

## Analysis of the $(p, 2p)$ Reaction on Deuterium at 600 MeV\*

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A calculation of the differential cross section ( $d\sigma/dE d\Omega^2$ ) for the  $D(p, 2p)$  process at 600 MeV has been carried out using as input the available nucleon-nucleon elastic scattering data and a model deuteron wave function. The calculation includes Glauber double-scattering and final-state-interaction corrections to the plane-wave impulse approximation (PWIA). The calculation agrees favorably with the recent  $D(p, 2p)$  experimental data at most residual neutron recoil momenta and is an improvement over the earlier PWIA analysis. The recently proposed  $D^*$  component of the deuteron appears to be unimportant in reactions of this kind.

### I. INTRODUCTION

Recently the  $D(p, 2p)$  reaction was observed up to rather large residual neutron recoil momenta by Perdrisat *et al.*<sup>1</sup> The differential cross-section data were analyzed in terms of the plane-wave impulse approximation (PWIA), which seemed unsatisfactory at the larger neutron recoil momenta where the theoretical cross section predictions were rather small. An analysis incorporating multiple-scattering processes, in addition to the single-scattering contribution of the PWIA, was suggested to clarify the situation. Taking this as our motivation, we have carried out a calculation of the  $D(p, 2p)$  differential cross section which includes Glauber double-scattering processes,<sup>2</sup> as well as final-state-interaction (FSI) corrections to the PWIA. As input we employed the nucleon-nucleon phase shifts of MacGregor, Arndt, and Wright<sup>3</sup> together with  $NN$  total and differential cross-section data<sup>4</sup> at energies where phase shifts were unavailable. As the calculation in the large-recoil-momentum region depends critically on the large-momentum components of the deuteron wave function, a careful choice of wave function is necessary. We have employed the best Moravcsik fit<sup>5</sup> to the Gartenhaus wave function,<sup>6</sup> which for our purposes is the same as the Hamada-Johnston wave function.<sup>7</sup> Excellent agreement with experiment was obtained up to a rather large neutron recoil momentum. At momenta above this point, the agreement was improved considerably over the PWIA results of Ref. 1. Multiple-scattering effects do indeed become important at large recoil momenta, although it is essential that one employ a reasonable deuteron wave function. It appears, however, that the incorporation of the  $D^*$  component of the deuteron<sup>8</sup> would not influence our results. An ambiguity which arises from approximating off-energy-shell nucleon-nucleon scattering amplitudes by experimental on-shell amplitudes limits to some extent the accuracy of our

calculation, particularly in the large-recoil-momentum region. Further investigation of this old and difficult problem seems warranted.

Section II contains a summary of the multiple-scattering expansion, which gives the amplitude for scattering on a deuteron in terms of the slightly off-energy-shell amplitudes for scattering on free protons and neutrons. In Sec. III we describe how the formulas of Sec. II are applied in our calculation, and in Sec. IV we present and discuss our results.

### II. MULTIPLE-SCATTERING EXPANSION

In this section we derive, following the work of Faddeev,<sup>9</sup> the multiple-scattering expansion for the case of inelastic scattering of a proton from a deuteron target. Our result is identical to that obtained by Everett using the ideas of Chew and Goldberger.<sup>10</sup> We call the projectile proton particle No. 1 and the target proton and neutron particles Nos. 2 and 3, respectively. The relativistic kinetic energy operators are denoted by  $K_j$ ,  $j = 1, 2, 3$ . Assuming the nuclear interaction is mediated by two-body forces, we let  $V_j$  represent the interaction of particles  $k$  and  $n$ , with  $j, k, n$  a permutation of 1, 2, 3. The Hamiltonian for our three-nucleon system is then

$$H = H_0 + V, \quad (1)$$

where  $H_0 = K_1 + K_2 + K_3$  and  $V = V_1 + V_2 + V_3$ . We let  $\mathcal{E}$  denote the energy of the system.

The cross section for scattering from a state described asymptotically by the wave function  $\varphi_i$  to the state  $\varphi_f$  is proportional to the absolute value squared of the transition amplitude

$$\langle f | \mathcal{T} | i \rangle = \langle \varphi_f | V_f | \Psi_i \rangle, \quad (2)$$

where  $V_f$  is the part of  $V$  not taken into account by  $\varphi_f$ , and  $\Psi_i$  is the outgoing scattering state of the system which originates from the state  $\varphi_i$ .

That is

$$\Psi_i = \lim_{\epsilon \rightarrow 0^+} \frac{i\epsilon}{\mathcal{E} - H + i\epsilon} \varphi_i.$$

For the interaction under consideration,  $\varphi_f = |\vec{k}_1\rangle |\vec{k}_2\rangle |\vec{k}_3\rangle |L\rangle$ , the three-particle plane-wave state. Here  $\vec{k}_i$  is the final-state momentum of particle  $i$  and  $|L\rangle$  the three-particle spin state ( $L$  is an index ranging from 1 to 8;  $V_f = V$ ; and  $\varphi_i = |\vec{p}_1\rangle |s\rangle |\vec{P}, N\rangle$ , where  $|\vec{p}_1\rangle |s\rangle$  is the plane-wave state of the projectile proton, which has momentum  $\vec{p}_1$  and spin projection  $s = \pm \frac{1}{2}$ , and  $|\vec{P}, N\rangle$  is the wave function for a deuteron with momentum  $\vec{P}$  and spin projection  $N = -1, 0, 1$ .) Following Faddeev<sup>9</sup> we may write the scattering state in the form  $\Psi_i = \psi^{(1)} + \psi^{(2)} + \psi^{(3)}$ . The  $\psi^{(j)}$  satisfy the matrix equation

$$\begin{pmatrix} \psi^{(1)} \\ \psi^{(2)} \\ \psi^{(3)} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ \varphi_i \end{pmatrix} + G \begin{pmatrix} 0 & \mathcal{T}_1 & \mathcal{T}_1 \\ \mathcal{T}_2 & 0 & \mathcal{T}_2 \\ \mathcal{T}_3 & \mathcal{T}_3 & 0 \end{pmatrix} \begin{pmatrix} \psi^{(1)} \\ \psi^{(2)} \\ \psi^{(3)} \end{pmatrix}, \quad (3)$$

where  $G = 1/(\mathcal{E} - H_0 + i\epsilon)$  and the  $\mathcal{T}_j \equiv V_j + V_j G V_j$  are the free-particle two-body transition matrices. The  $\mathcal{T}_j$  satisfy the Lippmann-Schwinger equation,<sup>11</sup>

$$\mathcal{T}_j = V_j + V_j G \mathcal{T}_j. \quad (4)$$

Iterating Eq. (3), substituting the resulting expression for  $\Psi_i$  into Eq. (2), and making use of Eq. (4), we obtain the multiple-scattering series

$$\begin{aligned} \langle f | \mathcal{T} | i \rangle &= \langle \varphi_f | \mathcal{T}_3 | \varphi_i \rangle + \langle \varphi_f | \mathcal{T}_2 | \varphi_i \rangle \\ &+ \langle \varphi_f | \mathcal{T}_3 G \mathcal{T}_2 | \varphi_i \rangle + \langle \varphi_f | \mathcal{T}_2 G \mathcal{T}_3 | \varphi_i \rangle \\ &+ \langle \varphi_f | \mathcal{T}_1 G \mathcal{T}_3 | \varphi_i \rangle + \langle \varphi_f | \mathcal{T}_1 G \mathcal{T}_2 | \varphi_i \rangle + \mathcal{O}(G^2). \end{aligned} \quad (5)$$

The first two terms on the right-hand side of Eq. (5) are single-scattering or impulse terms. (The first term is the PWIA term). The terms containing two factors of  $\mathcal{T}_j$  are double-scattering contributions. The remaining terms describe successively higher-order scattering processes. Because the successively higher-order terms in Eq. (5) become increasingly difficult to calculate, a simplification of the multiple-scattering series is necessary for application. Let us consider the case of elastic  $pD$  scattering. In particular we consider the Glauber approximation for the transition amplitude,<sup>2</sup> which has been successful in describing the elastic process at intermediate through high energies.<sup>12</sup> This approximation includes only the impulse terms and the energy-conserving parts of the double-scattering terms which appear in the multiple-scattering series. (The relation

$$1/(\mathcal{E} - H_0 + i\epsilon) = P[1/(\mathcal{E} - H_0)] - i\pi\delta(\mathcal{E} - H_0)$$

is used to split the double-scattering contributions into two parts). The principal-value part of double scattering exactly cancels the infinite sum of triple and higher-order scattering terms, which contain the complete eikonal propagator.<sup>13</sup> Now the elastic scattering and the deuteron-breakup processes essentially differ only in the final state of the scattering system. Hence, returning to the breakup process, we retain as a reasonable first correction to the impulse terms of Eq. (5) the energy-conserving parts of double scattering:

$$\begin{aligned} \langle f | \mathcal{T} | i \rangle &\cong \langle \varphi_f | \mathcal{T}_3 | \varphi_i \rangle + \langle \varphi_f | \mathcal{T}_2 | \varphi_i \rangle \\ &- i\pi[\langle \varphi_f | \mathcal{T}_3 \delta(\mathcal{E} - H_0) \mathcal{T}_2 | \varphi_i \rangle \\ &+ \langle \varphi_f | \mathcal{T}_2 \delta(\mathcal{E} - H_0) \mathcal{T}_3 | \varphi_i \rangle \\ &+ \langle \varphi_f | \mathcal{T}_1 \delta(\mathcal{E} - H_0) \mathcal{T}_3 | \varphi_i \rangle \\ &+ \langle \varphi_f | \mathcal{T}_1 \delta(\mathcal{E} - H_0) \mathcal{T}_2 | \varphi_i \rangle]. \end{aligned} \quad (6)$$

Equation (6) contains, except for principal-value double-scattering terms, which are in fact strongly suppressed by a mathematical cancellation, the same terms retained by Everett in his analysis of deuteron breakup at 145 MeV.<sup>10</sup>

In order to write an expression for the differential cross section for the  $D(p, 2p)$  reaction, we let  $E$  denote the final energy of one of the protons and  $\Omega_1, \Omega_2$  the solid angles at which the two protons emerge. Assuming the deuteron to be initially at rest, the differential cross section is

$$\frac{d\sigma}{dE d\Omega_1 d\Omega_2} = (\text{kinematic factor}) \times \frac{1}{8} \sum_{\text{spins}} |\langle g | T | i \rangle|^2,$$

where

$$\begin{aligned} \langle f | \mathcal{T} | i \rangle &= - (2\pi)^3 \delta^3(\vec{k}_1 + \vec{k}_2 + \vec{k}_3 - \vec{p}_1) \\ &\times \prod_{\text{initial}} \frac{1}{(2\mathcal{V}E_i)^{1/2}} \prod_{\text{final}} \frac{1}{(2\mathcal{V}E_f)^{1/2}} \langle f | T | i \rangle. \end{aligned} \quad (7)$$

( $\mathcal{V}$  is the quantization volume.) Here the identity of the two final-state protons is taken into account by using the antisymmetrized final state  $\langle g | = \langle f | - \langle f' |$ , where  $\langle f' |$  is the same as state  $\langle f |$  except that the roles of particles 1 and 2 are interchanged. The factor of  $\frac{1}{8}$  provides an average over initial spin states as there are six distinct combinations of  $s, N$ . For the experiment under consideration, which was performed with coplanar symmetric reaction kinematics in which the two final-state protons have equal energies and angles  $\theta$  to the beam direction so the neutron recoil momentum is parallel or antiparallel to the beam, we have

$$(\text{kinematic factor}) = \frac{\pi k E (\vec{p}_1)}{16 p_1^2 E (\vec{k}_3) M} \left[ 1 - \frac{k_3 \cos E(\vec{k})}{E(\vec{k}_3) k} \right]^{-1} \quad (8)$$

where  $E(\vec{p}) = (\vec{p}^2 + m^2)^{1/2}$ , with  $m$  the nucleon mass;  $M$  is the deuteron mass; and  $k \equiv |\vec{k}_1| = |\vec{k}_2|$ .

Now, if the neutron were to behave simply as a spectator, the target would, from momentum conservation, be essentially a proton traveling parallel or antiparallel to the beam. In this case, the experimental kinematics would be appropriate for the observation of an elastic  $90^\circ$  c.m. proton-proton collision at an energy depending on the target proton momentum in the deuteron laboratory sys-

tem. (The beam momentum is assumed fixed.) In fact for the experiment under consideration,  $k_3 \ll k < p_1$ , so the over-all influence on the neutron in this  $D(p, 2p)$  experiment is relatively slight. Hence, one might expect, as is indeed the case, the  $90^\circ$  c.m. two-nucleon transition amplitudes to be heavily weighted in Eq. (6).

Equations (6)–(8) are the basic formulas of our analysis. In the next section we shall show how they are applied.

### III. CALCULATIONAL DETAILS

Equation (6) can be written explicitly as

$$\begin{aligned} \langle f | T | i \rangle = & (2\pi)^{3/2} (2M)^{1/2} \left[ \langle L | T_3(\vec{p}_1, -\vec{k}_3 - \vec{k}_1, \vec{k}_2) \varphi_N(\vec{k}_3) | s \rangle + \langle L | T_2(\vec{p}_1, -\vec{k}_2 - \vec{k}_1, \vec{k}_3) \varphi_N(-\vec{k}_2) | s \rangle \right. \\ & + \frac{i}{8} \left( \frac{M}{\pi} \right)^{1/2} \left[ \int_{(\vec{k}_1, \vec{k}_3)_\perp} d^2q \frac{\langle L | T_3(\vec{k}_1 + \vec{k}_2 + \vec{q}, -\vec{q} - \vec{k}_1, \vec{k}_2) T_2(\vec{p}_1, \vec{q} - \vec{k}_1 + \vec{k}_2 + \vec{q}, \vec{k}_3) \varphi_N(\vec{q}) | s \rangle}{|(q_\parallel + |\vec{k}_1 + \vec{k}_2|)E(\vec{q}) + q_\parallel E(\vec{k}_1 + \vec{k}_2 + \vec{q})|} \right. \\ & + \int_{(\vec{k}_1 + \vec{k}_3)_\perp} d^2q \frac{\langle L | T_2(\vec{k}_1 + \vec{k}_3 - \vec{q}, \vec{q} - \vec{k}_1, \vec{k}_3) T_3(\vec{p}_1, -\vec{q} - \vec{k}_1 + \vec{k}_3 - \vec{q}, \vec{k}_2) \varphi_N(\vec{q}) | s \rangle}{|(q_\parallel - |\vec{k}_1 + \vec{k}_3|)E(\vec{q}) + q_\parallel E(\vec{k}_1 + \vec{k}_3 - \vec{q})|} \\ & + \int_{(\vec{k}_2 + \vec{k}_3)_\perp} d^2q \frac{\langle L | T_1(\vec{k}_2 + \vec{k}_3 - \vec{q}, \vec{q} - \vec{k}_2, \vec{k}_3) T_3(\vec{p}_1, -\vec{q} - \vec{k}_1, \vec{k}_2 + \vec{k}_3 - \vec{q}) \varphi_N(\vec{q}) | s \rangle}{|(q_\parallel - |\vec{k}_2 + \vec{k}_3|)E(\vec{q}) + q_\parallel E(\vec{k}_2 + \vec{k}_3 - \vec{q})|} \\ & \left. \left. + \int_{(\vec{k}_2 + \vec{k}_3)_\perp} d^2q \frac{\langle L | T_1(-\vec{q}, \vec{k}_2 + \vec{k}_3 + \vec{q} - \vec{k}_2, \vec{k}_3) T_2(\vec{p}_1, \vec{q} - \vec{k}_1, \vec{k}_2 + \vec{k}_3 + \vec{q}) \varphi_N(\vec{q}) | s \rangle}{|(q_\parallel + |\vec{k}_2 + \vec{k}_3|)E(\vec{q}) + q_\parallel E(\vec{k}_2 + \vec{k}_3 + \vec{q})|} \right] \right], \quad (9) \end{aligned}$$

where  $\varphi_N(\vec{q})$  is the deuteron internal wave function in momentum space and  $\int_{(\vec{k}_1, \vec{k}_3)_\perp} d^2q$  indicates a two-dimensional integration over the components of  $\vec{q}$  perpendicular to  $\vec{k}$ . The component of  $\vec{q}$  parallel to  $\vec{k}$ , denoted by  $q_\parallel$ , is determined at each integration point by the appropriate energy-conserving  $\delta$  function of Eq. (6). The relation between the two-body nucleon-nucleon amplitudes  $T_j(\vec{k}_1, \vec{k}_2 - \vec{p}_1, \vec{p}_2)$ , which are operators in spin-space, and the nucleon-nucleon differential cross section is

$$\left( \frac{d\sigma}{d\Omega_{\text{c.m.}}} \right)_{n \rightarrow n'}^j = \frac{1}{(8\pi W)^2} |\langle n' | T_j(\vec{k}_1, \vec{k}_2 - \vec{p}_1, \vec{p}_2) | n \rangle|^2,$$

where  $|n\rangle$  and  $|n'\rangle$  are the two-nucleon initial and final spin states, respectively, and  $W$  is the center-of-mass energy.

The first impulse term in Eq. (9) describes a process in which the projectile proton experiences a single  $90^\circ$  c.m. collision with the target proton. The second impulse term describes a single collision of the projectile proton with the target neutron. We have neglected this term, as it is strongly suppressed with the present kinematics.<sup>14</sup> The third term describes a Glauber double-scattering process<sup>2</sup> in which the projectile is scattered first from the target proton (near  $90^\circ$  c.m.) and subse-

quently from the neutron (near  $0^\circ$ ). The fourth term describes a similar process where the projectile interacts first with the neutron. The fifth term describes a process where the projectile is scattered from the target proton (near  $90^\circ$  c.m.), which then nearly forward scatters from the neutron. This may be thought of as a single-scattering process followed by a final-state interaction of the target nucleons. Finally, the sixth term describes a similar process where the projectile collides first with the neutron (near  $90^\circ$  c.m.), which then collides with the target proton, transferring most of its momentum to the proton. The various single- and double-scattering processes are illustrated diagrammatically in Fig. 1.

The  $T_j(\vec{k}_1, \vec{k}_2 - \vec{p}_1, \vec{p}_2)$  amplitudes required in Eq. (9) are slightly off-energy-shell. That is, the initial-state energy invariant  $(k_1 + k_2)^2$ , the final-state invariant  $(p_1 + p_2)^2$ , and the energy parameter of the  $T_j$  operator, which we have suppressed, correspond to slightly different energies of the two-particle system. To employ Eq. (9) we approximate the required off-shell  $T_j$  matrices by the physical on-shell matrices at energy  $s = (p_1 + p_2)^2$  or  $(k_1 + k_2)^2$  and momentum transfer  $t = (p_1 - k_1)^2$  or  $(p_2 - k_2)^2$ . For our kinematics the  $s$  and  $t$  varia-

tion of the on-shell amplitudes is sufficiently slow so that the choice of  $s$  and  $t$  is usually unimportant. However, at larger values of  $k_3$  a discernible ambiguity in the calculated cross section is introduced. This is illustrated in the next section. For given  $s$  and  $t$  the on-shell amplitudes are obtained from  $NN$  phase shifts<sup>3</sup> at energies where they are available. At the higher required energies, we renormalize the highest-energy phase-shift amplitudes using  $NN$  total and differential cross-section data.<sup>4</sup> In this way we employ Eq. (9) to calculate the  $T$  matrix for the  $D(p, 2p)$  process.

For given  $\theta$  the values of  $\vec{k}_1$  and  $\vec{k}_2$  to be used in Eq. (9) are obtained from momentum conservation using the corresponding  $\vec{k}_3$  value, which is known from experiment. ( $\vec{p}_1$  is known from the beam energy.) Having calculated the  $T$  matrix, we substitute it into Eq. (7) to obtain the desired differential cross section ( $d\sigma/dEd\Omega^2$ ).

#### IV. RESULTS AND DISCUSSION

In Fig. 2 we have plotted the results of our calculation of  $d\sigma/dEd\Omega^2$  for 600-MeV protons on deuterium with coplanar symmetric reaction kine-

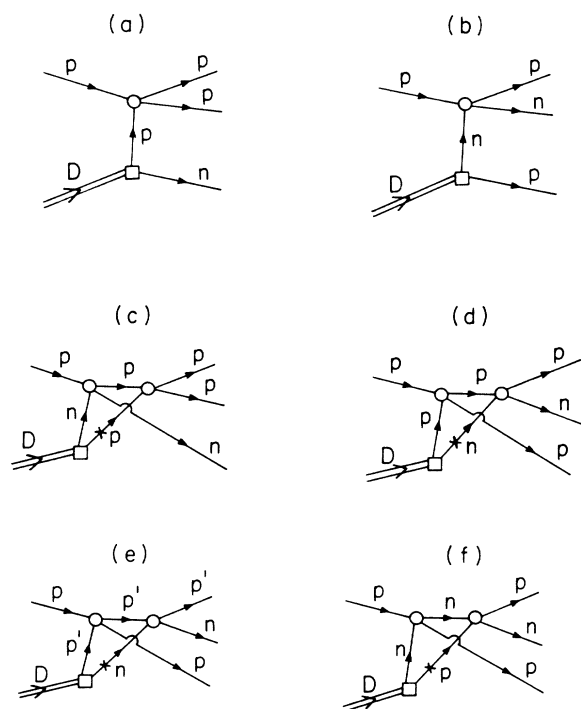


FIG. 1. The diagrammatic representation of : (a), (b) the impulse; (c), (d) the Glauber double scattering; and (e), (f) the final-state-interaction contributions to the  $D(p, 2p)$  reaction. The lines marked by crosses are on-energy-shell.

matics together with the experimental data of Perdrisat *et al.*<sup>15</sup> The angular limits  $\theta=28^\circ$  and  $\theta=56^\circ$  correspond to final-state neutron momenta  $k_3=180$  MeV/c antiparallel and  $k_3=370$  MeV/c parallel to the incident proton beam, respectively, intermediate angles corresponding to  $k_3$  between these two limits. We see that there is quite good agreement over most of the angular range, a divergence emerging at the large- and small-angle tails, where the neutron recoil momentum is very large. Our PWIA prediction is also indicated. We see that at the larger scattering angles, multiple scattering does become significant if not dominant, the contribution diminishing rapidly with decreasing  $\theta$  or, equivalently, decreasing recoil momentum.

We have indicated in Fig. 3 the ambiguity in our PWIA calculation arising from the choice of energy at which the on-shell  $pp$  amplitude is calculated. The ambiguity becomes larger with increasing neutron recoil momentum and introduces a discernible uncertainty in our results. A model for the off-energy-shell behavior of  $T_j$  in the 400-

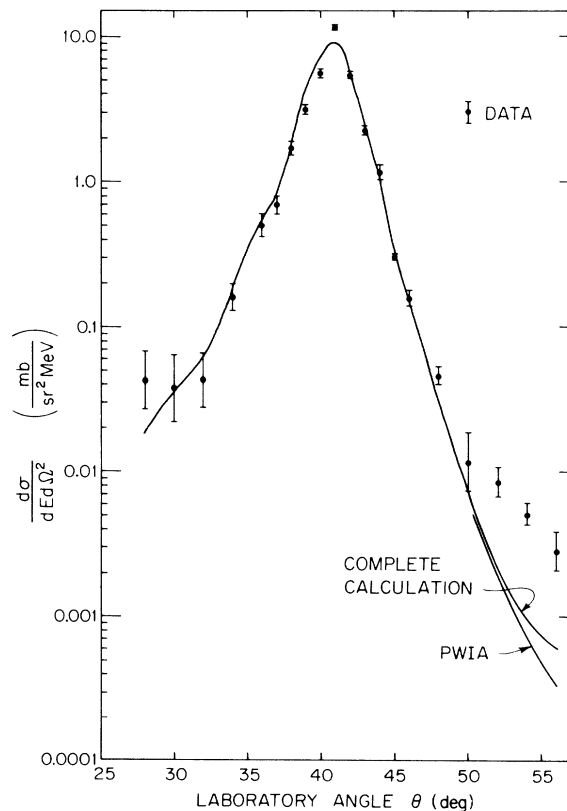


FIG. 2. The calculated  $D(p, 2p)$  differential cross section at 600 MeV together with the experimental data of Perdrisat *et al.* (Ref. 1). The lower curve is the PWIA result.

1000-MeV range required here would thus be most helpful.

To extract information about the deuteron wave function, one plots the single-particle momentum distribution  $\langle |\varphi_N(\vec{k})|^2 \rangle_{\text{spin}} = \rho(k)$ , which would give complete agreement between theory and experiment. Our results are shown in Fig. 4 together with the prediction of the Gartenhaus wave function.<sup>6</sup> We see here the excellent agreement of the Gartenhaus prediction with experiment up to rather large recoil momenta. The theory-experiment divergence in the tail region suggests a modification of the large-momentum components of the deuteron wave function. Such a modification is not unjustified, as our knowledge of the large-momentum, or equivalently small-internucleon-distance, behavior of the deuteron is limited. However, if we try to make a modification of the required magnitude, we arrive at a wave function which either has completely unconventional behavior at smaller internucleon distances, i.e., is far too large, or has a  $d$  state which accounts for considerably more than the typical 7% of the deuteron.<sup>16</sup> Hence a reasonable modification of the conventional wave function cannot in itself eliminate the tail problem. However, at large internal momenta, the probable 1–2%  $D^*$  component

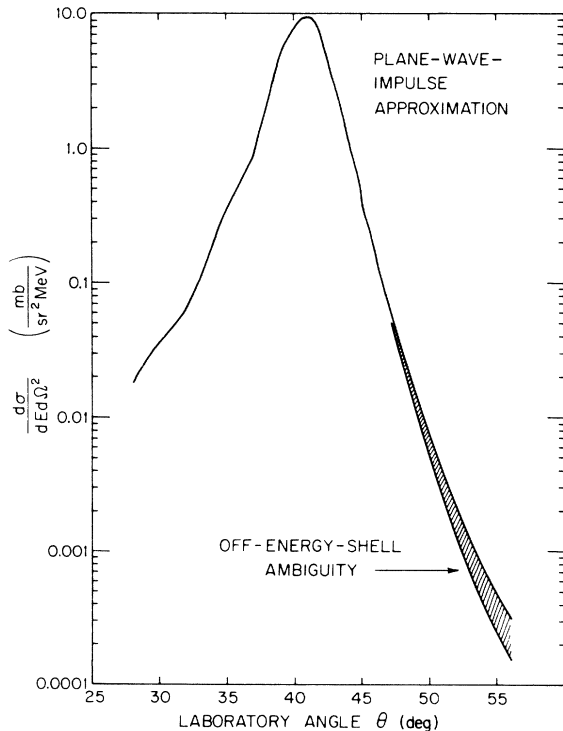


FIG. 3. The PWIA calculation of the  $D(p, 2p)$  differential cross section at 600 MeV. The off-energy-shell ambiguity is indicated by the cross-hatched area.

of the deuteron consisting of an  $NN^*(1688, \frac{5}{2}^+)$  bound state is thought to be important.<sup>8</sup> In the impulse approximation for a  $D^*$  target, the process  $N+N^* \rightarrow N+N$  contributes. Diagrammatic illustrations of the  $D^*$  impulse terms are shown in Fig. 5. The general structure of the impulse terms is  $T_I^* \sim T_{NN^* \rightarrow NN} \varphi^*$ , in analogy with the impulse terms of Eq. (9). If we invoke time-reversal invariance, the available data for  $NN \rightarrow NN^*(1688)$ <sup>4</sup> indicate that  $|T_{NN^* \rightarrow NN}|^2$  is at least an order of magnitude

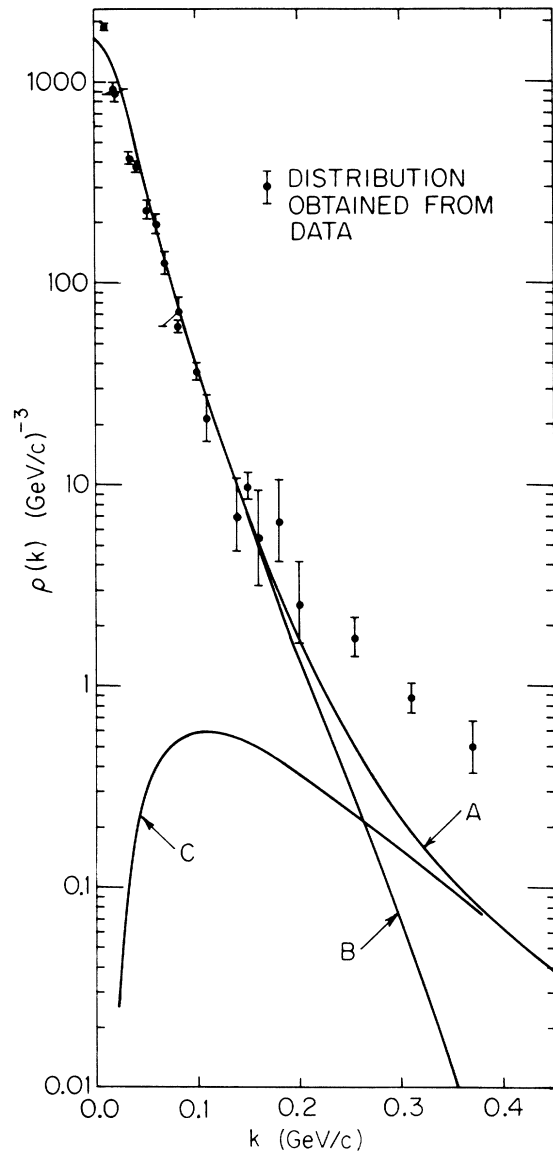


FIG. 4. The deuteron single-particle momentum distribution obtained from analysis of the data of Perdrisat *et al.* (Ref. 1). Curve A is the prediction of the complete Gartenhaus wave function. Curve B is the Gartenhaus  $s$ -state contribution, and curve C the  $d$ -state contribution.

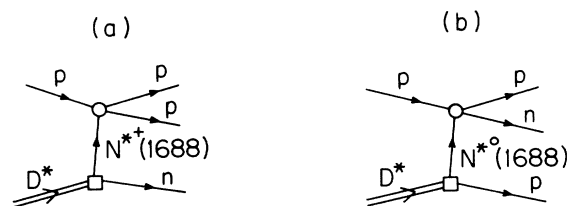


FIG. 5. The diagrammatic representation of the impulse contributions for a  $D^*$  target. Diagram (a) involves  $N^{*+}(1688)$  and diagram (b)  $N^{*o}(1688)$ .

smaller than the corresponding elastic scattering amplitude squared, barring extremely rapid off-shell variation for  $T_{NN^* \rightarrow NN^*}$ . The required large-momentum components of  $\varphi^*$ , the  $D^*$  wave function, are apparently of the same order of magnitude as the corresponding momentum components of the conventional deuteron function.<sup>8</sup> Thus, with the provision of not-too-rapid off-shell behavior, it appears that the incorporation of the  $D^*$  could not qualitatively effect our results. The disagreement between the calculation and experiment in the tail region remains, although the calculational input is rather uncertain here. Proper treatment

of the off-shell effects possibly together with a more sophisticated deuteron wave function, might well remove the tail difficulty.

Regardless of the current situation, our calculation does represent an encouraging improvement over the earlier PWIA analysis.<sup>1</sup> Excellent agreement with experiment was obtained over much of the angular range. Double-scattering processes are indeed significant, and perhaps higher-order scattering processes should be investigated. It is thus reasonable to hope that  $(p, 2p)$  data analyzed with the multiple-scattering theory described here may ultimately lead the way to information about the small-distance behavior of the nucleon-nucleon interaction.

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<sup>1</sup>C. F. Perdrisat *et al.*, Phys. Rev. **187**, 1201 (1969).

<sup>2</sup>R. J. Glauber, in *Lectures in Theoretical Physics*, edited by W. E. Brittin and L. G. Dunham (Interscience, New York, 1959); V. Franco and E. Coleman, Phys. Rev. Letters **17**, 827 (1966); V. Franco and R. J. Glauber, Phys. Rev. **142**, 1195 (1966); **156**, 1685 (1967); Phys. Rev. Letters **22**, 370 (1969).

<sup>3</sup>M. H. MacGregor, R. A. Arndt, and R. M. Wright, Phys. Rev. **169**, 1128, 1149 (1968); **173**, 1272 (1968); **182**, 1714 (1969).

<sup>4</sup>O. Benary, L. R. Price, and G. Alexander, University of California Radiation Laboratory Report No. UCRL-20000 NN, 1970 (unpublished).

<sup>5</sup>M. J. Moravcsik, Nucl. Phys. **7**, 113 (1958).

<sup>6</sup>S. Gartenhaus, Phys. Rev. **100**, 900 (1955).

<sup>7</sup>T. Hamada and I. D. Johnston, Nucl. Phys. **34**, 382 (1962).

<sup>8</sup>A. K. Kerman and L. S. Kisslinger, Phys. Rev. **180**, 1483 (1969); J. S. Vincent *et al.*, Phys. Rev. Letters **24**, 236 (1970).

<sup>9</sup>L. D. Faddeev, Zh. Eksperim. i Teor. Fiz. **39**, 1459 (1960) [transl.: Soviet Phys. - JETP **12**, 1014 (1961)].

<sup>10</sup>A. Everett, Phys. Rev. **126**, 831 (1962); G. F. Chew and M. L. Goldberger, *ibid.* **87**, 778 (1952).

<sup>11</sup> $T$  satisfies a Lippmann-Schwinger equation with the replacement  $V_j \rightarrow V$ .

<sup>12</sup>R. J. Glauber, in *High Energy Physics and Nuclear Structure*, edited by S. Devons (Plenum, New York, 1970).

<sup>13</sup>R. J. Glauber, Phys. Rev. **91**, 459 (1953); B. J. Malenka, *ibid.* **95**, 522 (1954); D. Harrington, *ibid.* **184**, 1745 (1969).

<sup>14</sup>In the experiment under consideration  $|\vec{k}_3| \leq 370$  MeV/c, whereas  $|\vec{k}_2| \sim 800$  MeV/c, and  $\varphi_N(\vec{k})$  is a rapidly decreasing function of  $|\vec{k}|$ .

<sup>15</sup>In the 36–45° region we have included only the data for transverse neutron momentum less than 10 MeV/c as presented in Ref. 1. These data correspond most precisely to the case of coplanar symmetric kinematics.

<sup>16</sup>A review of some of the deuteron wave functions may be found in J. E. Elias *et al.*, Phys. Rev. **177**, 2075 (1969).