reactions (term $b$) in the events and also of the fraction $d$ of AgBr events we measured:

$$b \simeq 15\%$$

and

$$d \simeq 4\%.$$

The purpose of this paper was to give a short survey of emulsion technique applied to a nuclear-reaction study; more details can be found in an internal report.[4]

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# Krauss-Kowalski Calculations of Nucleon-Deuteron Polarization*

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Nucleon-deuteron polarizations and differential cross sections have been calculated in the energy range from 11 to 23 MeV, with exactly the same approximation (the unitary first-order approximation) and exactly the same interactions and parameters as in the recent calculations of Krauss and Kowalski. All the results are entirely different, however, and in contrast to the previous results the calculated polarizations are in complete disagreement with experiment. It is pointed out that the procedure used in the previous calculations of cutting off the partial-wave expansion at $J=\frac{3}{2}$ is unsatisfactory, but this observation does not at all explain the differences between the results.

Recently Krauss and Kowalski[1,2] have calculated the nucleon polarization produced in elastic nucleon-deuteron scattering, at a number of energies from 11 to 40 MeV. Their results are not in detailed agreement with experimental polarizations, but they do reproduce their main qualitative features, with respect to both angular and energy dependence, and they are the first theoretical calculations that have been able to do so.

We have repeated these calculations, as an incidental part of a more general calculation, and have now to report entirely different results from those of Krauss and Kowalski, even though the two-body interactions and the approximation method were supposed to be exactly the same in both cases.

The basic approximation used by Krauss and Kowalski was the unitary first-order approximation,[3] and this they applied to the three-nucleon collision problem, using noncentral two-nucleon interactions, since the polarization is zero for central interactions. In principle, the unitary-approximation method[3] can be used with any two-body interactions, but in the interests of practical convenience Krauss and Kowalski used very simple interactions, namely separable interactions of the Yamaguchi[4,5] form in the $^1S$ and $(^3S+^3D)$ states. (The latter gives a bound deuteron with the correct $D$-state admixture.) The use of these separable interactions provides the link with our present work, since we are currently seeking the *exact* solution of the three-nucleon collision problem with these interactions, through the numerical solution of coupled one-dimensional integral equations[6] for the partial-wave scattering amplitudes. As an incidental part of this work, we have obtained results in the unitary first-order approximation,[3] by numerically iterating the integral equations once, and then deducing the unitary first-order partial-wave amplitudes by a simple extension of an argument given previously.[7]

The polarizations obtained in this way are shown at three different energies in Figs. 1–3, and are compared in each case with the calculations of Krauss and Kowalski[8] and with the experimental results of Faivre *et al.*[9] It is very

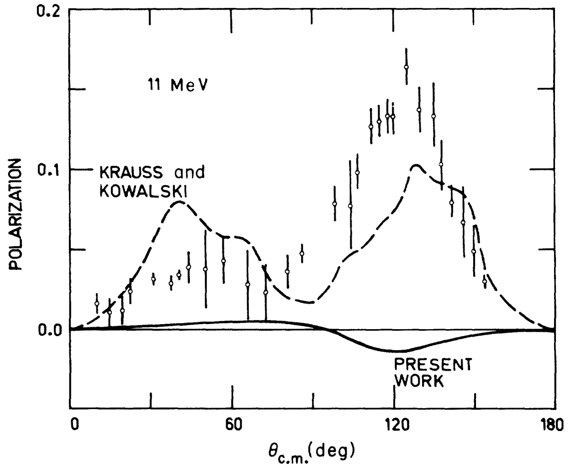


FIG. 1. Nucleon-deuteron polarization at 11 MeV. The present calculations (made at 10.85 MeV) used the same approximation and the same interactions as Krauss and Kowalski (Refs. 2 and 8). The experimental points are from Ref. 9.

clear that the two sets of theoretical calculations are entirely different, and that the new results, unlike the old, entirely disagree with experiment.

There is also a significant, though less spectacular, difference between the two theoretical calculations of differential cross sections, as we show in Figs. 4 and 5. In these figures we also show experimental $p$-$d$ results[10, 11] and the dif-

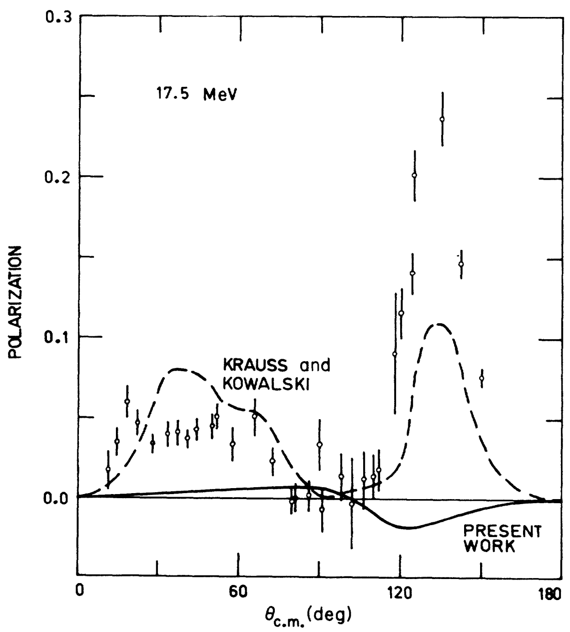


FIG. 2. Nucleon-deuteron polarization at 17.5 MeV. The present calculations used the same approximation and the same interactions as Krauss and Kowalski (Refs. 2 and 8). The experimental points are from Ref. 9.

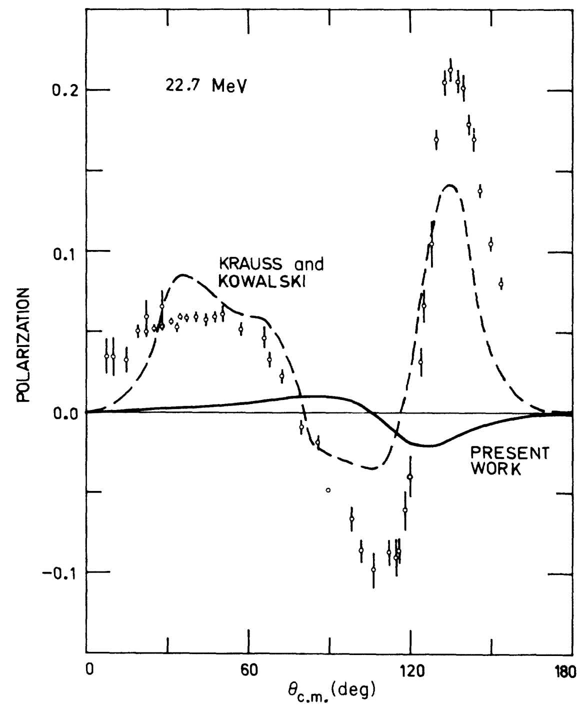


FIG. 3. Nucleon-deuteron polarization at 22.7 MeV. The present calculations used the same approximation and the same interactions as Krauss and Kowalski (Refs. 2 and 8). The experimental points are from Ref. 9.

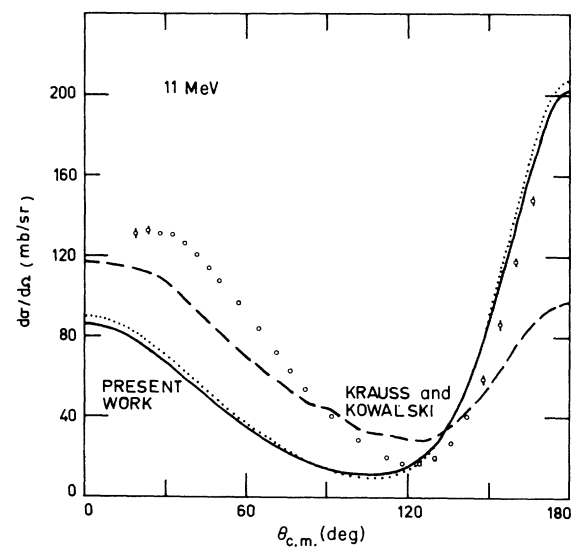


FIG. 4. Nucleon-deuteron differential cross sections at 11 MeV. The present calculations (made at 10.85 MeV) used the same approximation and the same interactions as Krauss and Kowalski (Refs. 2 and 8). The dotted curve was obtained (Ref. 12) by applying the same approximation to a central-force model (Ref. 13). The experimental points are 10.04-MeV $p$-$d$ results from Ref. 10.

ferential cross sections obtained[12] with a purely central-force separable-potential model[13] of the three-nucleon system (dotted curves). It is seen that the introduction of the simple noncentral force into this separable-potential model has made very little difference to the differential cross section, at least within the unitary approximation method. Coulomb forces have been neglected in all of the theoretical calculations, so that in comparing with experiment the small-angle Coulomb-interference region in the $p$-$d$ differential cross sections should be ignored.

We do not know the reason for the difference between the two calculations, but one feature of the previous calculations has a very large effect on the results, and deserves special consideration. This feature is the procedure followed in Ref. 2 of cutting off the partial-wave expansion at a certain low value of the total angular-momentum quantum number $J$, specifically at $J=\frac{3}{2}$ (i.e., all partial-wave amplitudes with $J>\frac{3}{2}$ are zero, whereas all those with $J=\frac{1}{2}$ or $\frac{3}{2}$ are nonzero, except by accident).

It is easy to see that $J=\frac{3}{2}$ is too small a value of the cutoff to give accurate results, and in fact this was recognized in Ref. 2. One might still be surprised, nevertheless, at the enormous magnitude of the error caused by following this procedure. In Fig. 6 we show the polarization obtained by cutting off our own 22.7-MeV calculation at $J=\frac{3}{2}$, as well as the original result already shown in Fig. 3. (The relevant curves in Fig. 6 are the solid ones. Note that the vertical scale has been changed to accomodate the new curve.) It can be seen that the effect of cutting off the partial-wave expansion at $J=\frac{3}{2}$ is truly enormous. In particular, the magnitude of the polarization now reaches the large maximum value of 0.5, compared to the proper maximum of only 0.02.

To help in understanding this result, we also show in Fig. 6 (dotted curve) the apparent polarization obtained by applying exactly the same cutoff procedure to calculations with the *central-force* model used previously, for which the true polarization is exactly zero. We see that far from being zero, the calculated polarization curve is in fact very close indeed to the analogous result with noncentral forces. In this case we have, therefore, an entirely spurious polarization curve, with the magnitude reaching as high as 0.5, half the polarization produced by a perfect polarizer. (It is perhaps worth mentioning that the central-force results in Fig. 6 have been checked at three angles by a completely independent hand calculation, so that we have unusual confidence in these possibly surprising results.)

The anomalous central-force results show clearly what is wrong with the procedure of cutting off the partial-wave expansion at $J=\frac{3}{2}$: It is not just that the cutoff value is too low, but rather that the cutoff procedure is itself unsatisfactory, since it destroys certain approximate but impor-

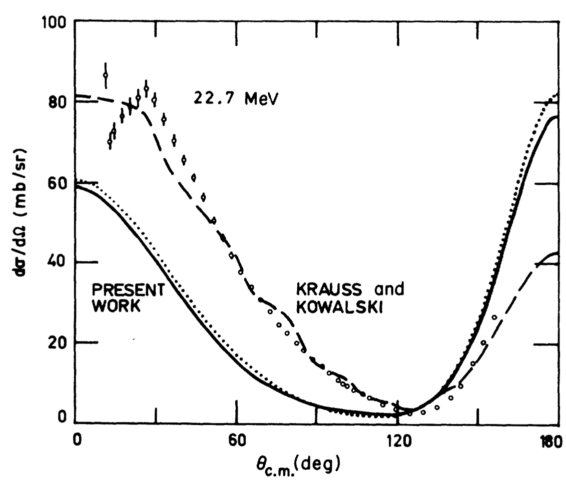


FIG. 5. Nucleon–deuteron differential cross sections at 22.7 MeV. The present calculations used the same approximation and the same interactions as Krauss and Kowalski (Refs. 2 and 8). The dotted curve was obtained (Ref. 12) by applying the same approximation to a central-force model (Ref. 13). The experimental points are 22.0-MeV $p$-$d$ results from Ref. 11.

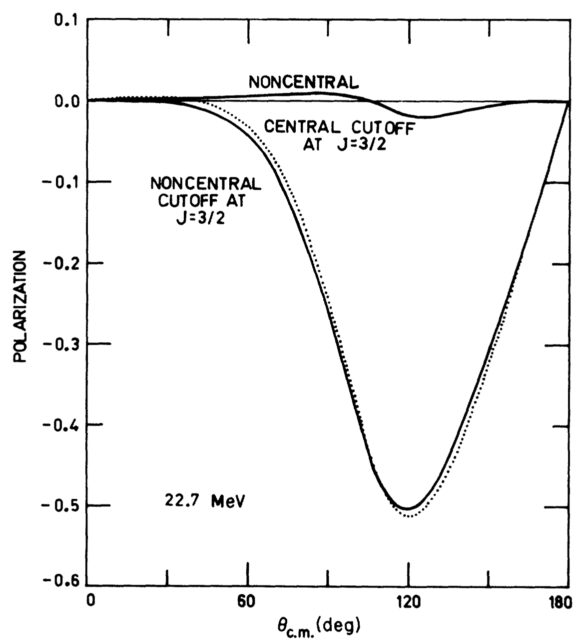


FIG. 6. Nucleon–deuteron polarization at 22.7 MeV, showing the effect of cutting off the partial-wave expansion at $J=\frac{3}{2}$. In the central-force case the true polarization is zero.

tant symmetries of the system, namely the invariance under separate rotations in configuration and spin space. For the central-force case these symmetries are of course exact, corresponding to the separate conservation of orbital angular momentum and total spin in this case, and it is, of course, the complete absence of any coupling between spin and orbital motion that makes the true polarization zero. The procedure of truncating in $J$ destroys the separate conservation of orbital and spin angular momentum in the central-force case, and the results in Fig. 6 show that the practical importance of this is very great.

The anomalous results shown in Fig. 6 can easily be avoided, by truncating the partial-wave expansion at a particular value of the orbial angular-momentum quantum number $L$, rather than at a particular value of $J$, since the rotational symmetries of the central-force case are thereby preserved. [In the calculations for Figs. 1–5 the partial-wave expansion was truncated at $L=7$, with both values of the total spin ($S=\frac{1}{2}$ and $\frac{3}{2}$) being included in all cases. The truncation was achieved in practice by first solving for the partial-wave amplitudes in the unitary approximation for all necessary $J$ values up to $J=\frac{17}{2}$, and then neglecting those with $L>7$.]

The foregoing argument, while it certainly provides a sufficient reason for the two theoretical calculations to differ greatly, does not at all explain the discrepancies that actually exist, since the 22.7-MeV polarization calculation of Ref. 2 (the dashed curve in Fig. 3) is totally different from the result obtained in Fig. 6 by applying the same truncation procedure. A similar, though less extreme, situation exists for the differential cross sections: The effect on our results (with both noncentral and central forces) of truncating the partial-wave expansion at $J=\frac{3}{2}$ is considerable, but does nothing at all to explain the discrepancies between the two calculations. At 22.7 MeV, for instance, the effect of the truncation is to *reduce* the differential cross sections in the forward direction to about half their original value, thus making the disagreement very much worse.

Finally, we consider the implications for the unitary first-order approximation[3] if the present calculations are correct. The differential-cross-section results obtained with the model of Ref. 2 are seen in Figs. 4 and 5 to be very similar to results obtained with a central-force model, and generally reinforce previous conclusions[7] that the approximation has some qualitative merit, but is not capable of giving detailed agreement with exact calculations or with experiment. The situation is not so clear, however, with the polarization results. Certainly the calculated polarizations are in complete disagreement with experiment, but from the results given so far it is not yet clear whether the error is caused mainly by the approximation method, or on the other hand by the crude assumptions made about the two-body interactions. Preliminary exact calculations with the same interactions suggest that in fact the larger part of the error comes from the inadequacy of the interactions, and that the unitary approximation itself has at least a degree of qualitative success, though again it is incapable of giving detailed agreement. The exact calculations will be described in a future publication.

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