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Adiabatic Deuteron Model and the $^{16}O(p, d)$ Reaction

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Distorted-wave calculations using the Johnson and Soper adiabatic deuteron potential have been carried out for the reaction $^{16}O(p,d)^{15}O$ at 31.82, 38.63, and 45.34 MeV. The calculated angular distributions describe the data well with no need for a lower radial cutoff. The resultant spectroscopic factors are plausible and relatively independent of energy. Various modifications of the distorted-wave calculations and the sensitivity to different proton and deuteron potentials are investigated.

I. INTRODUCTION

The adiabatic deuteron potential of Johnson and Soper[1] has been shown to be successful in improving distorted-wave (DW) fits to (p, d) reactions.[2-4] This paper presents a reanalysis of the $^{16}O(p, d)^{15}O$ data of Snelgrove and Kashy[5] to further test the applicability of this potential. The data were taken at four proton bombarding energies (25.52, 31.82, 38.63, and 45.34 MeV) and the angular distributions cover the angular range from 12 to 160° (c.m.). As shown in a previous study,[6] these data require severe damping of the contributions to the reaction from the nuclear interior in order to get resonable relative spectroscopic factors and fits to the angular distributions for the transfer of a $1p_{1/2}$ and a $1p_{3/2}$ neutron. The results of the present analysis using the adiabatic deuteron potential will be compared with the spectroscopic factors and the quality of the fits obtained using the radial damping terms.

II. OPTICAL POTENTIALS

The optical potentials used in the DW analyses had the usual form

$$U(r) = U_C(r) - \left[V_0 - V_{so}\left(\frac{\hbar}{m_\pi c}\right)^2 (\vec{\sigma}\cdot\vec{l})\frac{1}{r}\frac{d}{dr}\right] f(x_R) - i\left(W_V - 4W_D\frac{d}{dx_I}\right) f(x_I),$$

where

$$f(x) = (1 + e^x)^{-1}$$

and

$$x_R = \frac{r - r_R A^{1/3}}{a_R}, \quad x_I = \frac{r - r_I A^{1/3}}{a_I},$$

and U_C is the potential of a uniformly charged sphere of radius $r_C A^{1/3}$.

Two proton optical potentials were used in the analysis. They are presented in Table I. The set of parameters labeled SK came from Ref. 5. The set labeled WSS are the average proton parameters of Watson, Singh, and Segal.[7]

Three deuteron optical potentials were used. They are presented in Table II. Set SK is from Ref. 5. Set R is obtained from the proton and neutron potentials of Rosen[8] using the simplified pre-

TABLE I. Proton optical-model parameters.

	E_p (lab) (MeV)	V_0 (MeV)	$r_R=r_{so}$ (F)	$a_R=a_{so}$ (F)	V_{so} (MeV)	W_V (MeV)	W_D (MeV)	r_I (F)	a_I (F)	r_C (F)
SK [a]	31.82	45.5	1.12	0.69	7.0	0.0	5.31	1.44	0.490	1.15
	38.63	44.4	1.12	0.69	7.0	2.00	4.89	1.40	0.430	1.15
	45.34	42.7	1.12	0.69	7.0	3.11	5.65	1.28	0.415	1.15
WSS [b]	31.82	52.3	1.12	0.57	5.5	0.0	7.80	1.12	0.50	1.12
	38.63	50.4	1.11	0.57	5.5	4.3	7.41	1.11	0.50	1.11
	45.34	48.5	1.11	0.57	5.5	7.5	7.04	1.11	0.50	1.11

[a] See Ref. 5.

[b] See Ref. 7.

scription of Harvey and Johnson.[2] Set MPS is the effective adiabatic potential used by McAllen, Pinkston, and Satchler[3] in their analysis of $1p$ shell nuclei.

III. FORM FACTOR

The form factor for the local zero-range (LZR) calculations was taken to be a Woods-Saxon eigenfunction for which the well depth was adjusted to give the separation energy as the eigenenergy. The parameters of the well for the bound neutron were $r_0=r_{so}=1.12$ F, $a=a_{so}=0.69$ F, and a spin-orbit strength of $\lambda=25$ as in Ref. 9. The separation energy of the neutron was taken to be 15.663 MeV for a $1p_{1/2}$ neutron and 21.843 MeV for a $1p_{3/2}$ neutron as in Ref. 5.

Finite-range[10,11] and nonlocality[11,12] correction terms were included using standard radial damping factors. The range of the interaction was taken to be 1.5 F, and the nonlocality ranges, determined from the energy dependence of local optical potentials,[11,12] were 0.54 F for the deuteron and 0.85 F for the proton and the transferred neutron. These calculations are labeled FRNL.

As in Ref. 6, an effective density dependence has also been included in some of the calculations. Such a calculation is an attempt to take account of the fact that the matrix element responsible for the reaction, $\langle d|V_{pn}|p\rangle$, is not the same outside and inside nuclear matter. The specific density dependence used here and in Ref. 6 is one found by Green[13] for the triplet-even interaction. This yields the radial damping term $(1-1.845\rho^{2/3})$, where ρ is taken to be a Gaussian form. Specifically, for the oxygen region[13]

$$\rho=0.306e^{-0.224r^2}\ \text{nucleons/F}^3\,.$$

The calculations do not include the over-all renormalization of 1.623 given by Green. Calculations using the radial damping terms for the density-dependence, finite-range effects, and nonlocality effects are labeled DFRNL.

IV. RESULTS

In Fig. 1, DW calculations[9] using the adiabatic deuteron potential (MPS) are compared with those using the "standard" LZR approach and with those using the DFRNL damping terms. All three calculations use the proton optical parameters labeled

TABLE II. Deuteron optical-model parameters.

	E_d (lab) (MeV)	V_0 (MeV)	$r_R=r_{so}$ (F)	$a_R=a_{so}$ (F)	V_{so} (MeV)	W_V (MeV)	W_D (MeV)	r_I (F)	a_I (F)	r_C (F)
SK [a]	12	114.0	0.95	0.80	7.57	0.0	6.00	1.57	0.70	1.30
	20	104.0	0.98	0.80	7.57	0.0	7.05	1.50	0.70	1.30
	26	98.0	1.00	0.80	7.57	0.0	7.95	1.45	0.70	1.30
	33	92.8	1.03	0.80	7.57	0.0	8.84	1.41	0.70	1.30
R [b]	12	99.1	1.25	0.69	5.5	0.0	12.5	1.25	0.74	1.30
	20	96.5	1.25	0.69	5.5	0.0	12.5	1.25	0.74	1.30
	26	94.5	1.25	0.69	5.5	0.0	12.5	1.25	0.74	1.30
	33	92.1	1.25	0.69	5.5	0.0	12.5	1.25	0.74	1.30
MPS [c]	12	116	1.15	0.62	5.0	0.0	12.0	1.15	0.615	1.30
	20	114	1.15	0.62	5.0	0.0	12.0	1.15	0.615	1.30
	26	111	1.15	0.62	5.0	0.0	12.0	1.15	0.615	1.30
	33	109	1.15	0.62	5.0	0.0	12.0	1.15	0.615	1.30

[a] See Ref. 5.

[b] See Ref. 8.

[c] See Ref. 3.

SK, and the LZR and DFRNL calculations use the deuteron optical potentials labeled SK. These calculations are not shown for the 25.52-MeV data, since there are problems in understanding the analysis of the $p_{3/2}$ neutron transfer ($Q = -19.62$ MeV) for this case.[5,6] The LZR curves show no resemblance to the data. The DFRNL curves show that damping out the contributions from the interior of the nucleus is helpful, but these calculations infer that even more damping is required. Such effects can also be achieved using a lower radial cutoff, but then spectroscopic factors become rather arbitrary.[5,6] The adiabatic calculation appears to solve many of the problems. It fits the average slope and reproduces the general shape rather well. Only for the $p_{1/2}$ transfer at 45.34 MeV is the DFRNL calculation better.

Since FRNL effects are known to exist, calculations were made to see what would happen if one includes them along with the adiabatic deuteron

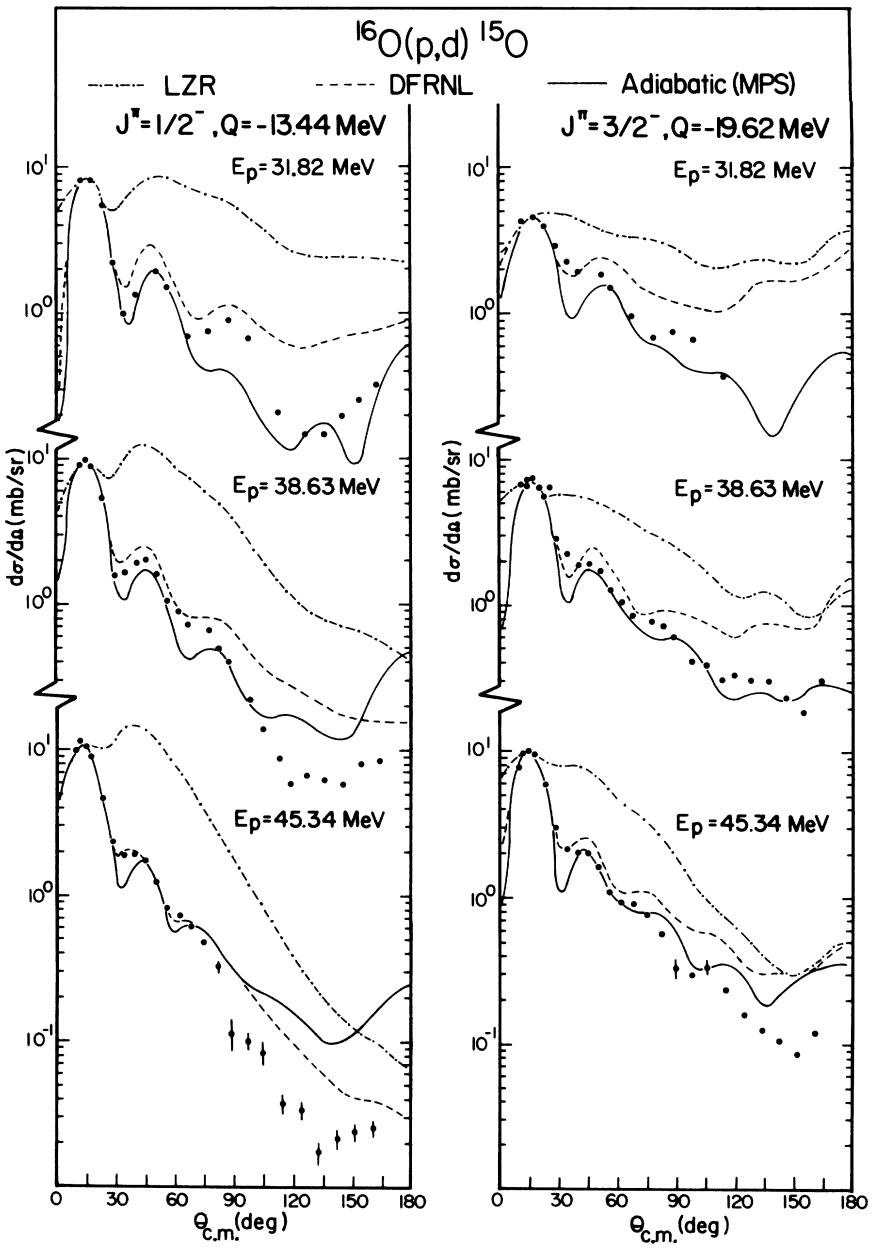

FIG. 1. Comparison of DW calculations: (a) using the local zero-range calculations (LZR); (b) including damping factors for density dependence, finite range, and nonlocality (DFRNL); and (c) using the adiabatic deuteron potential MPS.

potential. These calculations are presented in Fig. 2. The solid curves, labeled MPS-LZR, are the same adiabatic calculations as in Fig. 1. The dashed curves, labeled MPS-FRNL, use the MPS adiabatic deuteron potential and include the FRNL damping factors. The dot-dashed curves, labeled R-FRNL, use the R adiabatic deuteron potential and also include the damping terms. All three calculations use the proton optical potential labeled SK. It is seen that the inclusion of the known damping effects makes the fits worse in all cases (except possibly the $p_{1/2}$ transfer at 45.34 MeV). From these calculations it appears that the adiabatic potential is itself including in some way the effects due to the finite range of the interaction and the nonlocality of the potentials, as well as taking into account the deuteron breakup channels as proposed.[1] Including the effective density dependence in these calculations resulted in only a slight change at the larger angles.

In order to investigate the effects of the optical potentials on the calculations, various combina-

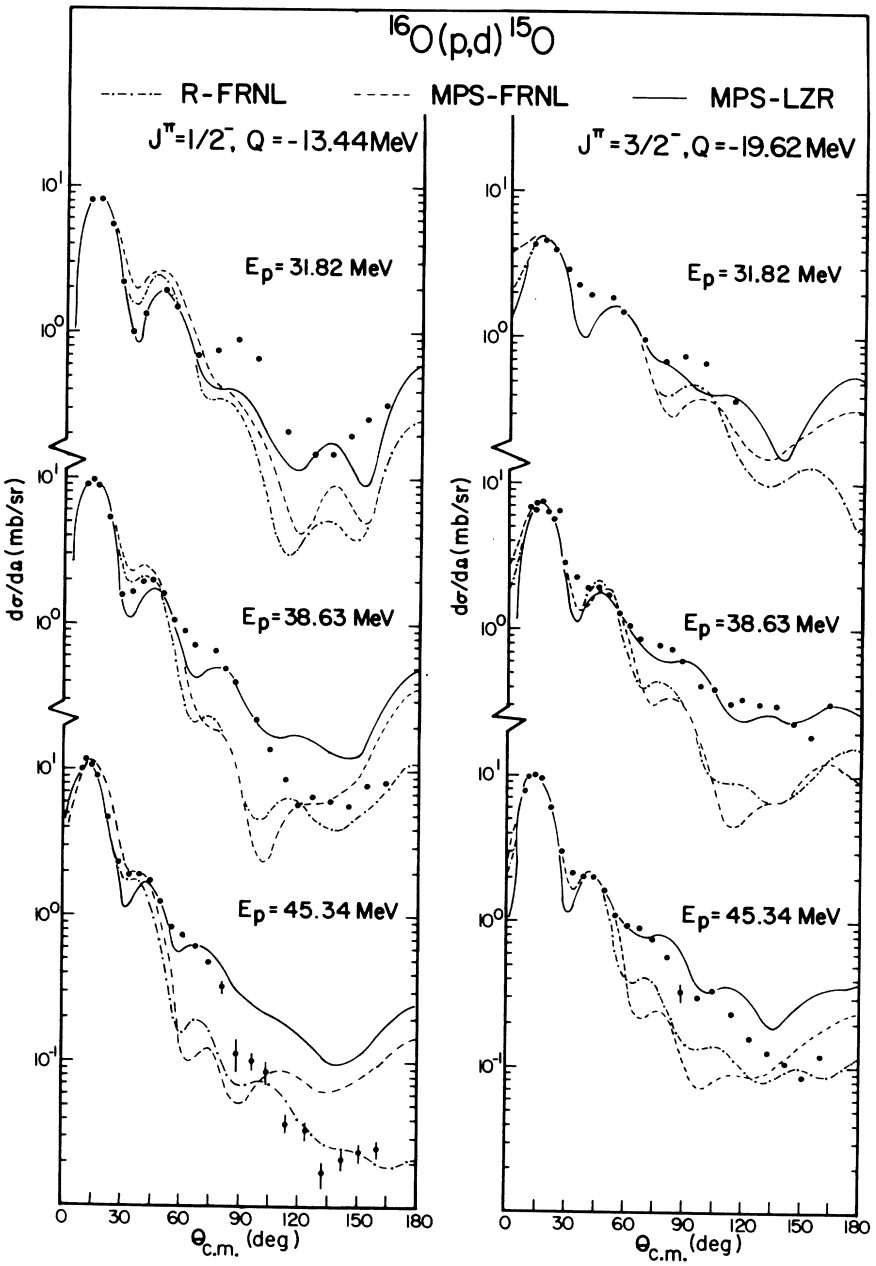

FIG. 2. Comparison of LZR DW and FRNL DW calculations using adiabatic deuteron potentials.

tions of proton and deuteron optical potentials were tried. In Fig. 3, three such calculations are shown. The labeling gives the proton parameters (SK or WSS) and the deuteron parameters (MPS or R). It is seen that calculations using the proton parameter set WSS always predict too much structure but are not very different from the calculations using the proton set SK, except possibly at 31.82 MeV. Similar calculations at 32 MeV were reported by McAllen, Pinkston, and Satchler[3] to have displayed even larger differences than shown here. The calculations using the adiabatic deuteron set R give shapes somewhat different than the data. Calculations using the proton set WSS and the adiabatic deuteron set R were also made, and it was found that the distributions were intermediate be-

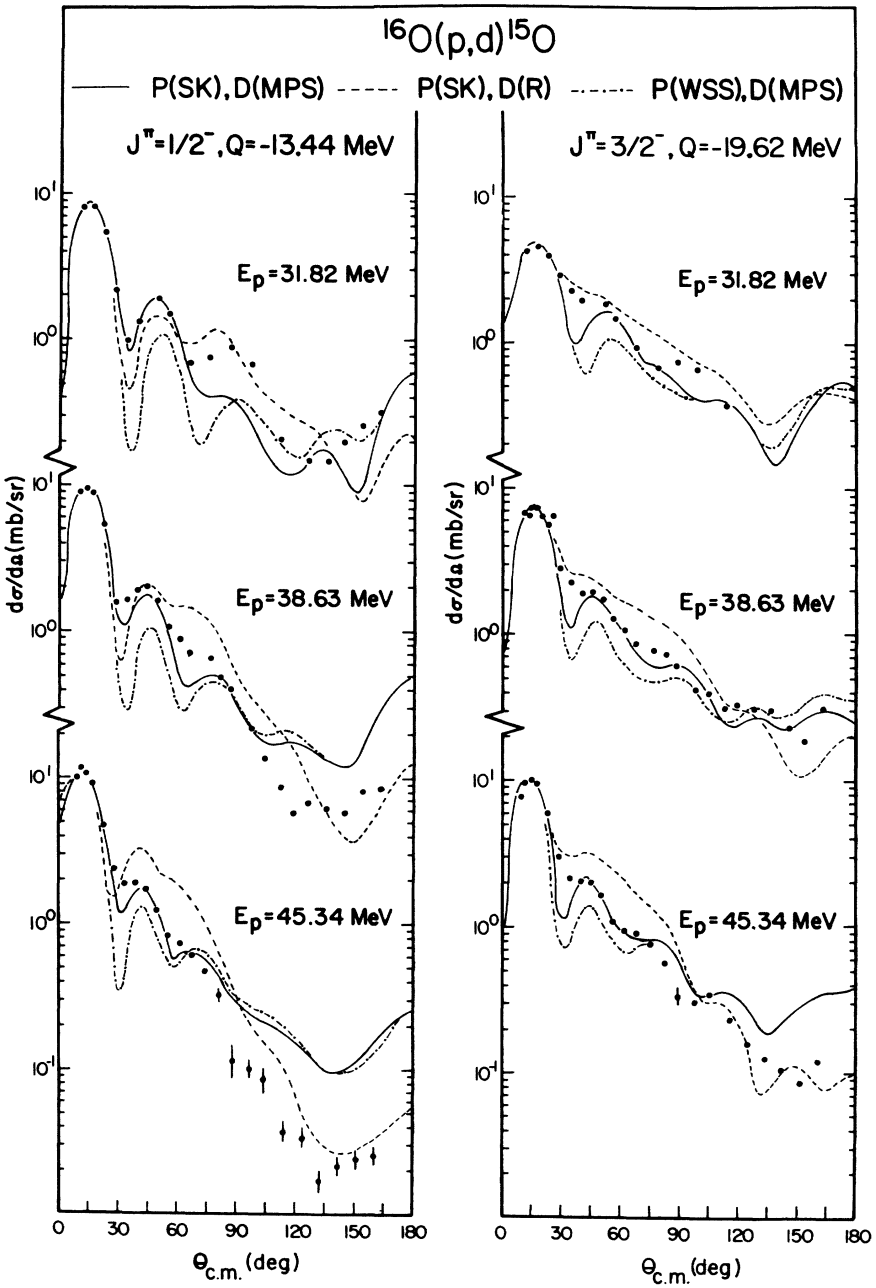

FIG. 3. Comparison of LZR DW calculations using the indicated combinations of proton and adiabatic deuteron potentials.

TABLE III. Spectroscopic strengths for $^{16}O(p, d)^{15}O$. Spectroscopic strengths are extracted from the data according to the expression $\sigma_{exp} = 2.25 S(nlj)\sigma_{DW}$. All calculations were normalized to the data at the first peak in the angular distribution.

	Incident energy (MeV)	P(SK), D(SK)		P(SK), D(MPS)		P(SK), D(R)		P(WSS), D(MPS)		P(WSS), D(R)	
		LZR	DFRNL [a]	LZR	FRNL	LZR	FRNL	LZR	FRNL	LZR	FRNL
$S(1p_{1/2})$	31.82	2.5	2.6	2.8	1.6	3.9	2.9	2.2	1.6	2.6	2.6
	38.63	2.8	2.3	2.3	1.6	3.6	2.8	2.0	1.6	2.8	2.8
	45.34	3.5	2.3	2.6	1.6	4.2	2.9	2.4	1.8	3.5	3.1
$S(1p_{3/2})$	31.82	2.0	5.7	5.7	2.9	4.9	7.2	4.2	3.2	3.1	6.0
	38.63	2.2	3.9	4.6	2.6	4.9	5.7	3.5	2.9	3.4	5.0
	45.34	2.2	3.8	4.3	2.7	5.3	4.9	3.8	2.9	4.3	5.3
$S(1p_{3/2})/S(1p_{1/2})$	31.82	0.80	2.2	2.0	1.8	1.3	2.5	1.9	2.0	1.2	2.3
	38.63	0.79	1.7	2.0	1.6	1.4	2.0	1.8	1.8	1.2	1.8
	45.34	0.63	1.7	1.7	1.7	1.3	1.7	1.6	1.6	1.2	1.7

[a] The normalization of 1.623 for the density-dependent interaction (Ref. 13) has not been included.

tween the P(WSS), D(MPS), and the P(SK), D(R) calculations for angles less than 60° and were very similar to the P(WSS), D(MPS) calculations for angles greater than 60°.

In order to test the usefulness of the adiabatic approach for obtaining nuclear-structure information, spectroscopic factors were extracted at each energy for the various calculations. These are presented in Table III. Comparing the relative $1p_{3/2}$ to $1p_{1/2}$ strengths, it is seen that for the calculations which do not use an adiabatic deuteron potential [P(SK), D(SK)] the LZR calculations fail to give even plausible strengths, whereas the DFRNL calculations work well for 38.63 and 45.34 MeV. The relative strengths for the adiabatic calculations all appear to be plausible and almost independent of bombarding energy, except possibly for the FRNL calculations using the deuteron potential R. An indication of what values of this ratio of neutron strengths might be reasonable can be obtained from the measured values for the proton configuration of the ^{16}O ground state. In the $^{16}O(d, {}^3He)^{15}N$ reaction,[14] the relative $p_{3/2}$-$p_{1/2}$ strength was determined to be 1.58 when a LZR DW calculation was used and 1.74 when finite-range effects were included.

V. CONCLUSIONS

Previous work has shown that the adiabatic model of Johnson and Soper has been successful in the analysis of (p, d) reactions from some light-,[3] medium-,[2] and heavy-weight[4] nuclei. This paper looks at the possible energy dependence of the model and also presents spectroscopic factors obtained using the model. It is reaffirmed that the model improves the shapes of the calculated angular distributions without the use of a radial cutoff in the integration. However, the fits at large angles (>90°) could in most cases be improved even more. It is also shown that the model must contain in some way the known damping effects due to the finite range of the interaction and the nonlocality of the potentials. The spectroscopic factors obtained using this model appear to be realistic and relatively independent of bombarding energy. In conclusion, this model goes a long way toward improving the reliability of spectroscopic information from (p, d) reactions.

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g Factor of the $\frac{19}{2}^-$ Isomeric State in $^{43}Sc^\dagger$

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The g factor of the $\frac{19}{2}^-$ state in ^{43}Sc is discussed within the framework of the conventional shell model. An inert ^{40}Ca core is assumed. The effective interaction derived for this mass region by Kuo and Brown is used. It is shown that the g factor can be well explained with a small configuration mixing. The g factor of the 6^+ state in ^{42}Ca is also discussed.

The explanation of the measured magnetic dipole moments provides a useful probe in the study of nuclear structure. With the j-j coupling shell model, the various features of the deviations of magnetic dipole moments from the Schmidt values were interpreted by configuration mixing for almost the whole region of nuclei.[1] Freed and Kisslinger[2] carried out the calculations using the same method, but within the framework of the pairing model. For the magnetic dipole moments of $p_{1/2}$-shell nuclei, the importance of the tensor force which causes the configuration mixing is emphasized in the explanation of the small deviations from the Schmidt values.[3] However, these calculations were restricted to the magnetic dipole moments of the ground state of odd-mass nuclei, where the mixed seniority-three configurations were assumed to be the initial nucleon of the seniority-one configuration coupled to the other two nucleons of the same kind having equal orbital angular momenta and $J = 1$. The deviations from the Schmidt values of the magnetic dipole moments of high-spin excited states have been studied in the ^{208}Pb region[4] and $A = 88$ region.[5] The anomalous g^{eff} factor of about 1.10 has been deduced under the assumption of the renormalized single-particle operator $\vec{\mu} = g_l^{\text{eff}}\vec{l} + g_s^{\text{eff}}\vec{s}$.

In addition, the g factor of the $\frac{19}{2}^-$ isomeric state in ^{43}Sc has recently been measured to be 0.331 ± 0.002 by both the time-differential, perturbed-angular-distribution, and the stroboscopic-resonance methods.[6] The configuration of this $\frac{19}{2}^-$ state was proposed to be an $f_{7/2}$ proton coupled to the 6^+ state in ^{42}Ca. A g factor for the $f_{7/2}$ proton was deduced from the measured value combined with the experimental value for the g factor of the 6^+ state in ^{42}Ca under the assumption of the additivity of g factors. However, there are two experimental values for the 6^+ state[5,7] which do not agree with each other. Nevertheless, the g factors of the $f_{7/2}$ proton deduced from both values were close to, or even larger than, the Schmidt value. By adopting the more recent datum in Ref. 5, Nakai *et al.*[6] have shown that the single-particle value, using the orbital g factor of the proton $g(\text{proton}) = 1.1$, agrees with the experimental value of the g factor of the $\frac{19}{2}^-$ state in ^{43}Sc. They also concluded that the anomaly of this magnitude (~10%) of g(proton) may indicate the same effect as those reported by Yamazaki *et al.* in the ^{208}Pb[4] and ^{90}Zr[5] region.