assumed that the $1s$ state population is equal to the $2p$-$1s$ x-ray yield. Both of these assumptions are incorrect, as indicated in Tables I and II.

[18]Conforming with the authors of Ref. 4 we have assumed the *ad hoc* nucleon-nucleon correlation parameter of the potential to be unity.

[19]We are grateful to D. K. Anderson, D. A. Jenkins, and R. J. Powers for the calculated predictions and for their permission to present these results.

[20]D. K. Anderson, private communication.

PHYSICAL REVIEW C VOLUME 5, NUMBER 3 MARCH 1972

Tensor Correlations in Nuclear Matter: Three-Body Effects*

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(Received 4 November 1971)

The Jastrow method has been extended to the treatment of tensor forces by the incorporation of suitable spin and isospin dependence into the correlation operators. Here the three-body contributions to the factor-cluster expansion of the energy expectation value are evaluated for nuclear matter, supplementing the evaluation of the one- and two-body contributions already reported in the first paper of this series. Complete numerical results are presented for three semirealistic hard-core potentials containing differing mixtures of central and tensor components. The three-body corrections result in a distinct improvement of the saturation behavior of the approximate energy per particle. Nevertheless, these calculations should be regarded as still exploratory rather than final in the sense that the most intelligent (i.e., a suitably restricted) choice of radial dependence has not yet been determined for the correlation functions.

I. INTRODUCTION

It is commonplace to remark on the complexity of the nucleon-nucleon force: its strong repulsion at short distances, its state dependence (i.e., its dependence on spin, isospin, and total angular momentum), and especially its strong tensor component. These complexities have received careful attention in the well-established Brueckner-Bethe-Goldstone (BBG) theory[1,2] of the ground-state energy of nuclear matter. The literature offers numerical investigations of the two-hole-line diagrams (as defined in BBG theory) for a variety of realistic and semirealistic nuclear potentials.[3] There are also calculations of the three-hole-line diagrams[4] based on the Bethe-Faddeev scheme.[5] However, among the studies of this type, only those of Dahlblom[6] and Day[7] (who also considers four-hole-line diagrams) include the effect of the tensor component in an adequate manner, and not just in terms of an "equivalent" central potential.

In order to study the effects of tensor correlations in nuclear matter – or finite nuclei – we have recently proposed an approach exploiting the original Jastrow idea, in the enlarged context of the method of correlated basis functions.[8] We have begun with a relatively simple (and surely limited) realization of the general scheme. It is our immediate purpose to make explicit the merits and deficiencies of this simple picture. Afterwards we can still proceed, if necessary, to more complicated structures; i.e., we can incorporate additional physical information (e.g., impose conservation of certain "sum rules" in each cluster order,[9] examine higher-order perturbative corrections with respect to the non-orthogonal basis of correlated wave functions,[10] change to a different ansatz for the Jastrow correlation factor, invoke a more highly summed cluster expansion,[11] etc.).

It may be well to stress at this point that the Jastrow approach is not limited merely to the rough evaluation of expectation values – to a "pair approximation" useful only for making qualitative estimates in nuclear physics. There exist highly informative and/or successful applications of this approach, in the enlarged context of the method of correlated basis functions, to a wide range of quantum many-body systems including liquid and solid He^3 and He^4, the charged Bose gas, α matter, hypernuclear matter, and the neutron gas.[12] Furthermore, the first steps toward establishing a clear connection with the Bethe-Faddeev scheme and other diagrammatic methods are currently being made.[13]

In this paper we continue with the exploration of the scheme outlined by Ristig, Ter Louw, and Clark[8] (hereafter called RTC). Our aim is the complete evaluation of the three-body-cluster contributions to the ground-state energy of symmetrical nuclear matter, including tensor effects. This is accomplished according to the same pattern as was established for the two-body effects in RTC. One can identify a "smallness parameter" in our theory[14] analogous to the "smallness parameter"[2] of the BBG or compact-cluster theory. In the latter method the smallness parameter is prescribed *a priori*, and is considered to be small enough that three- or more- hole-line contributions may be discarded. However, in our case the smallness parameter is not fixed by *a priori* considerations, with the consequence that (at least) the three-body clusters must necessarily be included in the analysis of the ground-state energy, even if, in the final issue, the numerical contribution of these clusters is small. I.e., the three-body clusters (and possibly the higher-order clusters as well) play an important role in the determination of the correlations and thus the smallness parameter. We *cannot* be satisfied merely with an estimate of the three-body clusters to check rapidity of convergence of the cluster expansion. In this connection, one should note that it is in the three-body clusters that nondiagonal two-body matrix elements of the two-body correlation factor first come into play.

In Sec. II we begin with the most general expression (from RTC) for the three-body-cluster contribution for a system with any number A of particles, and derive a simpler expression in the case of uniform extended matter by expanding in powers of $1/A$. The most general spin- and isospin-dependent, local three-body correlation operator is written down in Sec. III. Necessary and desired properties of this operator are discussed, and mathematically the simplest choice of correlation (the "asymptotic form") is selected for further investigations. For this choice the correlated A-body wave function can be written in a simple form. In Sec. IV we turn to the evaluation of the three-body clusters for the ground-state energy of nuclear matter for the class of noncentral potentials studied in RTC. We express all contributions conveniently either in terms of matrix elements of an effective three-body potential, or in terms of matrix elements of a product of the effective two-body potential of RTC and an effective two-body correlation operator. (Contributions of the latter type are the analogs of the two-body combination terms of Day.[7]) In Sec. V we choose three test potentials containing differing triplet-even-state tensor components (the same potentials as were employed in RTC) and calculate, without any approximation, their three-body contributions to the energy per particle. The preliminary character of the results is pointed out in Sec. VI, and some questions are raised in regard to how more judicious choices of correlation functions should be made. Until these questions are answered, it is not possible to come to any firm conclusion on the relative merits of the present approach compared to the BBG method or any other developed theory.[15]

II. THREE-BODY-CLUSTER CONTRIBUTION FOR UNIFORM EXTENDED MATTER

We begin with the Hamiltonian

$$H = T + V = \sum_{i=1}^{A} t(i) + \sum_{i<j}^{A} v(ij)\,, \tag{1}$$

and the general correlated wave function

$$\Psi = F\Phi\,. \tag{2}$$

Here $t(i)$ and $v(ij)$ are kinetic and potential operators for particles i, j; Φ is an independent-particle-model ground-state wave function (the unperturbed wave function), and F an appropriate Hermitian correlation operator.

The expectation value of the Hamiltonian H with respect to the trial function Ψ is given by the factor-cluster expansion derived in RTC:

$$E = E_0 + (\Delta E)_1 + (\Delta E)_2 + (\Delta E)_3 + \cdots + (\Delta E)_A\,. \tag{3}$$

The indices 1, 2, 3, ... label one-body, two-body, three-body, ... correlation corrections to the unperturbed energy E_0. For uniform extended matter, E_0 is the kinetic energy of the free particles, $(\Delta E)_1$ vanishes, and $(\Delta E)_2$ is the two-body-cluster contribution as evaluated in RTC. The three-body

contribution (per particle) is given by

$$\frac{1}{A}(\Delta E)_3 = \frac{1}{A}\sum_{i<j<k}\left[\frac{1}{I_{ijk}}\frac{\partial I_{ijk}}{\partial\beta} - \left(\frac{1}{I_{ij}}\frac{\partial I_{ij}}{\partial\beta}+\text{cycl.}\right)+\left(\frac{\partial I_i}{\partial\beta}+\text{cycl.}\right)\right]\Big|_{\beta=0}, \tag{4}$$

with

$$\begin{aligned} I_i &= 1, \\ I_{ij} &= \langle ij|F(12)\{\exp\beta[t(1)+t(2)+v(12)-\epsilon_i-\epsilon_j]\}F(12)|ij-ji\rangle, \\ I_{ijk} &= \langle ijk|F(123)\{\exp\beta[t(1)+t(2)+t(3)+v(12)+v(13)+v(23)-\epsilon_i-\epsilon_j-\epsilon_k]\}F(123)P_A|ijk\rangle. \end{aligned} \tag{5}$$

Here the $|i\rangle$ are spin- and isospin-dependent plane waves; $F(12)$, $F(123)$, ... are Hermitian two-, three-, ... body correlation operators; ϵ_i is the kinetic energy of plane-wave orbital i; P_A is the antisymmetrization operator $P_A|ijk\rangle = |ijk\rangle - |ikj\rangle + \cdots$; and the summation extends over all i, j, k, which are elements of the set $\{i, \ldots, i_A\}$ of A orbitals occupied in Φ.

Expanding the above three-body contribution in powers of $1/A$ (for the uniform, extended medium), we obtain

$$\frac{1}{A}(\Delta E)_3 = \frac{1}{A}\sum_{i<j<k}\left(\frac{\partial I_{ijk}}{\partial\beta}-\frac{\partial I_{ij}}{\partial\beta}-\frac{\partial I_{ik}}{\partial\beta}-\frac{\partial I_{jk}}{\partial\beta}\right)\Big|_{\beta=0} - \frac{1}{A}\sum_{i<j<k}\left\{[(I_{ik}-1)+(I_{jk}-1)]\frac{\partial I_{ij}}{\partial\beta}+\text{cycl.}\right\}\Big|_{\beta=0} + O\left(\frac{1}{A}\right). \tag{6}$$

Now we take $A\to\infty$ and thus drop all contributions to the energy per particle $O(1/A)$. Our result for $(1/A)(\Delta E)_3$ may conveniently be written in an explicit and suggestive manner upon introduction of the following two- and three-body operators:

$$W_2(12) = \tfrac{1}{2}[F(12),[t(1)+t(2),F(12)]]+F(12)v(12)F(12), \tag{7}$$

$$W_3(123) = \tfrac{1}{2}[F(123),[t(3),F(123)]]+F(123)v(12)F(123)-W_2+\text{cycl.} \tag{8}$$

We arrive at

$$\frac{1}{A}(\Delta E)_3 = \frac{1}{3!}\frac{1}{A}\sum_{ijk}\langle ijk|W_3(123)P_A|ijk\rangle - \frac{1}{A}\sum_{ijk}\langle ik|[F^2(12)-1]|ik-ki\rangle\langle ij|W_2(12)|ij-ji\rangle. \tag{9}$$

The second sum is built entirely out of diagonal matrix elements for the two-body subsystems alone. We therefore call them two-body combination terms (cf. Day's terminology[7]).

In order to obtain an expression for the energy per particle correct to first order in the "smallness parameter" ξ appropriate to the assumed correlation operators, it would presumably be necessary—as in the ordinary Jastrow treatment, where $F(1\cdots n)=\prod_{1\le i<j\le n} f(r_{ij})$, and this parameter is $\xi=\rho\int[f^2(r)-1]d\vec{r}$—to extract a double exchange contribution of first order in ξ from the *four-body* contribution.[9,14] Here we shall adhere strictly to a classification of contributions in terms of the number of bodies or (equivalently) the number of hole orbitals involved, rather than in terms of order in ξ or (equivalently) the number of *independent* hole orbitals. Application of the latter classification scheme must await a careful analysis of the four-body correction $(\Delta E)_4/A$ in the factor-cluster expansion.

III. CORRELATION OPERATOR FOR TENSOR FORCES

Consider a potential with a state-independent hard core and with spin- and isospin-dependent central and tensor components outside the hard core:

$$\begin{aligned} v(12) &= +\infty, \quad r=r_{12}\le c, \\ &= {}^1V_C^+(r)P_\tau^3P^1+{}^1V_C^-(r)P_\tau^1P^1 \\ &\quad +{}^3V_C^+(r)P_\tau^1P^3+{}^3V_C^-(r)P_\tau^3P^3 \\ &\quad +V_T^+(r)P_\tau^1S_{12}+V_T^-(r)P_\tau^3S_{12}, \quad r>c. \end{aligned} \tag{10}$$

Note that the parity projectors $P^\pm$ of RTC have been replaced by the appropriate isospin projectors P_τ^1, P_τ^3.

We seek to construct a correlation operator $F(123)$ for three particles, or $F(1\cdots A)$ for the complete nucleon system, which is suited to the above nucleon-nucleon potential. It will be helpful first to review the nature of the choice of two-body

correlation operators $F(12)$ that was made in RTC: there we set down the most general local but spin- and isospin-dependent two-body operator with certain *necessary* and *desired* properties. Namely, we took $F(12)$ to be a scalar, invariant under rotation, inversion, and translation, and to be totally symmetric in the particle labels 1, 2. Further, we required $F(12)$ to satisfy the hard-core boundary condition (it must vanish for configurations with $r_{12} \leq c$) and the asymptotic boundary condition implied by the cluster property (it must go to a constant, taken unity, for $r_{12} \to \infty$). In addition, we chose $F(12)$ to be positive everywhere except inside a hard core or at the origin, where it may vanish, in accordance with the desire to avoid a large kinetic energy.

The detailed calculations of RTC showed then that the major contribution to the energy $(\Delta E)_2$ may be obtained using a restricted set of correlation operators of the type

$$F(12) = f(r) + f(r)[g(r) - 1] P_\tau^1 P^3 (1 - Q), \tag{11}$$

where $Q = (\vec{S} \cdot \vec{r})^2 / r^2$, $\vec{S}$ being the total spin operator. (Again we have written $P_\tau^1 P^3$ in place of $P^+ P^3$ of RTC. Note that the second term comes into play only in triplet-even states.) We recognize that $f(r)$ [or rather, the deviation of $f(r)$ from unity] describes the "over-all" correlation, due mainly to the hard core and of rather short range, while $g(r)$ [or rather, the deviation of $g(r)$ from unity] describes the special correlations induced by the tensor force, which are of rather long range, but comparatively weak. The actual choices to be made for f and g, discussed in RTC, are unimportant at this point.

To ensure that $F(12)$ of Eq. (11) is positive definite, all we need is f, $g > 0$. For we can introduce

$$\begin{aligned} G(12) &= 1 + (g^{1/2} - 1) P_\tau^1 P^3 (1 - Q), \\ \Omega(12) &= (\ln g) P_\tau^1 P^3 (1 - Q), \end{aligned} \tag{12}$$

and write

$$\begin{aligned} F(12) &= f(r) G^2(12), \\ F(12) &= f(r) e^{-\Omega(12)}, \end{aligned} \tag{13}$$

either expression being manifestly positive definite under the given condition.

Now we are prepared to determine the most general local three-body correlation operator $F(123)$ subject to the same conditions as we have imposed on $F(12)$. The correlation operator $F(123)$ may depend on the three spin pseudovectors whose components are $\sigma_i(\alpha)$ ($\alpha = 1, 2, 3$ being the particle label), the isospin operators $\tau(\alpha)$ [in the form of projectors $P_\tau^{1,3}(\alpha, \beta)$], and the space coordinates. The algebra of the spin operators [in particular, the property that any product of more than three $\sigma_i(\alpha)$ can be reduced] implies that $F(123)$ may be cast in the form

$$\begin{aligned} F(123) = f(r_{12}) f(r_{13}) f(r_{23}) \\ \times \Big[T + \sum_{\alpha=1}^{3} T_i(\alpha) \sigma_i(\alpha) + \sum_{\alpha<\beta}^{3} T_{ij}(\alpha\beta) \sigma_i(\alpha) \sigma_j(\beta) \\ + T_{ijk}(123) \sigma_i(1) \sigma_j(2) \sigma_k(3) \Big], \end{aligned} \tag{14}$$

where summation over the repeated indices i, j, k is understood. This form calls to mind the symmetry decomposition of the three-body wave function underlying the Faddeev method. The various T's are functions of relative separations in coordinate space (i.e., functions of $\vec{r}_{12}, \vec{r}_{13}, \vec{r}_{12} \times \vec{r}_{13}$) and of the isospin projectors, with rather evident symmetry properties with respect to the particle labels $\alpha = 1, 2, 3$ to ensure that $F(123)$ is totally symmetric. Further, for $F(123)$ to be a scalar, T must be a scalar, the $T_i(\alpha)$ must form for each α the components of a pseudovector, the $T_{ij}(\alpha\beta)$ must form for each $\alpha\beta$ a tensor of second rank, and the $T_{ijk}(123)$ must form a third-rank pseudotensor. To fulfill the hard-core boundary condition in the three-particle configuration space, we have attached a Jastrow correlation factor to each term. The cluster property (asymptotic boundary condition) for three particles reads

$$F(123) \to F(12) \quad \text{for } 3 \to \infty. \tag{15}$$

This property holds only if

$$T_i(\alpha), \quad T_{ijk}(123) \to 0 \quad \text{for } 3 \to \infty. \tag{16}$$

I.e., T_i and T_{ijk} must introduce pure three-body correlations. Adopting the special form Eq. (11) for the operator $F(12)$, the remaining T's in Eq. (14) must obey the asymptotic boundary conditions

$$\begin{aligned} T &\to 1 + \tfrac{1}{4}[g(r_{12}) - 1] P_\tau^1(12), \\ T_{ij}(13), \; T_{ij}(23) &\to 0, \\ T_{ij}(12) &\to \tfrac{1}{4}[g(r_{12}) - 1] P_\tau^1(12) \\ &\quad \times \Big[\delta_{ij} - \frac{2}{r_{12}^2} x_i(12) x_j(12) \Big], \end{aligned} \tag{17}$$

for $3 \to \infty$, with $\vec{r}(12) = (x_1(12), x_2(12), x_3(12))$, $\vec{r}^2(12) = r_{12}^2$.

In contrast to expression (11) for the two-body correlation operator, expression (14) is not an orthogonal decomposition in spin-isospin space. Consequently, the condition that Eq. (14) be positive definite cannot be incorporated explicitly in a straightforward manner without first accomplishing a difficult diagonalization, although sufficient conditions are easily found.

Hopefully, it will not be necessary to exploit the

full flexibility of Eq. (14). Special choices of the T functions should, of course, be physically well motivated and, if possible, mathematically simple.

In the absence of precedent we shall adopt the simplest reasonable form. We extend Eq. (16) to all configurations and assume for the surviving terms in Eq. (14) the asymptotic forms corresponding to Eq. (17) (invoke cyclic permutation of 123 as necessary):

$$T = 1 + \tfrac{1}{4}[\,g(r_{12}) - 1]\,P_\tau^1(12) + \text{cycl.}\,, \tag{18}$$

$$T_{ij}(12) = \tfrac{1}{4}[\,g(r_{12}) - 1]\,P_\tau^1(12)\left[\delta_{ij} - \frac{2}{r_{12}^{\,2}}x_i(12)x_j(12)\right].$$

In the notation of Eq. (11), we thus arrive at the following three-body correlation operator:

$$F(123) = f(r_{12})f(r_{13})f(r_{23})\{1 + [\,g(r_{12}) - 1]\,P_\tau^1(12) \times P^3(12)[\,1 - Q(12)] + \text{cycl.}\}\,, \tag{19}$$

which we shall refer to as the asymptotic choice. Additional freedom allowing an improved description of three-body effects may be introduced by a simple generalization of Eq. (19),

$$F(123) = f(r_{12})f(r_{13})f(r_{23}) \times\{1 + h(r_{23})h(r_{13})[\,g(r_{12}) - 1]\,P_\tau^1(12) \times P^3(12)[\,1 - Q(12)] + \text{cycl.}\}\,, \tag{20}$$

where $h(r)$ is an independent function with the property $h(r) \to 1$ for $r \to \infty$. This generalization would (for example) provide a better chance to make $F(123)$ positive definite for given $g(r)$, or to meet other desired conditions such as a three-body Pauli condition.

Other special forms of $F(123)$ could be constructed by comparison with the Bethe-Faddeev scheme,[5,13] or (from a more mathematical point of view) by generalization of Eq. (13) to three particles via

$$\begin{aligned} &f(r) \to f(r_{12})f(r_{13})f(r_{23})\,,\\ &G(12) \to G(12) + G(13) + G(23) - 2\,,\\ &\Omega(12) \to \Omega(12) + \Omega(13) + \Omega(23)\,. \end{aligned} \tag{21}$$

But of course the price of any improvement on the asymptotic choice of $F(123)$ must be paid in terms of more difficult T functions.

To complete the specification of the problem in the framework of the "asymptotic-correlation ansatz," we observe that Eq. (20) may be generalized in a straightforward way to A particles; we thus propose the following A-particle correlation operator including tensor effects:

$$F(1\cdots A) = F_1 + F_2\sum_{i<j}^{A} u(ij)\,, \tag{22}$$

where F_1, F_2 are ordinary Jastrow correlation factors

$$F_{1,2} = \prod_{i<j} f_{1,2}(r_{ij})\,, \tag{23}$$

and the quantities $u(ij)$ incorporate the state dependence

$$u(12) = \frac{f_1(r_{12})}{f_2(r_{12})}[\,g(r_{12}) - 1]\,P_\tau^1(12)P^3(12)[\,1 - Q(12)]\,. \tag{24}$$

The corresponding trial A-body wave function is reminiscent of that of Feingold,[16] which was suggested by second-order perturbation theory. In the present study, only the two- and three-body parts of Eq. (22) are employed, so that what we will learn has to do only with these aspects of the A-body wave function. A good trial wave function for $A > 3$ nucleons may have to be much more complicated than Eq. (22), or at least different from it.

IV. EFFECTIVE POTENTIALS

Calculation of the three-body contribution, Eq. (9), to the energy per particle for given correlation operators $F(12)$, $F(123)$ requires evaluation of matrix elements of the operators $F^2(12) - 1$, $W_2(12)$, and $W_3(123)$. These latter operators involve at most spin scalars, spin vectors, and spin tensors. Note in particular that $W_3(123)$ has the same form as Eq. (14). According to Eq. (9) we need the spin trace of W_3P_A; from the theory of irreducible tensorial sets[17] it then follows that only the spin-scalar part of W_3 will give a nonvanishing contribution. This last fact holds for the entire expression (9); we need retain only the spin-scalar portions of the operators $F^2(12) - 1$, $W_2(12)$, $W_3(123)$. Thus we may replace these operators, for our purposes, by simpler *effective operators*,

$$\begin{aligned} &F^2(12) - 1 \to K(12)\,,\\ &W_2(12) \to w_2(12)\,,\\ &W_3(123) \to w_3(123)\,, \end{aligned} \tag{25}$$

which involve the spin operators only in the forms $\vec{\sigma}(\alpha)\cdot\vec{\sigma}(\beta)$, but satisfy

$$\frac{1}{A}(\Delta E)_3 = \frac{1}{3!}\frac{1}{A}\sum_{ijk}\langle ijk|\,w_3P_A\,|ijk\rangle - \frac{1}{A}\sum_{ijk}\langle ik|\,K\,|ik - ki\rangle\langle ij\,|w_2|\,ij - ji\rangle\,. \tag{26}$$

The construction of such effective operators proceeds via the definitions (7) and (8), the general forms for $F(12)$, $F(123)$, and the replacements

$$\sigma_i(\alpha) \to 0\,,$$
$$\sigma_i(\alpha)\sigma_j(\beta) \to 0\,, \quad i \neq j\,, \quad \alpha \neq \beta\,, \tag{27}$$
$$\sigma_i(1)\sigma_j(2)\sigma_k(3) \to 0\,, \quad \text{for any two of } ijk \text{ coincident}\,.$$

For all further reductions we have used the "asymptotic choice," Eqs. (11) and (19). In RTC the above procedure was already applied to obtain the two-body effective potential $w_2(12)$ for the most general $F(12)$. For the special choice Eq. (11) this result collapses to

$$w_2(12) = w_2^{(1)}(12) + w_2^{(2)}(12)\,, \tag{28}$$

with

$$w_2^{(1)}(12) = \frac{\hbar^2}{m}[\nabla_1 f(r_{12})]^2 + f^2(r_{12})V_C(12)\,, \tag{29}$$

$$w_2^{(2)}(12) = w_T(r_{12})P_\tau^1(12)P^3(12)\,. \tag{30}$$

Here V_C is the central part of the bare potential (10) and w_T is the "effective tensor potential":

$$V_C = {}^1V_C^+ P_\tau^3 P^1 + {}^1V_C^- P_\tau^1 P^1 + {}^3V_C^+ P_\tau^1 P^3 + {}^3V_C^- P_\tau^3 P^3\,, \tag{31}$$

$$w_T = \tfrac{1}{3}({}^3V_C^+ - 4V_T^+)f^2(g^2-1) + \frac{1}{3}\frac{\hbar^2}{m}[(\nabla g f)^2 - (\nabla f)^2] + \frac{4}{3}\frac{\hbar^2}{m}\frac{1}{r^2}f^2(g-1)^2\,. \tag{32}$$

For the "effective correlation" $K(12)$ we find simply

$$K(12) = [f^2(r_{12}) - 1] + \tfrac{1}{3}f^2(r_{12})[g^2(r_{12}) - 1]P_\tau^1(12)P^3(12)\,. \tag{33}$$

The first term in this expression will be referred to as the "over-all correlation," the second, the "effective tensor correlation."

We can follow the same pattern for the construction of the effective three-body potential $w_3(123)$. The procedure is elementary but the manipulations are lengthy; an outline is given in the Appendix. The final result is

$$w_3(123) = [w_3^{(1)}(123) + w_3^{(2)}(123) + w_3^{(3)}(123) + w_3^{(4)}(123) + w_3^{(5)}(123)] + [w_3^{(6)}(123) + w_3^{(7)}(123)] + w_3^{(8)}(123)\,, \tag{34}$$

with

$$w_3^{(1)}(123) = [f^2(r_{13})f^2(r_{23}) - 1]w_2^{(1)}(12) + \frac{\hbar^2}{4m}f^2(r_{12})[\nabla_3 f^2(r_{13})]\cdot[\nabla_3 f^2(r_{23})] + \text{cycl.}\,, \tag{35a}$$

$$w_3^{(2)}(123) = [f^2(r_{13})f^2(r_{23}) - 1]w_T(r_{12})P_\tau^1(12)P^3(12) + \frac{1}{12}\frac{\hbar^2}{m}f^2(r_{12})[g^2(r_{12}) - 1][\nabla_3 f^2(r_{13})]\cdot[\nabla_3 f^2(r_{23})]P_\tau^1(12)P^3(12) + \text{cycl.}\,, \tag{35b}$$

$$w_3^{(3)}(123) = \tfrac{1}{6}f^2(r_{23})f^2(r_{13})[g^2(r_{13}) - 1]P_\tau^1(13)P^3(13)w_2^{(1)}(12) + \frac{1}{24}\frac{\hbar^2}{m}f^2(r_{12})[\nabla_3 f^2(r_{23})]\cdot\{\nabla_3 f^2(r_{13})[g^2(r_{13}) - 1]\}P_\tau^1(13)P^3(13) + (1 \to 2) + \text{adj.} + \text{cycl.}\,, \tag{35c}$$

$$w_3^{(4)}(123) = \tfrac{1}{6}f^2(r_{23})f^2(r_{13})[g(r_{13}) - 1]w_T^{(1)}(r_{12})P_\tau^1(12)P^3(12)P_\tau^1(13)P^3(13) + \frac{1}{36}\frac{\hbar^2}{m}f^2(r_{12})[g(r_{12}) - 1][\nabla_3 f^2(r_{23})]\cdot\{\nabla_3 f^2(r_{13})[g(r_{13}) - 1]\}P_\tau^1(12)P^3(12)P_\tau^1(13)P^3(13) + (1 \to 2) + \text{adj.} + \text{cycl.}\,, \tag{35d}$$

$$w_T^{(1)}(r_{12}) = \tfrac{2}{3}[{}^3V_C^+(r_{12}) - 4V_T^+(r_{12})]f^2(r_{12})[g(r_{12}) - 1] + \frac{2}{3}\frac{\hbar^2}{m}[\nabla f(r_{12})]\cdot\{\nabla f(r_{12})[g(r_{12}) - 1]\}\,, \tag{35d'}$$

$$w_3^{(5)}(123) = V_5(123)P_\tau^1(13)P^3(13)P_\tau^1(23)P^3(23) + \text{adj.} + \text{cycl.}\,, \tag{35e}$$

$$w_3^{(6)}(123) = V_6(123)P_\tau^1(13)P^3(12)[P^3(23) - \tfrac{5}{6}]P^3(13) + (1 \to 2) + \text{adj.} + \text{cycl.}\,, \tag{36a}$$

$$w_3^{(7)}(123) = V_7(123)P_\tau^1(23)P^3(13)[P^3(12) - \tfrac{5}{6}]P^3(23) + (1 \to 2) + \text{adj.} + \text{cycl.}\,, \tag{36b}$$

$$w_3^{(8)}(123) = V_8(123)[P^3(12), P^3(13)][P_\tau^3(12), P_\tau^3(13)] + \text{cycl.} \tag{37}$$

The functions $V_5, \ldots, V_8$ depend only on spatial coordinates.

We note that the terms in the first bracket of Eq. (34), viz., Eqs. (35a)–(35e), have the following structure:
(a) $w_3^{(1)}$ is the "over-all" effective three-body operator, surviving intact for a spherically symmetric two-body Jastrow factor ($g \equiv 1$); indeed, the only term surviving in this case.
(b) $w_3^{(2)}$ contains the over-all correlation times the effective tensor potential $w_2^{(2)}$.
(c) $w_3^{(3)}$ contains the effective tensor correlation times the "over-all" two-body potential $w_2^{(1)}$.
(d) $w_3^{(4)}$ and $w_3^{(5)}$ involve pieces of the effective tensor correlation times the effective tensor potential.

These addends all possess direct parts. The remaining terms in Eq. (34), viz. Eqs. (36a), (36b), and (37) do not. In particular, $w_3^{(6)}$ only contributes through double exchange, i.e., via matrix elements joining $|ijk\rangle$ with $|jki\rangle$ or $|kij\rangle$.

The explicit forms for the functions $V_5, \ldots, V_8$ show clearly the difficulty of the three-body evaluation:

$$V_5(123) = \tfrac{1}{36} f^2(r_{23})[g(r_{23})-1] f^2(r_{13})[g(r_{13})-1]$$
$$\times \Big(4\frac{\hbar^2}{m}[\nabla_1 f(r_{12})]^2 + f^2(r_{12})\sin^2\gamma[3^1V_C^+(r_{12}) - {}^1V_C^-(r_{12}) - {}^3V_C^+(r_{12}) + 3^3V_C^-(r_{12}) + 4V_T^+(r_{12}) - 12V_T^-(r_{12})]$$
$$- f^2(r_{12})\cos^2\gamma[3^1V_C^+(r_{12}) - {}^1V_C^-(r_{12}) + 3^3V_C^+(r_{12}) - 9^3V_C^-(r_{12})]\Big)$$
$$+ \frac{1}{18}\frac{\hbar^2}{m} f^2(r_{12})\Big(f(r_{13})f(r_{23})\{\nabla_3 f(r_{13})[g(r_{13})-1]\}\cdot\{\nabla_3 f(r_{23})[g(r_{23})-1]\}$$
$$+ f(r_{13})[g(r_{13})-1] f(r_{23})[g(r_{23})-1][\nabla_3 f(r_{13})]\cdot[\nabla_3 f(r_{23})]\Big), \tag{38}$$

$$V_6(123) = -\tfrac{1}{6} f^2(r_{23}) f^2(r_{13})[g(r_{13})-1]$$
$$\times \Big(-6f^2(r_{12})(1-3\cos^2\alpha)V_T^+(r_{12}) + f^2(r_{12})[g(r_{12})-1](1-3\cos^2\alpha)[^3V_C^+(r_{12}) - 4V_T^+(r_{12})]$$
$$+ \frac{\hbar^2}{2m}\{\nabla_1 f(r_{12})[g(r_{12})-1]\}\cdot[(\nabla_1 - \nabla_2) f(r_{12})(1-3\cos^2\alpha)]\Big)$$
$$+ \tfrac{1}{64} f^2(r_{23}) f^2(r_{13})[g(r_{13})-1]^2 f^2(r_{12})[3^1V_C^+(r_{12}) + {}^1V_C^-(r_{12}) + 3^3V_C^+(r_{12}) + 9^3V_C^-(r_{12}) - 16^3V_C^+(r_{12})]$$
$$- \frac{1}{24}\frac{\hbar^2}{m} f^2(r_{12})[g(r_{12})-1](1-3\cos^2\alpha)[\nabla_3 f^2(r_{23})]\cdot\{\nabla_3 f^2(r_{13})[g(r_{13})-1]\}, \tag{39}$$

$$V_7(123) = -\tfrac{1}{48} f^2(r_{13})[g(r_{13})-1] f^2(r_{23})[g(r_{23})-1]$$
$$\times \Big(2\frac{\hbar^2}{m}[\nabla_1 f(r_{12})]\cdot[(\nabla_1 - \nabla_2) f(r_{12})(1-3\cos^2\gamma)] - 6\frac{\hbar^2}{m} f^2(r_{12})(\nabla_1\cdot\nabla_2\cos^2\gamma)$$
$$+ f^2(r_{12})\sin^2\gamma[3^1V_C^+(r_{12}) - {}^1V_C^-(r_{12}) - {}^3V_C^+(r_{12}) + 3^3V_C^-(r_{12}) + 4V_T^+(r_{12}) - 12V_T^-(r_{12})]$$
$$+ 2f^2(r_{12})\cos^2\gamma[3^1V_C^+(r_{12}) - {}^1V_C^-(r_{12}) + 3^3V_C^+(r_{12}) - 9^3V_C^-(r_{12})]\Big)$$
$$+ \tfrac{9}{64} f^2(r_{23}) f^2(r_{13})[g(r_{13})-1]^2 f^2(r_{12})[5^1V_C^+(r_{12}) - {}^1V_C^-(r_{12}) + 5^3V_C^+(r_{12}) - 9^3V_C^-(r_{12})]$$
$$- \frac{1}{24}\frac{\hbar^2}{m} f^2(r_{12})(1-3\cos^2\gamma)\Big(f(r_{13})f(r_{23})\{\nabla_3 f(r_{13})[g(r_{13})-1]\}\cdot\{\nabla_3 f(r_{23})[g(r_{23})-1]\}$$
$$+ f(r_{13})[g(r_{13})-1] f(r_{23})[g(r_{23})-1][\nabla_3 f(r_{13})]\cdot[\nabla_3 f(r_{23})]\Big), \tag{40}$$

$$V_8(123) = -\frac{1}{36}\frac{\hbar^2}{m} f^2(r_{12})\Big(\nabla_3 f^2(r_{23}) f^2(r_{13})[g(r_{13})-1]\cdot\{\nabla_2[g(r_{12})-1](1+4\cos^2\alpha)\}\Big) + (1\rightleftarrows 2). \tag{41}$$

In these expressions,

$$\cos\alpha \equiv \frac{\vec{r}_{12}\cdot\vec{r}_{13}}{r_{12}r_{13}}, \quad \cos\beta \equiv \frac{\vec{r}_{12}\cdot\vec{r}_{32}}{r_{12}r_{32}}, \quad \cos\gamma \equiv \frac{\vec{r}_{13}\cdot\vec{r}_{23}}{r_{13}r_{23}}. \tag{42}$$

V. NUMERICAL RESULTS FOR NUCLEAR MATTER

For numerical calculations it is convenient to decompose the three-body potential $w_3(123)$ according to

$$w_3(123) = w_3(12,3) + \text{cycl.}, \tag{43}$$

as already indicated in Eqs. (35)–(37). Also, it is useful to define the following spin- and isospin-averaged functions (μ, ν, κ label single-particle spin and isospin states) of the spatial coordinates:

$$\begin{aligned}
V_{\mathrm{I}} &= \tfrac{1}{2}\sum_{\mu\nu\kappa}\langle \mu\nu\kappa|\,[w_3(12,3) - K(13)w_2(12) - K(23)w_2(12)]\,|\mu\nu\kappa\rangle,\\
V_{\mathrm{II}} &= \sum_{\mu\nu\kappa}\langle \mu\nu\kappa|\,[w_3(12,3) - K(23)w_2(12)]\,|\mu\kappa\nu\rangle,\\
V_{\mathrm{III}} &= \tfrac{1}{2}\sum_{\mu\nu\kappa}\langle \mu\nu\kappa|\,[2w_3(12,3) - K(13)w_2(12) - w_2(12)K(23)]\,|\nu\kappa\mu\rangle,\\
V_{\mathrm{IV}} &= \tfrac{1}{2}\sum_{\mu\nu\kappa}\langle \mu\nu\kappa|\,[w_3(12,3) - K(13)w_2(12) - K(23)w_2(12)]\,|\nu\mu\kappa\rangle.
\end{aligned} \tag{44}$$

These definitions incorporate the two-body combination terms. If we wish, we may classify contributions to a given V_α ($\alpha = \mathrm{I}, \ldots, \mathrm{IV}$) in correspondence with the scheme set up for the structural decomposition of w_3 in Eq. (34) *et seq.* For instance, we can put the addend of V_α arising from the "over-all" term $w_3^{(1)}$ together with that addend arising from the "over-all" part of the two-body combination contribution, etc. Thus, we may write

$$V_\alpha = \sum_{i=1}^{8} V_\alpha^{(i)}, \quad \alpha = \mathrm{I}, \ldots, \mathrm{IV}. \tag{45}$$

Standard procedures now permit reduction of Eq. (26) to

$$\frac{1}{A}(\Delta E)_3 = \frac{1}{9}\frac{k_F^6}{2\pi^2}(g_{\mathrm{I}} - g_{\mathrm{II}} + g_{\mathrm{III}} - g_{\mathrm{IV}}), \tag{46}$$

where

$$\begin{aligned}
g_{\mathrm{I}} &= \int_0^\infty\int_0^\infty\int_{-1}^{1} V_{\mathrm{I}}\, d\tau,\\
g_{\mathrm{II}} &= \int_0^\infty\int_0^\infty\int_{-1}^{1} V_{\mathrm{II}} l^2(k_F r_{23}) d\tau,\\
g_{\mathrm{III}} &= \int_0^\infty\int_0^\infty\int_{-1}^{1} V_{\mathrm{III}} l(k_F r_{12})\, l(k_F r_{23}) l(k_F r_{13}) d\tau,\\
g_{\mathrm{IV}} &= \int_0^\infty\int_0^\infty\int_{-1}^{1} V_{\mathrm{IV}} l^2(k_F r_{12}) d\tau,
\end{aligned} \tag{47}$$

with

$$\begin{aligned}
d\tau &= r_{12}^2 r_{13}^2 d(\cos\alpha) dr_{12}\, dr_{13},\\
l(x) &\equiv \frac{3}{x^3}(\sin x - x\cos x).
\end{aligned} \tag{48}$$

It should be noted that g_{I} yields the direct, $-(g_{\mathrm{II}} + g_{\mathrm{IV}})$ the single-exchange, and g_{III} the double-exchange contribution to the three-body-cluster energy.

We have adopted for numerical study three examples of the class of noncentral potentials examined by Gammel and Thaler,[18] viz. the Gammel-Christian-Thaler (GCT) potential and the potentials with code Nos. 5200 and 5100. These are the same potentials as we used in RTC. The GCT po-

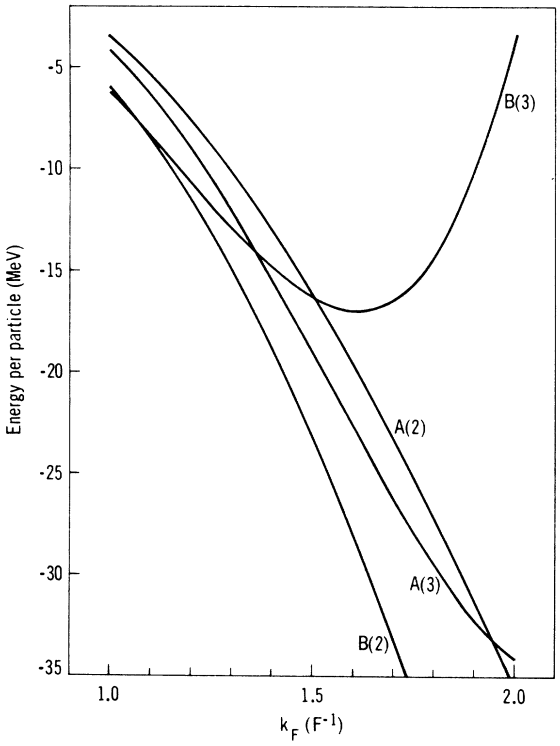

FIG. 1. Calculated energy per particle E/A as a function of Fermi wave number k_F for GCT potential. Curve labeled A(3): Three-body approximation $\mathcal{E}^{(3)} = [E_0 + (\Delta E)_2 + (\Delta E)_3]/A$ with tensor correlations absent ("over-all" result, $f \neq 1$, $g \equiv 1$). Curve labeled B(3): $\mathcal{E}^{(3)}$ with tensor correlations present ("complete" result, $f \neq 1$, $g \neq 1$). Curves labeled A(2) and B(2): "over-all" and "complete" results in two-body approximation $\mathcal{E}^{(2)} = [E_0 + (\Delta E)_2]/A$.

tential has a "weak" tensor component, the potential 5200 a tensor component of "moderate" strength, and the potential 5100 a "strong" tensor component. For a complete specification, see Tables I and II of RTC.

The two-body-cluster contributions to the ground-state-energy expectation value of nuclear matter being known for the choices of f and g made in RTC,[19] we have simply used in these functions the "optimal" correlation parameters λ, λ^+, and α^+ listed in Table III of RTC and calculated all the corresponding three-body contributions Eqs. (45)–(47) for k_F values 1.0 (0.1) 2.0 F^{-1}. To give some impression of the relative importance of the various contributions $U^{(i)}$, $i=1,\ldots,8$ to the three-body energy per particle [Eq. (46)] arising from the various $\sum_\alpha V_\alpha^{(i)}$, $i=1,\ldots,8$, we cite these results for the potential 5200 at $k_F=1.4$ F^{-1}: $U^{(1)}=-0.95$ MeV; $U^{(2)}=-2.12$ MeV; $U^{(3)}=-1.75$ MeV; $U^{(4)}+U^{(5)}=-1.84$ MeV; $U^{(6)}+U^{(7)}+U^{(8)}=5.38$ MeV. Our final results for the energy per particle in three-body approximation, $\mathcal{E}^{(3)}=[E_0+(\Delta E)_2+(\Delta E)_3]/A$, are displayed in Figs. 1–3 for potentials GCT, 5200, and 5100, respectively. The curves labeled $A(3)$ are the results including only over-all correlation effects; the curves labeled $B(3)$ are the complete results including the effects of tensor correlations. The curves $A(2)$, $B(2)$ are the corresponding results (over all, complete) in two-body approximation, $\mathcal{E}^{(2)}=[E_0+(\Delta E)_2]/A$.

The results for $\mathcal{E}^{(3)}$ exhibit a distinct qualitative improvement over the results (from RTC) for $\mathcal{E}^{(2)}$ with respect to the problem of saturation. It should be stressed, however, that it is not legitimate to interpret our present calculation as yielding reasonably good approximations to the correct energies per particle, or the correct energy expectation values per particle E/A, for the potentials in question – though the superficial appearance of the results might make it very tempting to do so. The correlation functions f and g used here and in RTC were chosen primarily on the basis of mathematical simplicity, with a view to exploring the general scheme in an unencumbered and efficient manner. The functions we have adopted are evidently unsatisfactory – at least beyond the saturation k_F of about 1.4 F^{-1} – because a good correlation function should (one would hope) already produce saturation in the two-body approximation to E/A. Indeed, $A(2)$ as well as $A(3)$ should already show saturation.

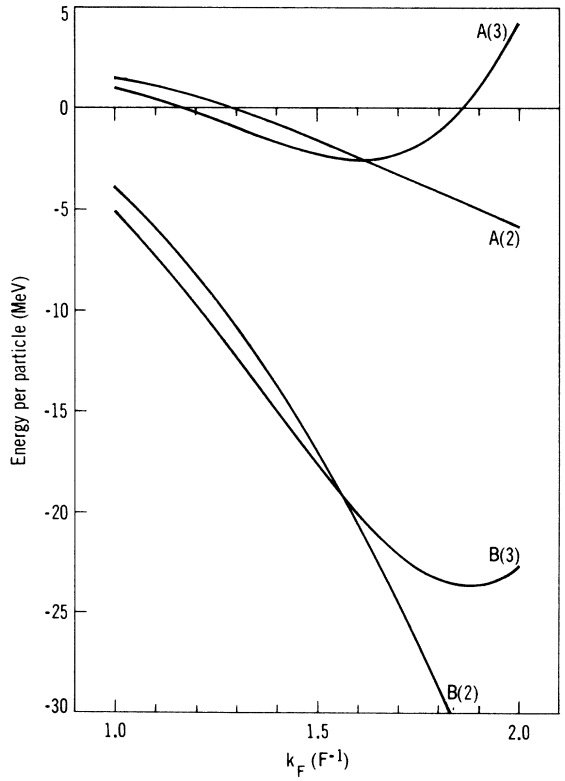

FIG. 2. Same as Fig. 1, for potential 5200.

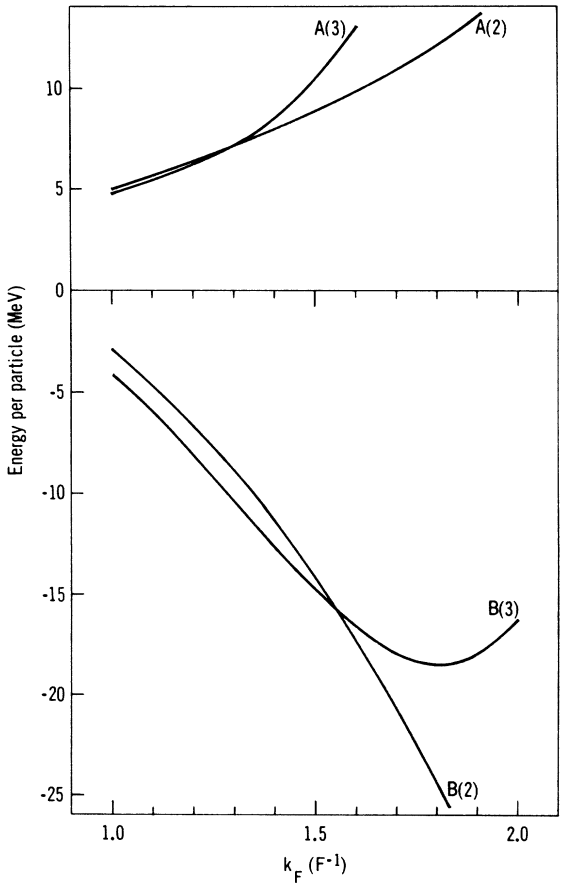

FIG. 3. Same as Fig. 1, for potential 5100.

On the other hand, it seems likely from our results that with better correlation functions the approach proposed here should be successful. Though the over-all and particularly the tensor-induced three-body corrections become rather large at high densities (casting doubt on the usefulness of the cluster expansion), this may well be associated with a tendency of the cluster expansion to repair the saturation behavior of the two-body approximation in the face of a poor (i.e., not adequately constrained) choice of correlation functions. The search for better correlation functions and their use in the present framework will be the subject of a more numerical paper.

We might remark additionally that for all densities $B(3)$ should lie below $A(3)$, and $B(2)$ below $A(2)$, provided the upper bound property holds for $\mathcal{G}^{(3)}, \mathcal{G}^{(2)}$, respectively (which it will if the cluster expansion in question converges rapidly enough), and the ansatz for the tensor correlation function g is a sensible one. These criteria are violated only for the GCT potential, and then only for $B(3)$ vs $A(3)$ at k_F beyond about 1.4 F^{-1}.

At $k_F = 1.3$ F^{-1} (just below the empirical saturation density), we find $B(3) \approx -13$, -12, and -10 MeV for potentials GCT, 5200, 5100, respectively – quite reasonable values. The order of these values is in line with the usual hypothesis about the dependence of the energy per particle on the central-tensor mixture,[8] but the effect is so weak that it may not be significant. The net three-body correction is found to be small in magnitude at this density, $|(\Delta E)_3/A| \approx 1-2$ MeV, but grows large and positive at high density.

VI. DISCUSSION

There are many problems connected with the extended Jastrow scheme which should be attacked. Some of these questions, it must be admitted, have yet to be properly formulated.

It would surely be useful to know the precise connection between the Jastrow approach and Brueckner theory, a connection which is still not completely elucidated. Consider, for example: Is there or should there be a dispersion effect in the Jastrow scheme as in reaction-matrix theory?[20] But the most pressing questions with regard to the method developed here are the following:

(a) What is the smallness parameter for the factor-cluster expansion if a correlated wave function of the type (22) is adopted, and what class of correlation functions produces a rapid convergence?[9,14]

(b) What kind of subsidiary conditions on the pair correlation functions f and g are preferred with respect to a good trial function and our choice of a factor-cluster expansion?

(c) Is it possible, within the general scheme, to separate "short-range" and "long-range" effects or at least certain particle-hole excitations[21] from the "true" short-range correlations? A clarification would certainly be necessary in face of the form of the wave function (22). In this wave function the tensor effects are described, roughly speaking, by a two-body operator which generates zero-particle–zero-hole and two-particle–two-hole excitations only.

Of course, these questions are not independent, but rather emphasize different aspects of the entire problem. They must be considered seriously before profound conclusions can be drawn with regard to the present numerical results or future refinements. The same questions already arise in the two-body version of the theory; their importance is only brought into clearer focus by our exploration of three-body effects. At this juncture, little can be added to the discussion of RTC.

It is our belief that a fruitful attack on these problems should begin with the imposition of the Pauli condition on the trial wave function (see the discussion of RTC). This condition seems a natural one in view of our original introduction of the correlation operator F as a true dynamical correlation, and our desire that all contributions of the "uncorrelated" ground state (represented by Φ) to the energy be collected in the first term E_0 of the cluster expansion (insofar as it is possible). (In other words, we want to avoid cluster expanding in the statistical or kinematic correlations of free fermions.) The Pauli condition in this general sense takes the form

$$\langle \Phi | F - 1 | \Phi \rangle = 0 , \tag{49}$$

which is presumably to be applied separately in each cluster approximation. In two-body approximation, Eq. (49) yields just the "average Pauli condition" of RTC.

ACKNOWLEDGMENTS

M.L.R. appreciates very much the hospitality of the Washington University Physics Department during the years 1969–1971. He enjoyed a rewarding time at the Center for Physics at Aspen, Colorado, where he was encouraged to carry out parts of this work. J.W.C. would like to acknowledge the hospitality of the Niels Bohr Institute, Copenhagen, and of the Physics Institute of Åbo Akademi during the summer of 1971. He also thanks Nordita, Copenhagen, for financial support.

APPENDIX

The derivation of Eqs. (34) *et seq.* involves the following steps:

(a) First we evaluate the commutators in Eq. (8). To this end it is helpful to bring them into the form

$$[F(123),[t(3),F(123)]]=\frac{\hbar^2}{m}[\nabla_3,F(123)]^2-\frac{\hbar^2}{2m}\{\nabla_3[F(123),[\nabla_3,F(123)]]+\text{adj.}\}. \tag{A1}$$

The first term is merely the square of a derivative of a local operator and, hence, is itself local. The curly bracket term seems to be linear in the momentum operator $-i\hbar\nabla_3$, but a careful examination shows that it contributes to the energy only through matrix elements connecting $|ijk\rangle$ with $|jki\rangle$ or $|kij\rangle$, i.e., only through double exchange. This fact can be used to bring the curly bracket term also into local form.

(b) The second step makes use of the following relations:

$$\begin{aligned}
&[\nabla_3, f(r_{13})]=[\nabla_3 f(r_{13})],\\
&[\nabla_3, Q(13)]^2=\frac{1}{r_{13}^{\,2}}[2P^3(13)-Q(13)],\\
&[\nabla_3,[\nabla_3, Q(13)]]=\frac{2}{r_{13}^{\,2}}[2P^3(13)-3Q(13)],\\
&i[Q(13),[\nabla_3, Q(13)]]=\frac{1}{4r_{13}^{\,2}}[\vec{\sigma}(1)+\vec{\sigma}(3)]\times\vec{r}_{13}.
\end{aligned} \tag{A2}$$

Insertion of Eqs. (A2) into Eqs. (A1) and (8) yields then an expression for W_3 convenient for the extraction of the spin scalars. This expression contains the spin operators only in the combinations indicated in step (c) below.

(c) We next reduce the operators occurring in Eq. (8) after insertion of (A1) and (A2):

$$\begin{aligned}
&P^3(12)[1-Q(12)]=\tfrac{1}{3}P^3(12)+\cdots,\\
&S_{12}P^3(13)[1-Q(13)]=(1-3\cos^2\alpha)P^3(12)[P^3(23)-\tfrac{5}{6}]P^3(13)+\cdots,\\
&[P^3(13)-Q(13)]P^1(12)[P^3(13)-Q(13)]=\tfrac{1}{12}P^3(13)+\cdots,\\
&[P^3(13)-Q(13)]P^3(12)[P^3(13)-Q(13)]=\tfrac{1}{4}P^3(13)+\cdots,\\
&[P^3(13)-Q(13)]S_{12}[P^3(13)-Q(13)]=0+\cdots,\\
&[P^3(13)-Q(13)]P^1(12)[P^3(23)-Q(23)]=\tfrac{1}{18}(1-2\cos^2\gamma)P^3(13)P^3(23)\\
&\qquad\qquad -\tfrac{1}{12}(1+\cos^2\gamma)P^3(13)[P^3(12)-\tfrac{5}{6}]P^3(23)+\cdots,\\
&[P^3(13)-Q(13)]P^3(12)[P^3(23)-Q(23)]=\tfrac{1}{18}(1+2\cos^2\gamma)P^3(13)P^3(23)\\
&\qquad\qquad +\tfrac{1}{12}(7\cos^2\gamma-1)P^3(13)[P^3(12)-\tfrac{5}{6}]P^3(23)+\cdots,\\
&[P^3(13)-Q(13)]S_{12}[P^3(23)-Q(23)]=\tfrac{1}{6}\sin^2\gamma\,P^3(13)[2P^3(12)-3]P^3(23)+\cdots.
\end{aligned} \tag{A3}$$

We keep only the spin-scalar parts, thus drop the terms represented by dots. Note that we could express the various P products in accordance with our original statements (14) and (27), but the formulation above is somewhat preferable physically.

(d) In addition, we exploit some properties of the isospin operators. For example,

$$\begin{aligned}
&P^1_\tau(13)P^3_\tau(12)P^1_\tau(13)=3P^1_\tau(13)P^1_\tau(12)P^1_\tau(13)=\tfrac{3}{4}P^1_\tau(13),\\
&P^1_\tau(13)P^3_\tau(12)P^1_\tau(23)=-3P^1_\tau(13)P^1_\tau(12)P^1_\tau(23)=\tfrac{3}{2}P^1_\tau(13)P^1_\tau(23),
\end{aligned} \tag{A4}$$

while

$$P^3_\tau(12)P^1_\tau(13)P^3(12)[P^3(23)-\tfrac{5}{6}]P^3(13)=0. \tag{A5}$$

The replacement (A5) holds because the sums needed in Eq. (9) will cancel any contribution to the energy given by the left side.

Completing steps (a) through (d) and collecting related terms according to the decomposition (34), we arrive at the expressions (34) *et seq.*

*Research supported in part by the National Science Foundation, under Grant No. GP-22564.

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[1]B. D. Day, Rev. Mod. Phys. 39, 719 (1967).

[2]B. H. Brandow, Phys. Rev. 152, 863 (1966); Rev. Mod. Phys. 39, 771 (1967); in *Lectures in Theoretical Physics: Quantum Fluids and Nuclear Matter,* edited by K. T. Mahanthappa and W. E. Brittin (Gordon and Breach, New York, 1969), Vol XI B; Ann. Phys. (N.Y.) 57, 214 (1970).

[3]P. J. Siemens, Nucl. Phys. A141, 225 (1970); and P. Banerjee and D. W. L. Sprung, Can. J. Phys. 49, 1899 (1971) are among the most recent. The former paper, incidentally, points the way to a local approximation to the Brueckner reaction matrix, i.e., toward a practical connection with the Jastrow approach and the method of correlated basis functions (see Ref. 8 and works cited therein).

[4]T. Dahlblom, K. G. Fogel, B. Qvist, and A. Törn, Nucl. Phys. 56, 177 (1964); M. W. Kirson, *ibid.* A99, 353 (1967); B. S. Bhakar and R. J. McCarthy, Phys. Rev. 164, 1343 (1967); Y. Akaishi, H. Bando, A. Kuriyama, and S. Nagata, Progr. Theoret. Phys. (Kyoto) 40, 288 (1968); P. A. Lawson, Nucl. Phys. A143, 65 (1970).

[5]H. A. Bethe, Phys. Rev. 138, B804 (1965); B. D. Day, *ibid.* 151, 826 (1966); R. Rajaraman and H. A. Bethe, Rev. Mod. Phys. 39, 745 (1967).

[6]T. K. Dahlblom, Acta Acad. Åboensis, B29, No. 6 (1969).

[7]B. D. Day, Phys. Rev. 187, 1269 (1969).

[8]M. L. Ristig, W. J. Ter Louw, and J. W. Clark, Phys. Rev. C 3, 1504 (1971).

[9]S.-O. Bäckman, D. A. Chakkalakal, and J. W. Clark, Nucl. Phys. A130, 635 (1969).

[10]J. W. Clark, Ph.D. thesis, Washington University, 1959 (unpublished); C.-W. Woo, Phys. Rev. 151, 138 (1966).

[11]C.-W. Woo and W. E. Massey, Phys. Rev. 177, 272 (1969).

[12]E. Feenberg, *Theory of Quantum Fluids* (Academic, New York, 1969); C.-W. Woo and W. E. Massey, Phys. Rev. 177, 272 (1969); R. A. Guyer, Solid State Commun. 7, 315 (1969); T. R. Koehler and N. R. Werthamer, Phys. Rev. A 3, 2074 (1971); D. K. Lee, Phys. Rev. Letters 26, 877 (1971); G. P. Mueller and J. W. Clark, Nucl. Phys. A155, 561 (1970); G. Mueller and J. W. Clark, *ibid.* B7, 227 (1968); J. W. Clark and G. Mueller, Nuovo Cimento 64B, 217 (1969); C.-H. Yang and J. W. Clark, Nucl. Phys. A174, 49 (1971).

[13]H.-K. Sim, C.-W. Woo, and J. R. Buchler, Phys. Rev. A 2, 2024 (1970); H.-K. Sim and C.-W. Woo, *ibid.* 2, 2032 (1970); B. D. Day, *ibid.* 4, 681 (1971); K. R. Lassey and D. W. L. Sprung, to be published.

[14]J. W. Clark and P. Westhaus, Phys. Rev. 141, 833 (1966). Their observations on the "smallness parameter," as well as those of Ref. 9, should still be valid for our more elaborate trial function, if appropriate generalizations are made.

[15]F. Coester, in *Lectures in Theoretical Physics: Quantum Fluids and Nuclear Matter,* edited by K. T. Mahanthappa and W. E. Brittin (Gordon and Breach, New York, 1969), Vol. XI B; E. O. Fiset, Phys. Rev. C 2, 85 (1970); G. A. Baker, Jr., M. F. Hind, and J. Kahane, *ibid.* 2, 841 (1970).

[16]A. M. Feingold, Phys. Rev. 101, 258 (1956).

[17]U. Fano and G. Racah, *Irreducible Tensorial Sets* (Academic, New York, 1959).

[18]J. L. Gammel and R. M. Thaler, in *Progress in Elementary Particle and Cosmic-Ray Physics,* edited by J. G. Wilson and S. A. Wouthuysen (North-Holland, Amsterdam, 1960), Vol. V.

[19]See, in RTC, Eqs. (27) and (28), Table III with α^- set zero, and the curves labeled B in Figs. 4–6 showing the energy per particle in two-body approximation. It should be remembered that in the present work we are omitting odd-state tensor correlations in $F(12)$ and $F(123)$ (which are expected to be small).

[20]C. W. Wong, Phys. Rev. C 3, 1058 (1971); Phys. Rev. Letters 26, 783 (1971).

[21]R. M. Dreizler, A. Klein, F. R. Krejs, and G. J. Dreiss, Nucl. Phys. A166, 624 (1971).