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Radiative np Capture and Polarization Effects

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For many years a 10% discrepancy has existed between theory and experiment for the total radiative thermal np capture cross section. This process is usually considered to be dominated by a $^1S \to d$ transition. Breit and Rustgi have shown that a significant amount of $^3S \to d$ transition would be detectable in experiments with polarized n and p. We show that the $^3S \to d$ transition can be described with a general transition current containing two parameters. On the assumption that the amplitude is large enough to explain the discrepancy, we give angular distributions for the γ-ray emission from polarized n on polarized p. On the other hand, we calculate theoretical values for the parameters which indicate that the $^3S \to d$ transition is probably not large enough to explain the discrepancy, and we discuss the correctness of the theory.

1. INTRODUCTION

A discrepancy of about 10% has long existed between experiment and the theory of radiative thermal np capture, $n+p\to d+\gamma$.[1,2] This process is one of the simplest nuclear interactions involving an electromagnetic "probe," and is doubly interesting because it is very closely related to 180° inelastic electron-deuteron scattering near threshold,[3] as well as photodisintegration, $\gamma+d\to n+p$. The continued lack of agreement between theory and experiment has led to intensive work for well over a decade. References 4–11 are only a few notable examples.

Many suggestions have been advanced as to how theory can be improved to agree with experiment. We will mention only a few. It was early suggested that nucleon magnetic moments are somehow altered by the proximity of other nucleons, the so-called "interaction" effect.[4] One may view this alteration in terms of a distortion of the nucleon's meson cloud due to exchanged mesons[2,3]; the interaction effect may then plausibly be viewed as a meson-exchange current as has been done by Adler, Chertok, and Miller (hereafter referred to as ACM) in Ref. 2. Exchange currents have been calculated for numerous kinds of mesons; pions yield an increase of about 2.3% in the radiative capture cross section, while other meson currents have a neglibible effect. Finite nucleon size should manifest itself as an extremely small *decrease* (the wrong direction) in the nucleon magnetic moments due to the momentum dependence of the nucleon form factors.[12] Virtual nucleon resonances should be present in the deuteron Hilbert space as well as virtual neutron-proton-meson states[9,10]; an estimate by Stranahan[9] of the effect of the 33 resonance yields only about a 2.9% increase in the radiative capture cross section. A double-counting problem exists here, since meson-exchange currents and nucleon-resonance contributions appear to be different representations of the same physical effect.[10] The dependence of the nucleon magnetic form factors on the virtual-nucleon mass is expected to be small; an estimate of this effect is difficult, but preliminary results should be forthcoming.[13]

Despite intense work, the best present experimental and theoretical values, we believe, are 334.2 ± 0.5 mb[11] versus 309.5 ± 5 mb.[2] Thus there remains a discrepancy of about 7%.

The $^1S\to d$ transition certainly dominates the cross section, while the $^3S\to d$ transition is generally considered to be highly suppressed; this is because one of the amplitudes for the $^3S\to d$ transition is approximately described by an overlap integral of the np bound and continuum wave functions in the ^3S configuration.[14] Such an overlap integral is zero by orthogonality if a Schrödinger or Rarita-Schwinger equation describes the physical or dressed nucleon dynamics. Indeed any Hermitian time-evolution operator clearly guarantees such orthogonality. Breit and Rustgi (BR)[15] have suggested that an anomalously large $^3S\to d$ transition could nevertheless explain the discrepancy in the total cross section, and also give rise to an observable angular dependence of the differential cross section if the n and p are both polarized. BR also discuss qualitatively some of the effects that might produce a lack of orthogonality and an anomalously large transition. Their discussion involves only one amplitude, what we will refer to as G_3 in this paper.

In the present work we will discuss the general problem of radiative np capture from $l=0$ np states. The general transition current for this process involves two parameters, G_3 and G_4, to describe the $^3S \to d$ transition and one parameter, $G_{12} \equiv G_1 + G_2$, to describe the $^1S \to d$ transition. These three parameters are easily related to the form factors which describe 180° inelastic electron-deuteron scattering near threshold, as discussed in Ref. 3. Indeed the transition currents have exactly the same form, as noted above.

The correspondence between radiative np capture and inelastic electron-deuteron scattering allows us to use the results of theoretical analyses of the latter. Much progress has been made in the theoretical understanding of general electron-hadron reactions using the concepts of quantum electrodynamics modified by the strong interaction through form factors. In particular, the vector-dominance model provides an adequate description of nucleon form factors at low four-momentum transfer, q^2.[12,16,17] On the other hand, the description of a composite system such as the deuteron still harbors some difficulties if one wishes to describe accurately the effects of binding, such as with the use of nonrelativistic wave functions.[18-21] Chief among the difficulties is the inclusion of relativistic effects, which presents a somewhat ambiguous problem.[21-24] Nevertheless, rather convincing analyses have been made that allow one to estimate and correct for relativistic effects in elastic e-d scattering at low momentum transfers.[23] Meson-exchange currents in elastic e-d scattering also have been studied and found to be amenable to convincing phenomenological analysis from several different but apparently consistent points of view.[21,25] In both elastic and inelastic scattering the exchange-current effects and the relativistic effects that have been studied have been found to be small, though not negligible. There is at present a satisfying consistency between theory and experiment at low q^2. At higher q^2, around 9 fm^{-1}, discrepancies do arise in inelastic electron-deuteron scattering,[3,26] so the present phenomenological theory is certainly not beyond reproach. It must also be noted that the experimental errors and theoretical model uncertainties are greater in typical scattering problems than in a "static" problem such as np capture.[26] Thus one can only consider theory and experiment to agree to, characteristically, about 10% in the low-q^2 region of 180° inelastic electron-deuteron scattering. The elastic scattering problem involves considerably better accuracy, especially at very low q^2, as exemplified by the recent experiment of Bumiller *et al.*[27] In summary, the status of the theory of elastic and inelastic electron-deuteron scattering at low q^2 appear to be quite good, though the experimental tests are of limited accuracy.

Let us return to the problem at hand. Using the general $np \to d$ transition current, to be obtained in Sec. 3, we will calculate the total radiative np capture cross section and the differential cross section for detection of the emitted γ ray. The latter cross section displays an angular dependence only if the n and p are *both* polarized.[15] We find that:

(1) Numerical estimates of the G parameters from a quantum electrodynamical analysis of the deuteron structure in terms of nonrelativistic wave functions[3] yield very small values for G_3 and G_4, and a value for G_{12} in agreement with previous theoretical work.[2] The angular distribution of photons is characteristically so close to isotropic as to make the effect of the $^3S \to d$ transition virtually unobservable, and the total cross section is, as in ACM,[2] 7% too low. Orthogonality of the deuteron and ^3S np continuum wave functions is assumed in the estimates for the amplitude G_3, and recoil and pion-exchange currents are included in the calculation.

(2) It may be assumed, contrary to the above, that the $^3S \to d$ transition is large enough to resolve the discrepancy.[15] A table of angular distributions for the photon is given in Sec. 9; it appears evident that the nonisotropy is measurable, in qualitative agreement with BR.[15] Polarization measurements on the γ ray would yield similarly interesting results.

Thus a measurement of the angular distribution of γ rays emitted in polarized np capture should give direct evidence on the size of the $^3S \to d$ transition and settle unambiguously the question of its role in np capture. This is in agreement with the qualitative results of BR, although they apparently make use of only a single amplitude to describe the $^3S \to d$ transition, instead of the two that we find necessary to include all effects.

We will not consider in the present work the effects of parity violation in the nuclear force. This, however, represents a related and highly interesting field of investigation.[28]

2. GENERAL FEATURES

The quantum numbers associated with thermal-neutron capture have been discussed elsewhere.[1,2,14] In this section we will briefly summarize the results.

Radiative capture of thermal neutrons is dominantly from the $l=0$ states, ^1S and ^3S. This is quantitatively verified by explicit calculation, as well as intuitively clear.[14,18] The ^1S state has $l=J=s=0$, and ^3S has $l=0$ and $J=s=1$. Transition from the ^1S to the deuteron ^3S state has been well studied,[2,4-10] and certainly contributes the major part of the cross section. We will discuss the $^3S \to d$ transition in more detail in Sec. 4. The effect of the deuteron D state is a simple reduction of the amplitude, since it is an unopen channel that influences the S-state normalization.[1]

Since the continuum ^1S state is an isovector and the deuteron is an isoscalar, the dominant $^1S \to d$ transition is isovector. It thus involves the isovector nucleon magnetic moment, which can be used to simplify our calculations. Similarly the $^3S \to d$ transition is isoscalar and involves the isoscalar magnetic moment and isoscalar charge.

We will be working in a very nonrelativistic kinematic regime, and may even ignore deuteron recoil energy. Then the γ ray emitted in the capture process will have energy essentially equal to the deuteron binding energy ϵ.

3. GENERAL TRANSITION CURRENT

Radiative capture may be described by a conventional S matrix of the form[2]

$$S = \frac{e(2\pi)^4\delta^4(Q_f + q - Q_i)\vec{J}\cdot\vec{\epsilon}}{\sqrt{2\omega}}, \tag{3.1}$$

as illustrated in Fig. 1. Here Q_f is the final deuteron four-momentum, Q_i is the initial np four-momentum, q is the photon four-momentum, ω is the deuteron binding energy, and $\vec{\epsilon}$ is the γ-ray polarization three-vector. We have utilized the radiation gauge to write the transition current $\vec{J}$ as a three-vector. The entire dynamics is thus contained in the three-vector $\vec{J}$.

We wish now to write down the most general transition current consistent with gauge invariance and the parity and rotational invariance of $\vec{J}\cdot\vec{\epsilon}$. We will assume that the final deuteron-spin state and the initial np-spin states are adequately described by two-nucleon Pauli spinors as is done in virtually all of low-energy nuclear physics. This assumption is well justified in similar electromagnetic problems where Dirac spinors are approximated by Pauli spinor functions.[3,21] Thus we shall write

$$\vec{J} = \chi_m^\dagger \vec{\mathscr{J}} \chi_i , \tag{3.2}$$

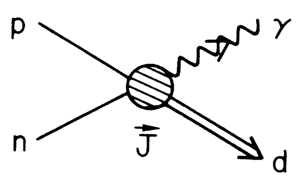

FIG. 1. The radiative capture process described by a transition current $\vec{J}$.

where $\vec{\mathscr{J}}$ is some outer product of Pauli-spin operators, and the χ_m is a conventional deuteron triplet spin function

$$\chi_{+1} = \alpha\alpha,\ \chi_{-1} = \beta\beta,\ \chi_0 = (\alpha\beta + \beta\alpha)/\sqrt{2}, \tag{3.3}$$

where α is a spin-up state, and β is a spin-down state. The χ_i is a spin function representing the initial np system in the singlet state, signified by s,

$$\chi_i = \chi_s = (\alpha\beta - \beta\alpha)/\sqrt{2}, \tag{3.4}$$

or one of the triplet states represented by the same spin functions that appear in (3.3). As is the usual convention, the first Pauli spinor in the above refers to the proton, the second to the neutron.

The available three-vectors are $\vec{q}$, $\vec{Q}_i$, $\vec{Q}_f$, and the relative np momentum $\vec{p}$. We will consider the situation where $\vec{Q}_i$ and $\vec{p}$ are very small, and delete them. For thermal neutrons both are about $\frac{1}{7}$ keV$/c \ll |\vec{q}|$. Then $\vec{q} = -\vec{Q}_f$ is the only three-vector in the problem. The spin operators $\vec{\sigma}_p$ and $\vec{\sigma}_n$ are pseudo vectors; thus the most general three-vector operator $\vec{\mathscr{J}}$ is

$$\vec{\mathscr{J}} = -iA_p(\vec{q}\times\vec{\sigma}_p) - iA_n(\vec{q}\times\vec{\sigma}_n) + B_p\vec{\sigma}_p(\vec{q}\cdot\vec{\sigma}_n) + B_n\vec{\sigma}_n(\vec{q}\cdot\vec{\sigma}_p), \tag{3.5}$$

where the A's and B's are scalar functions of the rotational invariant $\vec{q}^{\,2}$. Terms proportional to $\vec{q}$ are not included in $\vec{\mathscr{J}}$, since gauge invariance guarantees that they will not contribute to the invariant amplitude $\vec{J}\cdot\vec{\epsilon}$. The factors of $-i$ have been included for later convenience. Since we are ignoring the recoil energy of the deuteron, the magnitude of $\vec{q}$ is fixed at ϵ, the deuteron binding energy. Thus the A's and B's are constants. It is now clear that no more than four parameters are needed to describe the $^1S \to d$ transition and similarly for the $^3S \to d$ transition.

We can simplify the current $\vec{J}$ further. For the $^1S \to d$ transition the identity

$$\vec{\sigma}_p\cdot\vec{\sigma}_n = -3 \quad \text{(on singlet } np \text{ state)}, \tag{3.6}$$

and the symmetry properties of the spin functions allow us to write

$$\vec{J}_s = \chi_m^\dagger[-i(\vec{q}\times\vec{\sigma}_p)(A_p^s + B_p^s) - i(\vec{q}\times\vec{\sigma}_n)(A_n^s + B_n^s)]\chi_s . \tag{3.7}$$

Moreover, the isovector nature of the transition allows us to delete the neutron term entirely if proton quantities are replaced by isovector quantities. Thus

$$\vec{J}_s = \chi_m^\dagger[-i(\vec{q}\times\vec{\sigma}_p)]\chi_s(A_p^s - A_n^s + B_p^s - B_n^s) \tag{3.8}$$

is the general $^1S \to d$ transition current, and only a single parameter is needed to represent the dynamics.

Similar considerations applied to the $^3S \to d$ transition current leave us with two parameters,

$$\vec{J}_t = \chi_m^\dagger [-i(\vec{q}\times\vec{\sigma}_p)]\,\chi_{m'}(A_p^t + A_n^t) + \chi_m^\dagger [\vec{\sigma}_p(\vec{q}\cdot\vec{\sigma}_n)]\,\chi_{m'}(B_p^t + B_n^t)\,. \tag{3.9}$$

Thus three parameters describe both of the $l = 0$ to deuteron transition currents.

Precisely the same transition currents as above occur in 180° inelastic electron-deuteron scattering.[3] Thus we will rewrite the transition currents in analogy with the scattering currents of Ref. 3 as

$$J_s^j = \chi_m^\dagger(-i\epsilon^{jkl}q^k\sigma_p^l)\chi_s\frac{G_1+G_2}{M}\,,$$

$$J_t^j = \chi_m^\dagger(-i\epsilon^{jkl}q^k\sigma_p^l)\chi_{m'}\frac{G_3}{\sqrt{2}M} + \chi_m^\dagger(q^l\sigma_p^j\sigma_n^l)\chi_{m'}\frac{G_4}{\sqrt{2}M}\,. \tag{3.10}$$

The three parameters of np capture now appear as the values of the three form factors of inelastic e-d scattering, G_1+G_2, G_3, and G_4, evaluated at $\vec{q}^{\,2} = \epsilon^2$. We can thus directly use the results of Ref. 3 in discussing theoretical values of the three np capture parameters.

In summary, the currents in (3.10) are quite general for describing the capture of very-low-velocity neutrons on stationary protons with the standard assumption that the nucleon spins may be described by Pauli spinors. The parameter G_1+G_2, or G_{12}, represents the $^1S \to d$ transition (which dominates the cross section), while G_3 and G_4 represent the $^3S \to d$ transition, which is at most a small correction which we will discuss in the next section.

4. G PARAMETERS

In the preceding section we obtained a general current, under rather mild assumptions, for the np-to-deuteron transition. With it we can express any cross section associated with np capture. For example in Sec. 7 we will obtain a differential cross section for the emitted γ ray if both n and p are polarized. The measurement of such a cross section can give information on various combinations of the G parameters, as we will discuss. Before proceeding we wish to discuss theoretical attempts at calculating the G parameters. It is the apparent failure of these attempts that motivates the present work.

As indicated in the preceding section the current for np capture is the same as that which occurs in 180° inelastic electron-deuteron scattering. In the scattering problem $\vec{q}^{\,2}$ is the three-vector momentum transfer squared, and is an experimental variable. In np capture it represents the photon three-momentum squared, and is therefore fixed at the deuteron binding energy squared, ϵ^2. In this section we will summarize the results of Refs. 2 and 3, which deal with the dynamics of radiative np capture and the e-d scattering problem.

The calculations of Refs. 2 and 3 are based on conventional methods in wide use for hadron-photon interactions and electron scattering theory.[18-24] An S-matrix phenomenology is utilized which allows one to write an amplitude corresponding to a Feynman diagram: The npd "vertex" is described by a nonrelativistic wave function as is the np initial- or final-state interaction.[2,3,21-23] It is possible to obtain amplitudes corresponding to the so-called impulse approximation as shown in Fig. 2, and to the meson-exchange currents in Fig. 3. We will here merely summarize the results of Refs. 2 and 3.

The parameter $G_1+G_2=G_{12}$ representing the $^1S \to d$ transition has received a large amount of attention. The quantitatively reliable calculations of many authors are in substantial agreement with each other, but not with experiment. Because of the smoothness of the function $G_{12}=G_1+G_2$ near $\vec{q}^{\,2}=0$ it is very accurate to evaluate this parameter at $\vec{q}^{\,2}=0$, as is conventionally done. This is equivalent to ignoring the effect of recoil on the final deuteron wave function. For the impulse approximation the result is

$$(G_{12})_{ia} = \tfrac{1}{2}(\mu_p - \mu_n)H_s\,, \quad H_s = \int_0^\infty z_s(y,0)u(y)dy\,, \tag{4.1}$$

as obtained in Eqs. (4.14) and (4.15) of ACM.[2] The symbols are as follows: u is the deuteron S-wave function, μ_p and μ_n are the nucleon magnetic moments, and z_s is the zero-energy singlet np wave

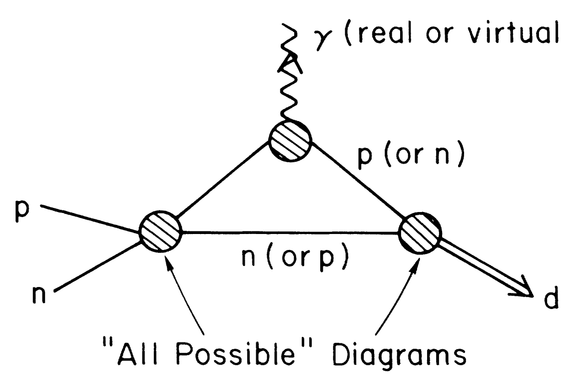

FIG. 2. The impulse-approximation diagram. In the present problem the initial-state np interaction and the npd vertex are described by nonrelativistic wave functions. The amplitude takes the form of a simple three-dimensional configuration-space matrix element. At the low energies we are considering, the nucleon electromagnetic current is well described by the nucleon static properties.

function, normalized by

$$\frac{z_s(y,p)}{y} \to \sqrt{(4\pi)}\frac{\sin(py+\delta_s)}{py}\,, \quad \text{as } y\to\infty\,. \tag{4.2}$$

Here δ_s is the singlet phase shift at relative np momentum p. Only the $\pi\pi$ meson-exchange current contributes significantly to G_{12}. Equations (7.12) and (7.14) of ACM give

$$(G_{12})_{\pi\pi} = (G^2/32M)\eta_s\,, \tag{4.3}$$
$$\eta_s = \frac{8}{3\pi}\int_0^\infty \frac{e^{-m_\pi y}}{y}\,(1-\tfrac{1}{2}m_\pi y)z_s(y,0)u(y)dy\,,$$

where G is the π-nucleon coupling constant. Numerical values (in natural units) are 680 $\text{fm}^{3/2}$ for the impulse contribution and 8 $\text{fm}^{3/2}$ for the $\pi\pi$-exchange current contribution. These values are not very model-dependent and lead to a theoretical value for the thermal np capture of 310 mb, which is about 7% below the experimental value of 334.2 mb.

In the context of the np capture reaction G_3 and G_4, representing the $^3S\to d$ transition, have received little serious attention. BR[15] have speculated on the magnitude of G_3. However, for the scattering problem, results are available.[3] For the impulse approximation

$$\frac{G_3(\vec{q}\,^2)_{ia}}{\sqrt{2}} = \tfrac{1}{2}(\mu_p+\mu_n)(H_t+J_t)\,,$$
$$\frac{G_4(\vec{q}\,^2)_{ia}}{\sqrt{2}} = 3(\mu_p+\mu_n)J_t - 12K_t\,,$$
$$H_t(\vec{q}\,^2) = \int_0^\infty u(y)z_t(y,0)j_0(\tfrac{1}{2}qy)dy\,, \tag{4.4}$$
$$J_t(\vec{q}\,^2) = \frac{1}{\sqrt{8}}\int_0^\infty w(y)z_t(y,0)j_2(\tfrac{1}{2}qy)dy\,,$$
$$K_t(\vec{q}\,^2) = \frac{1}{\sqrt{8}}\int_0^\infty [yz_t'(y,0)-z_t(y,0)]w(y)\frac{j_2(\frac{1}{2}qy)}{q^2y^2}\,dy\,.$$

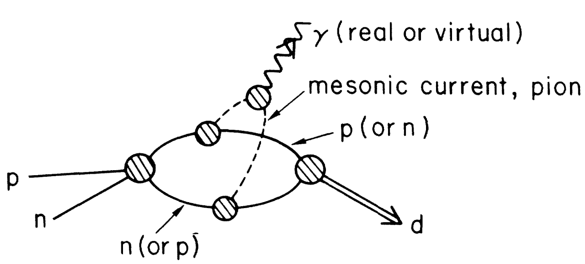

FIG. 3. The meson-exchange current, in this case with pions, which are the only relevant contribution. The np and npd blobs, as in Fig. 2, are described by nonrelativistic wave functions, while the π-nucleon and the π-γ vertices are described by conventional point coupling constants, as discussed in detail in Refs. 2 and 21. Again a simple nonrelativistic matrix element describes the process.

Evidently J_t vanishes at $\vec{q}\,^2=0$. Moreover, since u and z_t are, in general, supposed to be solutions of a Schrödinger or Rarita-Schwinger equation for different energy eigenvalues, they are orthogonal. Thus H_t also vanishes at $\vec{q}\,^2=0$. This will be true independent of the details of the dynamics if u and z_t are representations of the dressed two-nucleon eigenstates of a Hermitian Hamiltonian operator. We will thus retain only the lowest-order nonvanishing terms in ϵ^2:

$$H_t \simeq \frac{\epsilon^2}{24}\int_0^\infty z_t(y,0)u(y)y^2dy\,,$$
$$J_t \simeq \frac{\epsilon^2}{60\sqrt{8}}\int_0^\infty z_t(y,0)w(y)y^2dy\,, \tag{4.5}$$
$$K_t \simeq \frac{1}{60\sqrt{8}}\int_0^\infty [z_t'(y,0)y - z_t(y,0)]w(y)dy\,.$$

The orthogonality of z_t and u has been assumed, as motivated by the above paragraph. However, it seems to be a distinct possibility that our understanding of nucleon-nucleon dynamics is not adequate to allow us to drop the zeroth-order term of H_t on the grounds of orthogonality. This point has been stressed by BR[15] and is closely related to the normalization problem in the Bethe-Salpeter equation.[29] We will speculate on the size of H_t further in Sec. 9.

An approximate result for the lightest exchange current contribution to G_3 and G_4 can be easily inferred from Ref. 21 if the D wave of the deuteron is ignored. The lowest mass exchange current allowed by G parity is the $\rho\pi$ system, which is treated in Ref. 21 for elastic electron-deuteron scattering. We can convert the results for elastic scattering to a statement about G_3 as follows: comparing the current (1.2) of Ref. 21 with the current (3.10) of the present work we can make the correspondence $G_3/\sqrt{2} \leftrightarrow G_{Md}/2$. This actually becomes an identity if one neglects the D state and substitutes the function z_t for one of the functions u that occurs in G_{Md}. Explicitly we obtain from Eq. (3.11) of Ref. 21,

$$\frac{(G_3)_{\rho\pi}}{\sqrt{2}} = \frac{1}{2}\,\frac{8\Gamma}{3\pi(m_\rho^{\,2}-m_\pi^{\,2})}\eta_t\,,$$
$$\eta_t = \int_0^\infty \left(m_\rho^{\,2}\frac{e^{-m_\rho y}}{y} - m_\pi^{\,2}\frac{e^{-m_\pi y}}{y}\right)u(y)z_t(y,0)dy\,, \tag{4.6}$$
$$\frac{(G_4)_{\rho\pi}}{\sqrt{2}} = 0\,.$$

The constant G is the pion-nucleon coupling constant, $g_{\rho\pi\gamma}$ is the $\rho\pi\gamma$ coupling constant, and a is the ρ-nucleon coupling constant.

We desire only a crude estimate of the parameters G_3 and G_4, since our purpose is to show that they are very small compared to $G_{12}=688\ \text{fm}^{3/2}$. Accordingly, we will adopt simple analytic wave functions as used in effective-range theory[1,2,30]:

$$u(y)=N_g(e^{-\alpha y}-e^{-\beta y}),$$
$$z_t(y,0)=\sqrt{4\pi}(y-a_t+a_t e^{-\xi_t y}), \tag{4.7}$$
$$w(y)=N_g\eta(e^{-\alpha y}-e^{-\beta y})\left(1+\frac{3}{\alpha y}+\frac{3}{\alpha^2y^2}\right).$$

The functions u and z_t are well known and in common use, while w is an *ad hoc* modification of the usual asymptotic D wave. The w function behaves like $1/y$ at small y instead of y^3 as it should, but the integrals in which w occurs, J_t and K_t, receive their main contribution from large y. Our estimates should therefore be reasonable.

The value of $g_{\rho\pi\gamma}$ in Eq. (4.6) is not well known, so for a crude estimate of η_t we will assume that $g_{\rho\pi\gamma}a/e\sim 1$. With the numerical values of the effective-range parameters quoted in Ref. 30 we then obtain the following approximate values:

Impulse, S-wave contribution,

$$G_3=-1.25\times10^{-2}\ \text{fm}^{3/2};$$

Impulse, D-wave contribution,

$$G_3=5.96\times10^{-5}\ \text{fm}^{3/2};$$

Exchange, S-wave contribution, (4.8)

$$G_3=-9.28\times10^{-2}\ \text{fm}^{3/2};$$

Total,

$$G_3=-0.105\ \text{fm}^{3/2};$$

and

Impulse, D-wave contribution,

$$G_4=-1.46\ \text{fm}^{3/2}. \tag{4.9}$$

Evidently G_3 and G_4 are very small in magnitude compared to $G_{12}=G_1+G_2=686\ \text{fm}^{3/2}$. As discussed in the Ref. 3 erratum and in Sec. 7, Eq. (7.5) of this work, the combination $(G_{12})^2+G_3{}^2+G_4{}^2$ determines the total cross section for radiative np capture. Thus, the above numerical results may be interpreted to mean that, in the context of the conventional nonrelativistic theory in Ref. 3, the $^3S\to d$ transition has a negligible effect on the total np capture cross section. The validity of this statement will be discussed further in Sec. 9.

Before continuing we wish to note the features of the preceding theoretical results that we feel are on safe ground and those that we feel might be considered suspect. Certainly the treatment of the np and deuteron center-of-mass motion in terms of momentum eigenstates is on very safe ground. This is a common feature of a diagrammatic phenomenology. Only the internal-structure description is open to serious doubt. The retention of only the diagrams we have considered (impulse and π exchange) has been rather well justified in numerous previous works.[19-23] A small ambiguity does exist in the form of the Dirac electromagnetic current of the nucleons, since the nucleons are not exactly on the mass shell.[31] Certainly the q^2 behavior of the nucleon current is expected to be of very little consequence; however, one may question the off-mass-shell behavior of the isovector magnetic moment, and the behavior of the third nucleon form factor that does not occur at all on the mass shell.[32,33] Both effects could be significant in capture from the ^1S state. As mentioned in the Introduction, these difficult questions are now under consideration.[13] Lastly, the nucleon-nucleon dynamics may possess no nonrelativistic limit, in which case an exact orthogonality theorem is not justified. This is the point stressed by BR.[15] We will discuss it further in Sec. 9.

5. PHASE SPACE

The S matrix for radiative capture is given in Eq. (3.1). To obtain a differential cross section we must take its absolute value squared, as a measure of the transition probability, then perform appropriate sums over unobserved spins and momenta. In this section we will suppose the capture γ ray is to be observed, and perform the integral over unobserved momenta. The sum over spins constitutes the heart of a polarization calculation; we will defer that problem to Sec. 6.

The absolute square of S, divided by the neutron flux and the number of proton targets, is

$$\frac{e^2(2\pi)^4\delta^4(Q_f+q-Q_i)\,|\vec{J}\cdot\vec{\epsilon}\,|^2}{2\omega v_n}, \tag{5.1}$$

where v_n is the neutron velocity. If this is now summed over appropriate spins and integrated over unobserved momenta, we obtain (see Fig. 4)

$$d\sigma=\int\frac{d^3Q_f}{(2\pi)^2}(q^2dq\,d\Omega)\delta^4(Q_f+q-Q_i)\frac{e^2|\mathfrak{M}|^2}{2\omega v_n}, \tag{5.2}$$

where

$$|\mathfrak{M}|^2=\sum(J^kJ^{l*}\epsilon^k\epsilon^l)=\sum|\vec{J}\cdot\vec{\epsilon}\,|^2 \tag{5.3}$$

is the spin sum to be performed in the next section. The integrals over $\vec{\Omega}_f$ and $\vec{q}$ are easy if we ignore deuteron recoil and set $q=\omega=\epsilon$, the deuteron binding energy:

$$\frac{d\sigma}{d\Omega}=\frac{e^2\epsilon\,|\mathfrak{M}|^2}{2(2\pi)^2v_n}. \tag{5.4}$$

Since we ignore the deuteron recoil momentum, there is no angular dependence in $d\sigma/d\Omega$ due to phase space. We will find in the following sections that there is an angular dependence due to the dynamics of the $^3S\to d$ capture process.

6. SPIN SUMS WITH POLARIZED NUCLEONS

Our task is to evaluate the invariant $|\mathfrak{M}|^2$ in (5.3) for the ^1S and ^3S initial states. If the transition current for either initial state is written as

$$\vec{J}=\chi_m^\dagger\vec{\mathcal{J}}\chi\,, \tag{6.1}$$

then $|\mathfrak{M}|^2$ will be a sum over such terms dotted into $\vec{\epsilon}$, with the initial spin states written as

$$\chi=\alpha\alpha,\ \alpha\beta,\ \beta\alpha,\ \beta\beta\,. \tag{6.2}$$

Let P_p be the probability that the proton spin is up, and P_n that the neutron spin is up. Then the state $\alpha\alpha$ occurs with the probability P_pP_n, the state $\alpha\beta$ occurs $P_p(1-P_n)$ and so forth. The sum over nucleon spins in $|\mathfrak{M}|^2$ is then given explicitly by

$$|\mathfrak{M}|^2=\sum[P_pP_n|\chi_m^\dagger(\vec{\mathcal{J}}\cdot\vec{\epsilon})\alpha\alpha|^2+(P_p-P_pP_n)|\chi_m^\dagger(\vec{\mathcal{J}}\cdot\vec{\epsilon})\alpha\beta|^2+(P_n-P_nP_p)|\chi_m^\dagger(\vec{\mathcal{J}}\cdot\vec{\epsilon})\beta\alpha|^2+(1-P_p)(1-P_n)|\chi_m^\dagger(\vec{\mathcal{J}}\cdot\vec{\epsilon})\beta\beta|^2]\,. \tag{6.3}$$

The remaining sum is over the deuteron-spin label m and, if desired, the γ polarization.

Dynamically the singlet and triplet states are appropriate to the evaluation of $|\mathfrak{M}|^2$, not the states in (6.2). Thus we replace the states in (6.2) by the set of total spin angular momentum eigenstates

$$\begin{aligned}&\chi_s=(\alpha\beta-\beta\alpha)/\sqrt{2}\,,\\&\chi_m=\alpha\alpha,\ (\alpha\beta+\beta\alpha)/\sqrt{2},\ \beta\beta\,.\end{aligned} \tag{6.4}$$

In terms of these, $|\mathfrak{M}|^2$ may be written more conveniently as

$$\begin{aligned}|\mathfrak{M}|^2=\sum\Big\{&P_pP_n|\chi_m^\dagger(\vec{\mathcal{J}}\cdot\vec{\epsilon})\chi_{+1}|^2+(1-P_n)(1-P_p)|\chi_m^\dagger(\vec{\mathcal{J}}\cdot\vec{\epsilon})\chi_{-1}|^2+\left(\frac{P_p+P_n}{2}-P_pP_n\right)|\chi_m^\dagger(\vec{\mathcal{J}}\cdot\vec{\epsilon})\chi_0|^2\\&+\left(\frac{P_p+P_n}{2}-P_pP_n\right)|\chi_m^\dagger(\vec{\mathcal{J}}\cdot\vec{\epsilon})\chi_s|^2+(P_p-P_n)\mathrm{Re}[\chi_m^\dagger(\vec{\mathcal{J}}\cdot\vec{\epsilon})\chi_0\chi_s(\vec{\mathcal{J}}\cdot\vec{\epsilon})\chi_m^\dagger]\Big\}.\end{aligned} \tag{6.5}$$

The last term vanishes when the deuteron-spin sum is taken, so we will drop it henceforth.

For the moment consider the special case wherein the $^3S\to d$ transition is to be ignored, and no nucleon-spin polarization is present; that is, $P_p=P_n=\frac{1}{2}$. This corresponds to the ordinary np capture problem, and $|\mathfrak{M}|^2$ reduces to the usual form

$$|\mathfrak{M}|^2=\tfrac{1}{4}\sum|\chi_m^\dagger(\vec{\mathcal{J}}\cdot\vec{\epsilon})\chi_s|^2\,, \tag{6.6}$$

where the factor $\frac{1}{4}$ may be interpreted as the probability of n and p being in the singlet state.

Trace techniques allow us to evaluate the deuteron-spin sum very easily; we will assume that no deuteron-polarization measurement is to be made. Then the sum over the spin label m of any term in $|\mathfrak{M}|^2$ is easily evaluated. For example, the first term is

$$\begin{aligned}\sum_m|\chi_m^\dagger(\vec{\mathcal{J}}\cdot\vec{\epsilon})\chi_{+1}|^2&=\sum_m[\chi_{+1}^\dagger(\vec{\mathcal{J}}^\dagger\cdot\vec{\epsilon})\chi_m\chi_m^\dagger(\vec{\mathcal{J}}\cdot\vec{\epsilon})\chi_{+1}]\\&=\chi_{+1}^\dagger[(\vec{\mathcal{J}}^\dagger\cdot\vec{\epsilon})\sigma_t(\vec{\mathcal{J}}\cdot\vec{\epsilon})]\chi_{+1}\\&\equiv\chi_{+1}^\dagger T\chi_{+1}\,.\end{aligned} \tag{6.7}$$

That is, the sum over m is equivalent to inclusion of the triplet spin projection operator,[34]

$$\sigma_t=\tfrac{1}{4}(3+\vec{\sigma}_p\cdot\vec{\sigma}_n) \tag{6.8}$$

between the two current operators.

To evaluate the various matrix elements of J we will again use standard trace techniques; it is only necessary to have a projection operator for the singlet state and for each of three triplet states. These are easy to obtain for the standard states given in (6.4); utilizing the triplet projection operator σ_t as previously noted, and $\sigma_\pm$ defined by

$$\begin{aligned}&\sigma_\pm=\tfrac{1}{2}(1\pm\sigma^3)\,,\\&\sigma_+\alpha=\alpha,\quad\sigma_+\beta=0,\quad\sigma_-\alpha=0,\quad\sigma_-\beta=\beta\,,\end{aligned} \tag{6.9}$$

we arrive at

$$\begin{aligned}P_{+1}&=\tfrac{1}{4}(1+\sigma_p{}^3+\sigma_n{}^3+\sigma_p{}^3\sigma_n{}^3)\\P_0&=\tfrac{1}{4}(1+\vec{\sigma}_p\cdot\vec{\sigma}_n-2\sigma_p{}^3\sigma_n{}^3)\\P_{-1}&=\tfrac{1}{4}(1-\sigma_p{}^3-\sigma_n{}^3+\sigma_p{}^3\sigma_n{}^3)\\P_s&=\tfrac{1}{4}(1-\vec{\sigma}_p\cdot\vec{\sigma}_n)\,.\end{aligned} \tag{6.10}$$

These are conveniently rewritten with an arbitrary direction $\vec{p}$ chosen as the spin axis, instead of the z axis:

$$P_{\pm 1}=\tfrac{1}{4}[1\pm(\vec{\sigma}_p\cdot\vec{\rho})\pm(\vec{\sigma}_n\cdot\vec{\rho})+(\vec{\sigma}_p\cdot\vec{\rho})(\vec{\sigma}_n\cdot\vec{\rho})],$$
$$P_0=\tfrac{1}{4}[1+(\vec{\sigma}_p\cdot\vec{\sigma}_n)-2(\vec{\sigma}_p\cdot\vec{\rho})(\vec{\sigma}_n\cdot\vec{\rho})]. \qquad (6.11)$$

P_s, of course, is unchanged.

The task of expressing and evaluating the matrix elements which occur in $|\mathfrak{M}|^2$ is now quite simple. The first term in (6.5), for example, becomes

$$|\chi_m^\dagger(\vec{\mathcal{J}}\cdot\vec{\epsilon})\chi_{+1}|^2=\chi_{+1}^\dagger[(\vec{\mathcal{J}}^\dagger\cdot\vec{\epsilon})\sigma_t(\vec{\mathcal{J}}\cdot\vec{\epsilon})]\chi_{+1}$$
$$=\chi_{+1}^\dagger T\chi_{+1}=\mathrm{Tr}(\chi_{+1}\chi_{+1}^\dagger T)=\mathrm{Tr}(P_+T), \qquad (6.12)$$

and the other terms are equally simple. Now $|\mathfrak{M}|^2$ may be rewritten as

$$|\mathfrak{M}|^2=P_pP_n\mathrm{Tr}(P_{+1}T)+(1-P_p)(1-P_n)\mathrm{Tr}(P_{-1}T)+\left(\frac{P_p+P_n}{2}-P_nP_p\right)[\mathrm{Tr}(P_0T)+\mathrm{Tr}(P_sT)]. \qquad (6.13)$$

To complete our evaluation of $|\mathfrak{M}|^2$ we need only evaluate T for the triplet and singlet transition currents, and calculate $\mathrm{Tr}(PT)$ for each projection operator. We obtain for the singlet transition

$$T=\frac{\vec{q}^2G_{12}{}^2}{4M^2}[6\hat{q}^2+2(\vec{\epsilon}\cdot\hat{q}\times\vec{\sigma}_p)(\vec{\epsilon}\cdot\hat{q}\times\vec{\sigma}_n)], \qquad \hat{q}^2=1, \qquad (6.14)$$

and for the triplet

$$T=\frac{\vec{q}^2}{4M^2}[\tfrac{1}{2}(3G_3{}^2+3G_4{}^2+2G_3G_4)-(G_4{}^2+3G_3G_4)(\hat{q}\cdot\vec{\sigma}_n)(\hat{q}\cdot\vec{\sigma}_p)$$
$$+\tfrac{1}{2}(G_4{}^2-G_3{}^2)(\vec{\sigma}_p\cdot\vec{\sigma}_n)+(G_3G_4-G_4{}^2)(\vec{\epsilon}\cdot\vec{\sigma}_n)(\vec{\epsilon}\cdot\vec{\sigma}_p)+(G_3{}^2-G_3G_4)(\vec{\epsilon}\cdot\hat{q}\times\vec{\sigma}_p)(\vec{\epsilon}\cdot\hat{q}\times\vec{\sigma}_n)]. \qquad (6.15)$$

Evaluation of $\mathrm{Tr}(PT)$ is straightforward. We use the following easily proved trace theorems,

$$\mathrm{Tr}P=1, \qquad \mathrm{Tr}(P_{\pm 1}\sigma_p^l\sigma_n^t)=\rho^l\rho^t, \qquad \mathrm{Tr}(P_0\sigma_p^l\sigma_n^t)=\delta^{lt}-2\rho^l\rho^t, \qquad (6.16)$$

and obtain

$$\mathrm{Tr}(P_sT)=\vec{q}^2G_{12}{}^2/M^2,$$
$$\mathrm{Tr}(P_{\pm 1}T)=\mathrm{Tr}(P_{+1}T)=\frac{\vec{q}^2}{4M^2}[(2G_4{}^2+G_3{}^2+G_3G_4)-(G_4{}^2+3G_3G_4)(\hat{q}\cdot\vec{\rho})^2+(G_3G_4-G_4{}^2)(\vec{\epsilon}\cdot\vec{\rho})^2+(G_3{}^2-G_3G_4)(\vec{\epsilon}\cdot\hat{q}\times\vec{\rho})^2], \qquad (6.17)$$
$$\mathrm{Tr}(P_0T)=\frac{\vec{q}^2}{4M^2}[(2G_3{}^2-2G_3G_4)+(2G_4{}^2+6G_3G_4)(\hat{q}\cdot\vec{\rho})^2-(2G_3G_4-2G_4{}^2)(\vec{\epsilon}\cdot\vec{\rho})^2-(2G_3{}^2-2G_3G_4)(\vec{\epsilon}\cdot\hat{q}\times\vec{\rho})^2].$$

This is a very general result. We have allowed arbitrary polarization of both nucleons, we have used the most general transition currents, and we have not as yet specified the γ polarization. In the following sections we will extract experimentally interesting results from the expression (6.13) for $|\mathfrak{M}|^2$ and the expressions in (6.17).

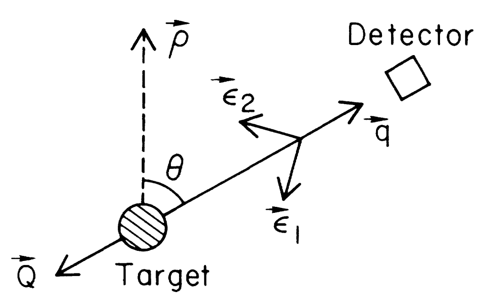

FIG. 4. Kinematics of the capture process for polarized n and p. The γ ray is detected; $\vec{q}$ is its momentum, and $\vec{\epsilon}$ its polarization. The $\vec{\rho}$ vector is the proton and neutron polarization reference vector, and the deuteron recoil momentum is $\vec{Q}$.

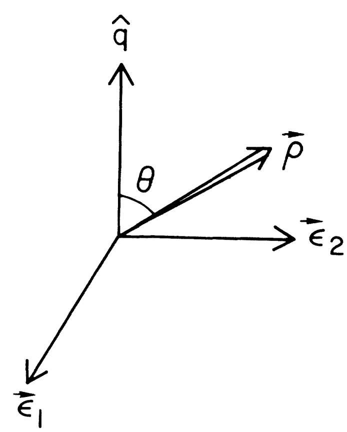

FIG. 5. Choice of polarization and γ-momentum vectors.

7. ANGULAR DISTRIBUTION OF PHOTONS

Let us consider an experiment where the photon polarization $\vec{\epsilon}$ is not detected. As is clear in Figs. 4 and 5, we may choose a coordinate system in which the relevant three-vectors are simple:

$$\begin{aligned} \hat{q} &= (0, 0, 1); \\ \vec{p} &= (0, \sin\theta, \cos\theta); \\ \vec{\epsilon}_1 &= (1, 0, 0); \\ \vec{\epsilon}_2 &= (0, 1, 0). \end{aligned} \tag{7.1}$$

Then we need merely note that

$$\begin{aligned} &(\hat{q}\cdot\vec{p})^2 = \cos^2\theta, \quad (\vec{\epsilon}_1\cdot\vec{p})^2 = 0, \quad (\vec{\epsilon}_2\cdot\vec{p})^2 = \sin^2\theta, \\ &(\vec{\epsilon}_1\cdot\hat{q}\times\vec{p})^2 = \sin^2\theta, \quad (\vec{\epsilon}_2\cdot\hat{q}\times\vec{p})^2 = 0, \end{aligned} \tag{7.2}$$

and the algebra necessary to sum over γ polarization is complete. The invariant $|\mathfrak{M}|^2$ is now, from (6.13) and (6.17),

$$\begin{aligned} |\mathfrak{M}|^2 = \frac{\vec{q}^2}{2M^2}\Big\{ &2G_{12}^2(P_p + P_n - 2P_pP_n) + G_3^2 + G_4^2 \\ &+ (4P_pP_n - 2P_p - 2P_n + 1) \\ &\times\left[(G_4^2 + G_3^2 + 6G_3G_4)\frac{\sin^2\theta}{2} - 2G_3G_4\right]\Big\}. \end{aligned} \tag{7.3}$$

From (5.4), the cross section is

$$\begin{aligned} \frac{d\sigma}{d\Omega} = \frac{e^2\epsilon^3}{16\pi^2M^2v_n}\Big\{ &2(P_p + P_n - 2P_pP_n)G_{12}^2 + G_3^2 + G_4^2 \\ &+ (4P_pP_n - 2P_p - 2P_n + 1) \\ &\times\left[(G_4^2 + G_3^2 + 6G_3G_4)\frac{\sin^2\theta}{2} - 2G_3G_4\right]\Big\}. \end{aligned} \tag{7.4}$$

A number of features of this result should be noted. (a) The singlet transition (the G_{12} term) has no angular dependence. The $\sin^2\theta$ dependence is due entirely to the supposedly small triplet-transition factors G_3 and G_4. (b) For the case of no polarization, $P_p = P_n = \frac{1}{2}$, the coefficient of $\sin^2\theta$ vanishes. The cross section becomes independent of angle, and is

$$\begin{aligned} \frac{d\sigma}{d\Omega} &= \frac{e^2\epsilon^3}{16\pi^2M^2v_n}(G_{12}^2 + G_3^2 + G_4^2), \\ \sigma_T &= \frac{e^2\epsilon^3}{(2\pi)2M^2v_n}(G_{12}^2 + G_3^2 + G_4^2). \end{aligned} \tag{7.5}$$

This agrees with previous results on np capture and electrodisintegration.[2,3] (c) If either P_p or P_n is $\frac{1}{2}$, the value of the other is irrelevant and (7.5) again results. (d) For *total* polarization, $P_p = P_n = 1$, the coefficient of G_{12} vanishes and

$$\frac{d\sigma}{d\Omega} = \frac{e^2\epsilon^3}{16\pi^2M^2v_n}\times\left[(G_4 - G_3)^2 + \tfrac{1}{2}(G_3^2 + G_4^2 + 6G_3G_4)\sin^2\theta\right]. \tag{7.6}$$

8. PHOTON POLARIZATION ASYMMETRY

In this section we wish to calculate the difference between $d\sigma/d\Omega$ for the two different polarization directions shown in Figs. 4 and 5. The difference will be proportional to the difference between $|\mathfrak{M}|^2$ for the two directions. This is easily read from (6.13), (6.17), and (7.2);

$$\begin{aligned} \Delta|\mathfrak{M}|^2 &= |\mathfrak{M}|^2_{\epsilon_1} - |\mathfrak{M}|^2_{\epsilon_2} \\ &= \frac{\vec{q}^2}{4M^2}\left[(4P_pP_n - 2P_p - 2P_n + 1)\,(G_3 - G_4)^2\sin^2\theta\right]. \end{aligned} \tag{8.1}$$

Thus the difference between differential cross sections is proportional to $\sin^2\theta$ and is explicitly

$$\left(\frac{d\sigma}{d\Omega}\right)_{\epsilon_1} - \left(\frac{d\sigma}{d\Omega}\right)_{\epsilon_2} = \frac{e^2\epsilon^3}{16\pi^2M^2v_n}\left[(G_3 - G_4)^2 \times(4P_pP_n - 2P_p - 2P_n + 1)\frac{\sin^2\theta}{2}\right]. \tag{8.2}$$

It would seem, at present, a very difficult task to measure the polarized γ cross section. Hence, we will merely note here that the difference depends only on the ^3S transition form factors.

9. MEASURABLE EFFECTS

In the preceding sections we have shown that the angular distribution of emitted γ rays and the polarization asymmetry depend only on the $^3S\to d$ transition, through the parameters G_3 and G_4. We wish to consider the possibility of a measurable angular distribution in the differential cross section.

First consider the total radiative np capture cross section with no polarization. As previously noted in Secs. 1 and 4, present theory gives an answer that is about 7% too low; the contribution of the $^3S\to d$ transition is generally ignored in calculations. From (7.5) we can write the cross section, including the $^3S\to d$ transition, as

$$\begin{aligned} \sigma_T &= \frac{e^2\epsilon^3G_{12}^2}{(2\pi)2M^2v_n}[1 + R_3^2 + R_4^2] \\ &= \sigma_s[1 + R_3^2 + R_4^2], \quad R_3 = \frac{G_3}{G_{12}}, \quad R_4 = \frac{G_4}{G_{12}}, \end{aligned} \tag{9.1}$$

where the factor σ_s outside the brackets is the usual theoretical cross section of 310 mb. Our estimates of G_3 and G_4 from Sec. 4, Eqs. (4.8) and (4.9), yield, with $G_{12} = 688\ \text{fm}^{3/2}$,

$$R_3 = -1.5\times 10^{-4},$$
$$R_4 = -2.1\times 10^{-3}; \tag{9.2}$$
$$R_3^2 + R_4^2 = 4.5\times 10^{-6}.$$

These are crude estimates, but they indicate that the $^3S\to d$ transition is several orders of magnitude too small to explain the discrepancy in the total capture cross section. Indeed, if $^3S\to d$ were responsible for the discrepancy, $R_3^2 + R_4^2$ would need to be 0.07. The total cross section for the $^3S\to d$ transition is, from Eqs. (9.1) and (9.2), about 1.4 μb.

As noted in Sec. 4 the possibility certainly exists that the $^3S\to d$ transition is a great deal larger than our estimates indicate. We will consider the angular dependence of the cross section for γ emission for two cases: the first is that in which our numerical estimates of G_3 and G_4 from Sec. 4 are used, and the second is that for which G_3 and G_4 "explain" the 7% discrepancy, i.e., $R_3^2 + R_4^2 = 0.07$.

The differential cross section, Eq. (7.4), can be written in terms of R_3 and R_4 as

$$\frac{d\sigma}{d\Omega} = \frac{\sigma_s}{4\pi}\Big\{2(P_p + P_n - 2P_pP_n) + R_3^2 + R_4^2 + (4P_pP_n - 2P_p - 2P_n + 1)\times\left[(R_3^2 + R_4^2 + 6R_3R_4)\frac{\sin^2\theta}{2} - 2R_3R_4\right]\Big\}. \tag{9.3}$$

Thus the dependence of $d\sigma/d\Omega$ on $\sin^2\theta$ is via the quantity $R_3^2 + R_4^2 + 6R_3R_4$.

For example, if we use $P_p = P_n = 0.75$ and suppose $R_3 = 0$, we obtain

$$\frac{d\sigma}{d\Omega} = \frac{\sigma_s}{4\pi}\left(0.75 + \frac{R_4^2}{8} + \frac{R_4^2}{8}\sin^2\theta\right). \tag{9.4}$$

If our estimates of Sec. 4 are correct, the angular distribution is proportional to

$$(0.75 + 0.56\times 10^{-6}\sin^2\theta); \tag{9.5}$$

whereas if R_4 is sufficiently large to explain the discrepancy, $R_4^2 = 0.07$, then the distribution is

$$(0.76 + 0.0087\sin^2\theta). \tag{9.6}$$

The second distribution would appear to be measurable, the first not.[15] In Table I we give values for the bracket in (9.3) for various combinations of R_3, R_4, P_p, and P_n, as calculated from (9.3) with $R_3^2 + R_4^2 = 0.07$. It is clear from the table that a measurement of an "anomalously" large $^3S\to d$ should be possible, although the roles of G_3 and G_4 cannot be separated because of their symmetric appearance.

A measurement of the γ polarization, as calculated in Sec. 8, would appear to be intrinsically more difficult. We will not discuss it further except to note that the dependence on the G_3 and G_4 parameters is symmetric and similar to the above.

It is interesting to note that if the np capture

TABLE I. Values of E and F for $R_3 = R_4 = \sqrt{0.07/2}$, and for $R_3 = 0$ and $R_4 = \sqrt{0.07}$ or vice versa. The cross section is written as

$$\frac{d\sigma}{d\Omega} = \frac{\sigma_s}{4\pi}(E + F\sin^2\theta),\quad \sigma_s \cong 309.5 \pm 5.0\ \text{mb},$$

with E given by the top number in each case, F by the bottom number. These are calculated from

$$E = 2(P_p + P_n - 2P_pP_n) + \delta - 2R_3R_4(4P_pP_n - 2P_p - 2P_n + 1),$$
$$F = \tfrac{1}{2}(4P_pP_n - 2P_n - 2P_p + 1)(\delta + 6R_3R_4),$$

where δ is the discrepancy in np capture, here taken to be 0.07. The table is symmetric in P_p and P_n.

$P_n \backslash P_p$	0.5	0.6	0.7	0.8	0.9	1.0
			$R_3 = R_4 = \sqrt{0.07/2}$			
0.5	1.07 0.00					
0.6	1.07 0.00	1.03 0.006				
0.7	1.07 0.00	0.984 0.011	0.899 0.022			
0.8	1.07 0.00	0.942 0.017	0.813 0.034	0.685 0.050		
0.9	1.07 0.00	0.899 0.022	0.728 0.045	0.556 0.067	0.385 0.090	
1.0	1.07 0.00	0.856 0.030	0.642 0.056	0.428 0.084	0.214 0.112	0.000 0.140
			$R_3 = 0,\ R_4 = \sqrt{0.07}$			
0.5	1.07 0.00	1.07 0.00	1.07 0.00	1.07 0.00	1.07 0.00	1.07 0.00
0.6		1.03 1.03	0.990 0.003	0.950 0.004	0.910 0.006	0.870 0.007
0.7			0.910 0.006	0.830 0.008	0.750 0.011	0.670 0.114
0.8				0.710 0.013	0.590 0.017	0.470 0.021
0.9					0.430 0.022	0.270 0.028
1.0						0.070 0.035

discrepancy is attributable to nonorthogonality of the wave functions z_t and u, then H_t is easily estimated from the demand $R_3^2 \cong 0.07$ and the relations (4.1) and (4.4),

$$H_t \cong H_s\left(\frac{\mu_p - \mu_n}{\mu_p + \mu_n}\right)\frac{R_3}{\sqrt{2}} \cong H_s. \tag{9.7}$$

Thus the necessary orthogonality breaking is very large indeed; it is comparable to the overlap between u and z_s. This seems highly unlikely, although obviously not impossible.

10. CONCLUSIONS

A 7% discrepancy exists between theory and experiment in *np* capture. Theoretical analyses have included the effects of realistic wave functions, meson-exchange currents, finite nucleon size, nucleon resonances, recoil and semirelativistic effects, and the $^3S \to d$ transition. If the theoretical treatment of the $^3S \to d$ transition should be incorrect, so that the discrepancy is actually accounted for by $^3S \to d$, then a significant angular dependence should be observed in the following reaction: (polarized p) + (polarized n) $\to d$ + (detected γ ray). The distribution is given in Table I for various values of polarization.

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