Shell-Model Structure of ^{210}Bi: ^{209}Bi(d, p) at 17 MeV*

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The reaction ^{209}Bi$(d, p)^{210}$Bi has been studied at 17.0-MeV incident deuteron energy. Reaction protons were detected with an energy resolution of 7–9 keV using an Enge split-pole spectrograph. Accurate excitation energies (±0.2%) were obtained by a direct comparison to the well-known states of ^{209}Pb populated in the ^{208}Pb(d, p) reaction. The ground-state Q-value difference of ^{209}Bi(d, p) relative to ^{208}Pb(d, p) was found to be $+0.662 \pm 0.004$ MeV. Angular distributions, neutron l-transfer assignments, and spectroscopic strengths were obtained for 53 of the 60 proton groups observed up to an excitation energy of 3.3 MeV in the residual nucleus. It is found that agreement between experimental angular distributions and the predictions of the distorted-wave approximation is markedly improved over previous work in the lead region by the use of real optical-model well parameters which closely satisfy the "well-matching" criterion $V_d(r) \approx V_p(r) + V_n(r)$. In particular, the extreme oscillations predicted for $l = 0$ neutron transfer with some conventional parameters are damped out in agreement with experiment. The experimental excitation energies and spectroscopic strengths are compared with the predictions of Kuo and Herling as well as with those of Kim and Rasmussen. Good agreement is found for most of the strong transitions studied.

I. INTRODUCTION

Up to about 2.6 MeV (the first collective excitation of ^{208}Pb) the odd-odd ^{210}Bi nucleus, with two particles outside a closed ^{208}Pb core, is expected to be rather well described by reasonably simple shell-model calculations. This is certainly true for the 10 lowest-lying states observed by Erskine, Buechner, and Enge,[1] which are known to be nearly pure $\pi h_{9/2} \nu g_{9/2}$ configurations.[2-4] The theoretical interpretation of the higher-lying states listed in Ref. 1 has been hampered by the lack of definite l-value assignments and spectroscopic factors.

At present there is renewed interest in these levels. Cline, Alford, and Gove[5] have proposed a multiplet classification from their study of the ^{209}Bi(d, p) reaction at 19 MeV. A group at Los Alamos National Laboratory[4] has attempted to identify the $\pi h_{9/2} \nu i_{11/2}$ states from (γ, γ) coincidence studies following thermal-neutron capture on ^{209}Bi. An analysis of all states up to 3.3 MeV observed in the $(d, p\gamma)$ reaction is underway.[3,6] Finally, a new set of shell-model structure calculations for ^{210}Bi has recently become available.[7]

In the present paper, we will present a detailed comparison between the theoretical predictions of Kim and Rasmussen[2] and Kuo and Herling,[7] and the states observed in a high-resolution study of the reaction ^{209}Bi$(d, p)^{210}$Bi. Points of comparison will include the number, spectroscopic strengths, and excitation energies of states which can be placed in a particle-particle multiplet on the basis of neutron l-transfer assignments.

II. EXPERIMENTAL METHOD

The experiment was performed with a 17.00-MeV deuteron beam from the University of Pittsburgh three-stage Van de Graaff accelerator. This beam was focused through a 0.5-mm-wide by 2-mm-high collimating slit (followed by an antiscattering slit) placed about 2 cm from the target position in the scattering chamber. The beam-handling system and scattering chamber have been described in detail elsewhere.[8,9] Charge collection and elastically scattered deuterons, detected by NaI(Tl) scintillators at ±38° relative to the beam direction, were used to monitor the beam.

The target was a 100-μg/cm^2 film of chemically pure natural ^{209}Bi metal evaporated onto a 20-μg/cm^2 C backing. The low melting point of Bi metal made it necessary to increase the effective beam spot size by vibrating the target. This procedure eliminated the tendency of the target to melt and allowed use of up to 1 μA of beam current. Target thickness was measured by elastic scattering of 10- and 17-MeV deuterons. At the lower energy, the cross section at 38° was assumed to be given by the Rutherford formula. The elastic scattering at 17 MeV was compared with optical-model predictions using standard parameters.[10] The two methods agreed to better than 10%, but because of systematic errors possible in these measurements it is estimated that the absolute cross sections may be uncertain to ±20%. The scattered protons were detected in Kodak NTB 50-μ nuclear emulsions placed in the focal plane of an Enge slit-pole

spectrograph. Typical reaction spectra are shown in Figs. 1 and 2. The peak-to-background ratio is between 10^2 and 10^3 for groups of average strength up to an excitation energy of 3.3 MeV in the residual nucleus. The states above this energy are too weak and too close together for accurate analysis. Light-element contaminant peaks which are kinematically defocused by the spectrograph are also visible in Fig. 1. These broad groups obscured the peaks of interest at several angles resulting in some gaps in the angular distributions. The full width at half maximum (FWHM) of the Bi proton groups is 7–9 keV, which is still not adequate to separate four known or suspected doublets. Major contributions to the experimental energy resolution come from beam divergence (≈4 keV) and spot size (≈4 keV), differential energy loss in the target (≲3 keV), and plate-scanning resolution (≈3 keV).

Emphasis was placed on obtaining accurate excitation energies in order to facilitate a correlation between states observed in this work and those deduced from recent γ-ray experiments.[3,4,6] The spectrograph calibration was obtained by a direct comparison[11] with the well-known[12] states of ^{209}Pb excited in the ^{208}Pb(d,p) reaction with the same incident beam energy, magnetic field setting, and focal-plane setting as for ^{209}Bi$(d,p)^{210}$Bi. The excitation-energy assignments, listed in Table I, are believed to be accurate to within ±2 keV below 2 MeV, and to ±3 keV above 2 MeV in ^{210}Bi. As a by-product of this calibration procedure, the ground-state Q-value difference of ^{209}Bi(d,p) relative to ^{208}Pb(d,p) was determined to be +0.662 ±0.004 MeV. From the tabulated ^{208}Pb(d,p) ground-state Q value,[13] we compute $Q_0 = 2.382$ ±0.010 MeV for the reaction ^{209}Bi$(d,p)^{210}$Bi.

Angular distributions for 53 of the 60 proton groups observed, arranged according to neutron l-transfer assignment, are shown in Figs. 3–7. The error bars are primarily due to statistics (standard deviation), but include an estimate of systematic errors in background subtraction for weak states and in separating doublets. The random monitoring error was 5% and is the dominant error for the strongest groups.

III. ANALYSIS AND RESULTS

The experimental angular distributions were compared with predictions of the distorted-wave Born approximation (DWBA) using the code

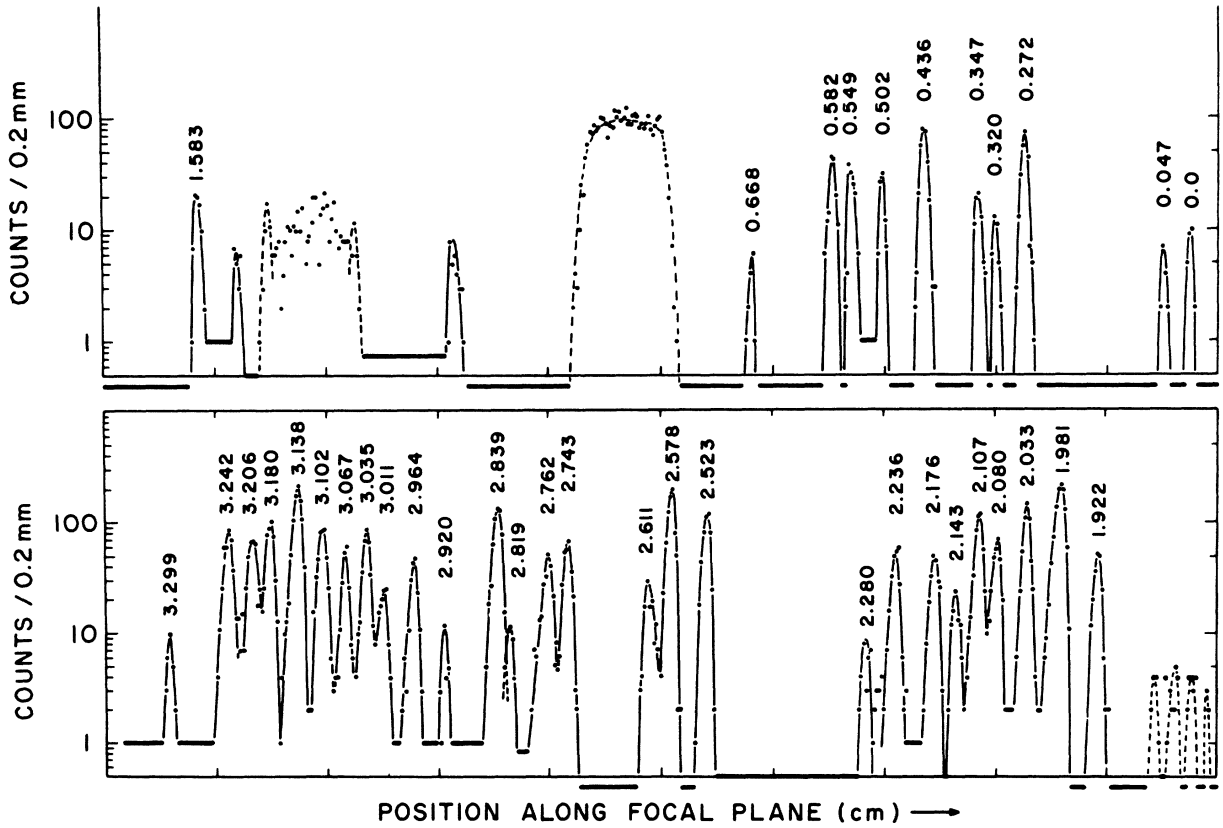

FIG. 1. Energy spectrum of protons from the reaction ^{209}Bi$(d,p)^{210}$Bi taken at $\theta_{lab} = 45°$. The excitation energies for most proton groups are labeled in MeV. Note the broad groups from light-element contaminants.

DWUCK.[14] Finite-range and nonlocality corrections were computed in the local-energy approximation.[15] The finite-range parameter was 0.62,[16] and the nonlocality parameters were $\beta_d = 0.54$ and $\beta_p = 0.85$.[14, 17] Nonlocal corrections were not applied to the bound-state wave functions.[17]

Deuteron optical-model parameters were taken from Refs. 10 and 17. They are similar to those used in previous investigations[17, 18] of the (d, p) reaction on the Pb isotopes. Two sets of proton optical-model parameters were employed. The first was obtained from Perey,[19] and the second from the work of Becchetti and Greenlees.[20] All optical-model parameters are listed in Table II, along with the parameters of the bound-state well.

The angular distributions predicted by the DWBA with the three combinations of optical-model parameters are compared with experiment in Figs. 3–6. The superiority of the Perey-Becchetti and Perey-Perey sets is striking. Note in particular the damping of the strong oscillations characteristic of conventional calculations[17, 18] for $l = 0$, and the successful prediction of rapid falloff at forward angles for $l = 4$ transitions. The improvement in the $l = 2$ transfers, which happen to be more nearly momentum-matched, is not as marked. Similar improvements in the shape of the DWBA angular distributions have previously been reported for two-nucleon-transfer analysis[21] and ascribed to the use of the "well-matching" criterion. Following the work of Stock *et al.*[22] on the (^{3}He, α) reaction, it is suggested that the most reliable DWBA predictions might be obtained by making the real well depths and geometries for the optical-model and bound-state wave functions as similar as possible, provided that they are consistent with elastic scattering data. The Perey-Becchetti set of parameters in the present study satisfies this criterion best and provides support for its application to the (d, p) reaction. Momentum mismatch in (d, p) is not ordinarily singled out as a serious problem. However, it is quite

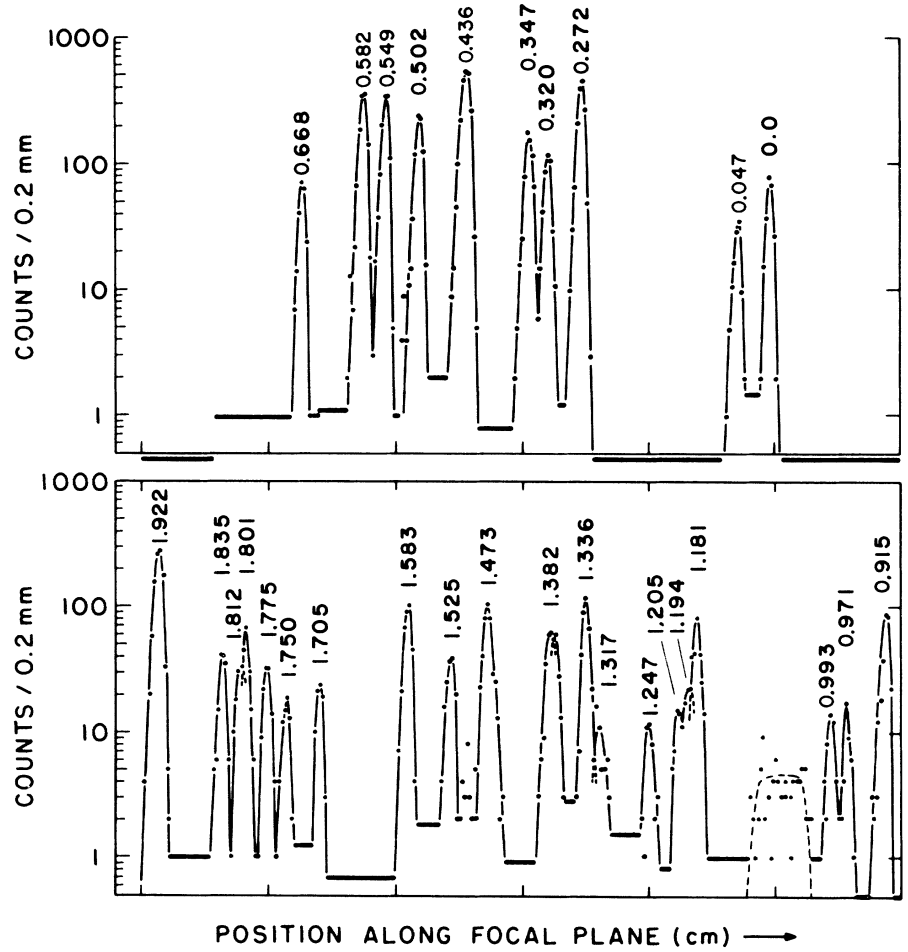

FIG. 2. Energy spectrum of protons from the reaction ^{209}Bi$(d, p)^{210}$Bi taken at $\theta_{lab} = 80°$ and for 10 times the exposure of Fig. 1 to emphasize the weak states below $E_x = 2.0$ MeV.

TABLE I. Experimental excitation energies, l transfers, and spectroscopic strengths for ^{209}Bi$(d,p)^{210}$Bi. Suggested J^π values for states in ^{210}Bi are also listed, and compared with existing assignments. Less certain states are enclosed in parentheses.

E_x [a] (MeV)	l	$(2J_f+1)S$	Suggested J^π [b] Present study	J^π [c] Previous assignments
0.0	4	2.9	1^-	1^-
0.047	4	1.0	0^-	0^-
0.272	4	18.5	9^-	9^-
0.320	4	5.0	2^-	2^-
0.347	4	7.4	3^-	3^-
0.436 D	4	27.5	5^-+7^-	5^-+7^-
0.502	4	10.0	4^-	4^-
0.549	4	11.8	6^-	6^-
0.582	4	14.6	8^-	8^-
0.668	6	28	10^-	10^-
0.915	4	2.8	(8^-)	8^-
0.971	2	0.1	2^-	2^-
0.993	6	8	3^-	3^-
1.181	6	37		
(1.194)				6^-
1.205	4 or 2	0.8		4^-
1.247	4 or 2	0.5		
1.317				
1.336	6	47		5^-
1.382 D	6	47		
1.473 D	6	58		
1.525	(6)	26		
1.583	2	1.2	2^-	
1.705	7	13	6^+	
1.750	7	10	5^+	
1.775	7	17	8^+	
1.801	7	24	12^+ or 11^+	
1.812	7	15	7^+	
1.835	7	19	10^+ or 9^+	
1.922	2	2.8	2^-	
1.981 D	2	17	7^-+3^-	
2.033	2	8.1	6^-	
2.080	2	4.0	4^-	
2.107	2	7.5	5^-	
2.143	2	1.6	6^-	
2.176	2	3.1	4^-	
2.236	2	3.8	3^-	
2.280	2	0.2	5^-	
2.314				
(2.340)				
2.464				
2.523	0	5.4	4^-	4^-
2.578	0	7.1	5^-	5^-
2.611	(2+4)	0.9+(1.0)		
2.734	(2+4)	2.8+(1.5)		
2.762	2	2.5		
2.819	2+4	2.0+(8.0)		
2.839	(2+4)	2.5+(15)		
2.920	2	1.9		
2.964	(2+4)	1.6+(2.0)		

TABLE I *(Continued)*

E_x [a] (MeV)	l	$(2J_f+1)S$	Suggested J^π [b] Present study	J^π [c] Previous assignments
3.011	(2+4)	0.8+(1.5)		
3.035	(2+4)	3.4+(1.5)		
3.067	(2+4)	1.7+(2.0)		
3.102	2+4	2.3+(9.0)		
3.138	(2+4)	9.0+(4.0)		
3.180	2	4.5		
3.206	(2+4)	3.5+(2.0)		
3.242	(2+4)	3.4+(2.5)		
3.299				
3.330				

[a] Uncertainty is ±2 keV below 2 MeV and ±3 keV above 2 MeV. D= doublet.

[b] Tentative J^π assignments consistent with observed strength. Ambiguous to the extent that they depend on the theoretical predictions.

[c] Accumulated assignments of Refs. 1, 3, and 4. Not all states are assigned in any one reference.

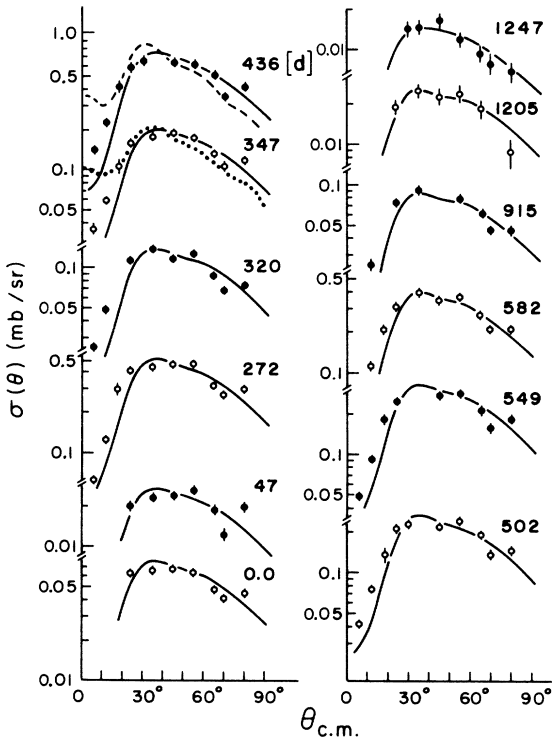

FIG. 3. Angular distributions for $l=4$ transitions compared with the DWBA calculations. The dotted curve represents the calculation with Bassel-Perey parameters. The dashed curve refers to the Perey-Perey parameter set, and the solid curves were computed with Perey-Becchetti parameters (Set II). Known or suspected doublets are indicated by [d]. Excitation energies in keV are indicated. Error bars include statistical errors and an estimate of errors in background subtraction and in separating close doublets.

easy to see that for the energies and target involved, the condition $\vec{P}_{in}=\vec{P}_{transf.}+\vec{P}_{out}$ cannot be fulfilled for all observed l transfers if the reaction is to occur at or near the nuclear radius ($R\approx 7$ fm). A reported failure[23] of the DWBA to predict consistent cross sections in the tin region for 12–17-MeV deuterons and $l\gtrsim 4$ has not been explained to date. It would be of interest to investigate if use of the well-matching criterion would reduce the reported discrepancies.

Spectroscopic factors were derived by fitting the DWBA predictions to the experimental angular distributions. The fits are shown in Figs. 3–7. The stripping cross section is proportional to $(2J_f+1)$, where J_f is the total angular momentum of the final state and is generally not known. Therefore, the quantity $(2J_f+1)S$ was computed using the conventional[17] (d,p) normalization factor $1.53D_0^2$ (where D_0^2 is the normalization in code JULIE), and is listed in Table I for all the transitions investigated. Detailed comparisons of these spectroscopic strengths with the predictions of Kim and Rasmussen[2] and Kuo and Herling[7] are made for the various particle-particle multiplets in the following sections.

A. $\pi h_{9/2}\nu g_{9/2}$ Multiplet

The spin sequence for this multiplet has recently been determined,[3,4] and the spectroscopic strengths listed in Table I are in good agreement with these assignments if the states are assumed to be almost pure. Both Refs. 2 and 7 predict a nearly pure ground-state multiplet. A major discrepancy with theory is the very strong $l=4$ transition to the 915-keV state, which seems to contain the missing 8^- strength. Spin and parity of

this state were determined as 8^- by the Los Alamos group,[4] who assign it to the $\pi h_{9/2}\nu i_{11/2}$ multiplet. This assignment seems inappropriate on the basis of the observed typical $l=4$ angular distribution (Fig. 3). While it is true that these multiplets can mix, and that the intrinsic stripping cross section of $l=4$ transitions is an order of magnitude greater than that of $l=6$, an 8^- state of the observed strength requires an $l=4$ amplitude admixture of about 0.4. Neither calculation predicts a second 8^- state with such large $l=4$ strength and $\pi h_{9/2}$ parentage, whatever the basic neutron configuration; and conversely, neither calculation predicts the observed weakening of the 8^- level at 582 keV (Table I).

The total $l=4$ stripping strength for the ground-state multiplet is listed in Table III. Since these states are reasonably pure, one expects to see all of the $g_{9/2}$ strength, as in fact is the case. It is clear that the improved fits to the angular distributions from the "well-matched" parameters have not been obtained at the expense of strength predictions.

The spectroscopic strengths and excitation energies for the 10 strong states in this multiplet are compared to theoretical predictions in Fig. 8.

Two level crossings are required in the comparison with Ref. 7 (the calculation of Kim and Rasmussen[2] predicts the correct ground state as it was designed to do). The spreading width of the multiplet is slightly underestimated by Ref. 2 and overestimated by Ref. 7. Nevertheless, the general agreement is impressive.

B. $\pi h_{9/2}\nu i_{11/2}$ and $\pi h_{9/2}\nu j_{15/2}$ States

Although shell-model calculations[2,7] predict relatively little admixture (amplitude $\lesssim 0.2$) of the $\pi h_{9/2}\nu i_{11/2}$ multiplet with states of other neutron configurations coupled to an $h_{9/2}$ proton, the intrinsic strength of $l=6$ transfers at 17 MeV is an order of magnitude smaller than that for $l=4$ and 2 orders of magnitude weaker than $l=0$ or $l=2$ transfers. It is therefore expected that the observed angular distributions for these states will not necessarily be characteristic of pure $l=6$ transfer. This is indeed the case (Fig. 4). However, in a two-particle space the 10^- state can only be formed from a $\pi h_{9/2}\nu i_{11/2}$ configuration, so that it must necessarily be pure. The angular distribution of the group at 668 keV, which is most probably the 10^- state proposed in Ref. 4, is close-

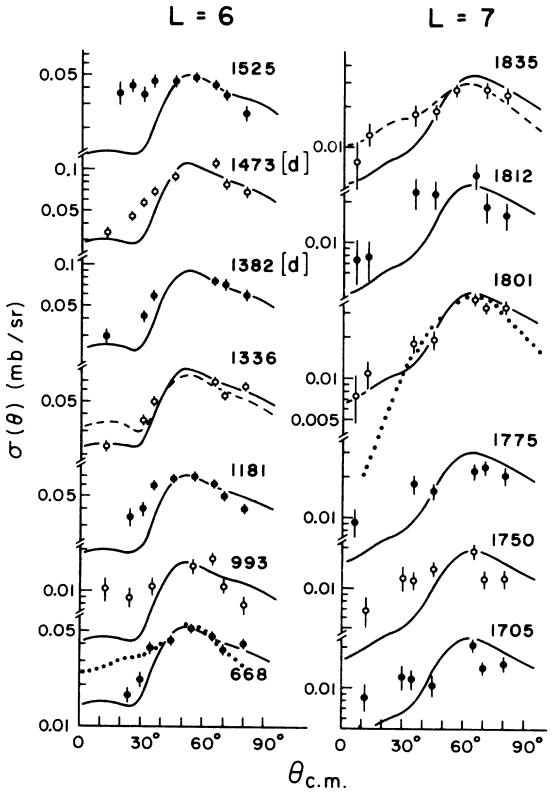

FIG. 4. Angular distributions for $l=6$ and $l=7$ transitions. See also Fig. 3 caption.

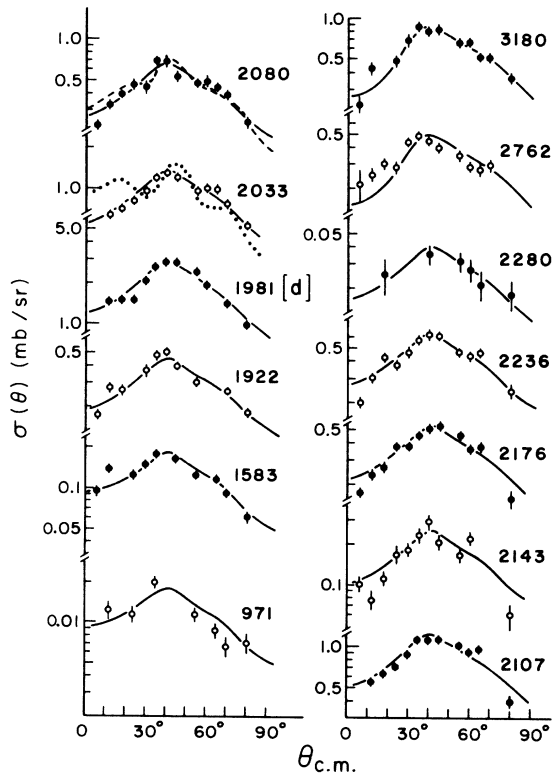

FIG. 5. The $l=2$ transfer angular distributions. See also Fig. 3 caption.

ly matched by a pure $l=6$ DWBA calculation.

The excitation energies of seven levels and their spectroscopic strengths extracted under the assumption of pure $l=6$ transfer are compared with theoretical predictions in Fig. 9. The discrepancies are large enough to prohibit even tentative J^π assignments to most of these states. The very large spectroscopic strengths are probably due in large part to admixtures of $l=0$, 2, and 4 neutron configurations. Note, however, that even the pure 10^- state has nearly 35% more strength than expected, so that some of the difficulty must be traced to the DWBA calculations for $l=6$ transfer.

There are three other states which may belong to this multiplet although they do not exhibit $l=6$ angular distributions. The 971-keV $l=2$ transition (Fig. 5) can probably be identified with the 2^- state predicted at 1050 keV with $(2J_f+1)S_{5/2}=0.1$ in Ref. 7 and at 1295 keV with $(2J_f+1)S_{5/2}=0.05$ in Ref. 2. This state has previously been assigned[4] $J^\pi=2^-$. (Kuo and Herling predict a second 2^- state with approximately the same $l=2$ spectroscopic strength at 1310 keV, which perhaps corresponds to the 1317-keV state for which we have no angular distribution. The spin and parity of this state have not previously been assigned.)

The weak transition to the states at 1205 and 1247 keV are compared with $l=4$ calculations in Fig. 3. However, $l=2$ cannot be ruled out, since the forward-angle data are missing. Kim and Rasmussen predict a predominant $l=4$ transition of about the correct strength to an 8^- state at 939 keV. No such state is predicted by Kuo and Herling who, on the other hand, predict weak $l=2$ transitions of about the observed strength to 4^-, 5^-, 6^-, and 7^- states between 1430 and 1520 keV. The 1205-keV state was assigned 4^- and the 1194-keV state was assigned 6^- in Ref. 4. The 1247-keV state was not seen.

The situation is clearer for the $\pi h_{9/2}\nu j_{15/2}$ states, which are expected to be nearly pure. We see six of the predicted 10 states with reasonable spectroscopic strengths. The excitation energies and strengths are compared with theoretical predictions in Fig. 8. Kim and Rasmussen seem to give

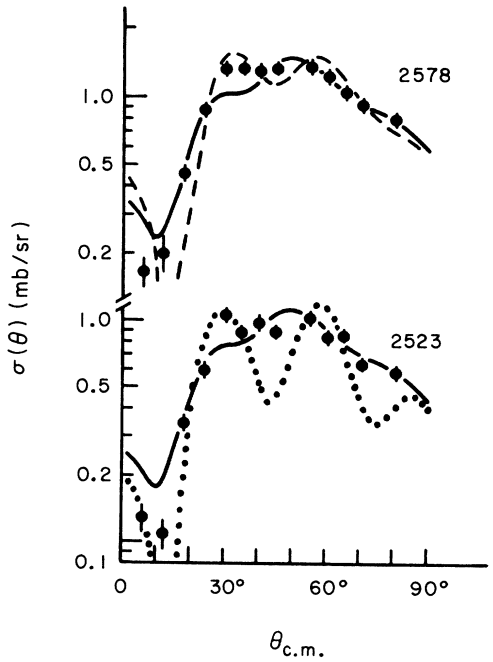

FIG. 6. The two $l=0$ angular distributions. See also Fig. 3 caption. Note the extreme oscillations predicted with the Bassel-Perey optical-model parameters.

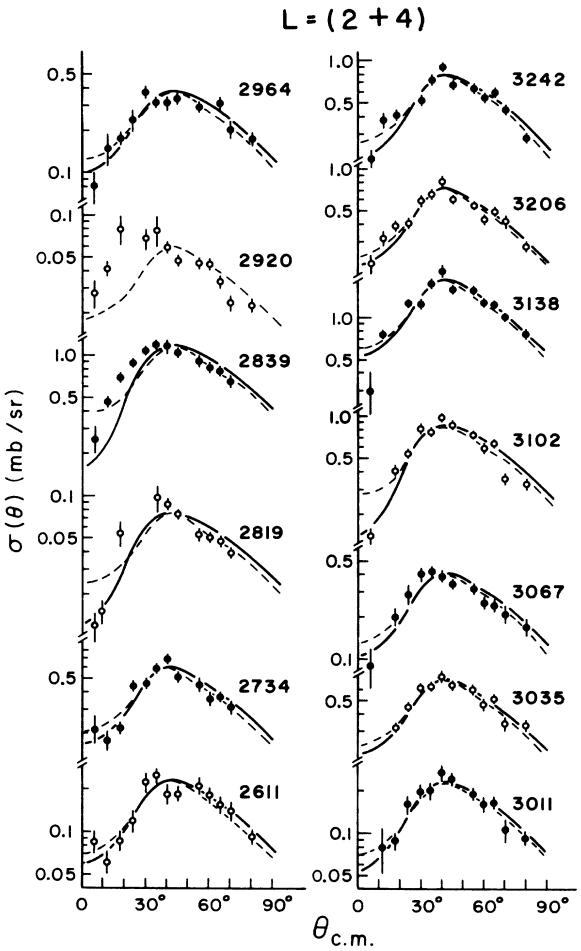

FIG. 7. Mixed $l=2$ and $l=4$ angular distributions. The solid curves are empirical admixtures of $l=2$ and $l=4$ DWBA predictions with Becchetti-Perey parameters. The corresponding strengths for the two components are listed in Table I. The dashed curves are for pure $l=2$ transfer.

TABLE II. Optical-model parameter sets used in the ^{209}Bi$(d,p)^{210}$Bi DWBA calculations.

	V (MeV)	$r_0=r_c$ (F)	a (F)	W (MeV)	W_D (MeV)	r_I (F)	a_I (F)	V_{so} (MeV)	r_{so} (F)	a_{so} (F)
Incident channel ($d+^{209}$Bi)										
Ref. 10	105.0	1.15	0.81	0	13.0	1.34	0.68	0		
Ref. 17	100.0	1.14	0.89	0	13.8	1.33	0.75	0		
Exit channel ($p+^{210}$Bi)										
Set I (Ref. 19)	52.0	1.25	0.65	0	7.5	1.25	0.76	0		
Set II (Ref. 20)	$58.4+0.32E_x$	1.17	0.75	$1.59-0.22E_x$	$9.45+0.25E_x$	1.32	0.66	6.2	1.01	0.75
Bound neutron										
Set I	a	1.20	0.65					$\lambda=25$		
Set II	a	1.17	0.75					$\lambda=25$		

[a] Adjusted to give the proper neutron separation energy.

a better account of the close grouping of these states, but this may in part be due to the fact that $l=7$ states above 1.9 MeV could not be seen because of the extremely strong $l=2$ transitions in this region. In addition, it is more difficult to identify the weak 3^+ and 4^+ members of this multiplet. On the basis of these considerations, we have suggested spins and parities for these states which are consistent with the observed spectroscopic strengths (Fig. 8). It is encouraging that

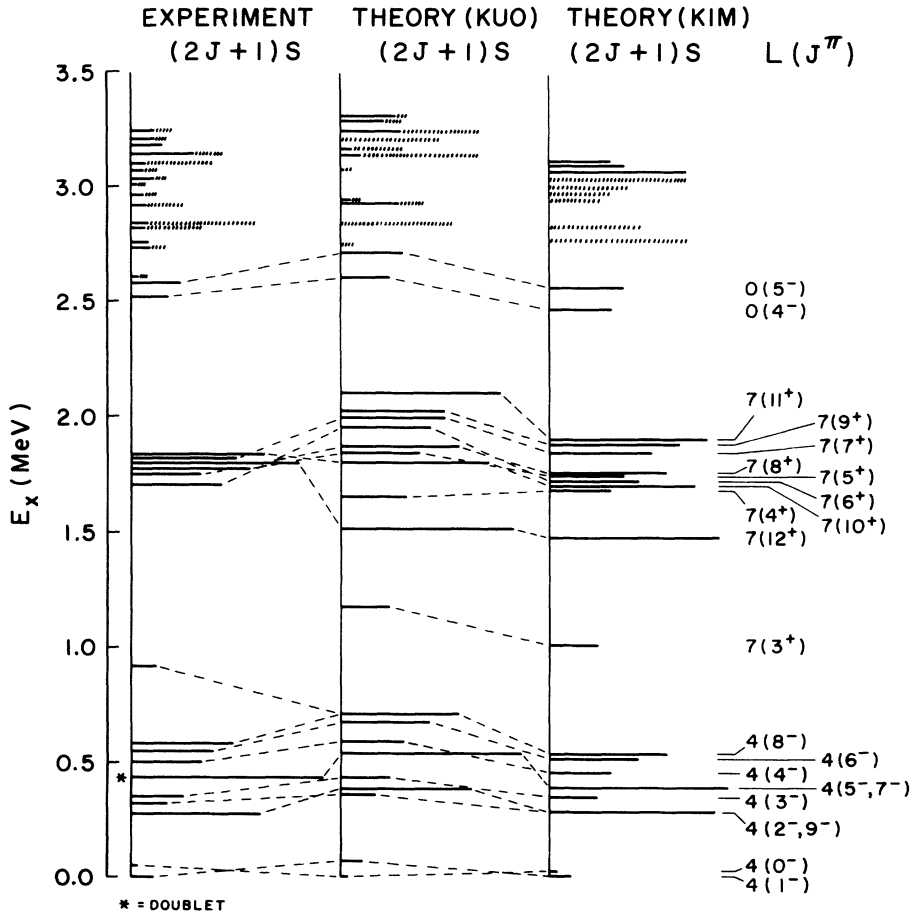

FIG. 8. Comparison of experimental excitation energies and spectroscopic strengths with the predictions of Kuo (Ref. 7) and Kim (Ref. 2) for the $g_{9/2}$, $j_{15/2}$, $s_{1/2}$, $g_{7/2}$, and $d_{3/2}$ neutron configurations coupled to a $h_{9/2}$ proton. For the latter two (mixed) multiplets, the $l=2$ strength is indicated by the solid bars and the $l=4$ strength by the dotted bars. Suggested J^π assignments are indicated where possible.

TABLE III. Total observed spectroscopic strength for the various multiplets. Obtained with Perey deuteron parameters and Becchetti proton parameters (Set II). $\Sigma_{11/2^+}$ contains contributions from lower (kinematically favored) even l values and represents an expected overestimate. $\Sigma_{15/2^-}$ represents the sum over six out of ten expected levels. This number would be further reduced if the Perey-Perey parameters (Set I) were used. Compare Table IV.

Configuration	Σ [a] (expt)	Σ [a] (sum-rule limit)
$\pi h_{9/2}\ \nu g_{9/2}$	10.2	9.8
$\pi h_{9/2}\ \nu i_{11/2}$	(25)	11.8
$\pi h_{9/2}\ \nu j_{15/2}$	9.8	15.6
$\pi h_{9/2}\ \nu d_{5/2}$	5.0	5.85
$\pi h_{9/2}\ \nu s_{1/2}$	1.3	1.96
$\pi h_{9/2}\ \nu g_{7/2}$	(5)	7.8
$\pi h_{9/2}\ \nu d_{3/2}$	4.3	3.9

[a] $\Sigma \equiv (2J_i+1)^{-1} \sum_{\text{all states}} (2J_f+1)C^2S$.

these $l=7$ strengths are in such good agreement with the theoretical predictions. This agreement provides evidence that the major difficulties with the $l=6$ strengths should not be ascribed to a failure of the DWBA for high angular momentum transfer.

C. $\pi h_{9/2}\nu d_{5/2}$ Multiplet

Nine strong proton groups with $l=2$ angular distributions, one of which is a known doublet, are assigned to this configuration. Since only six states are expected from a pure multiplet, it is clear that the $\nu d_{5/2}$ strength is fragmented. Shell-model calculations[2,7] indicate that the great majority of the mixing occurs with states of other proton configurations, and in particular with the $\pi f_{7/2}\nu i_{11/2}$ multiplet. Under such circumstances, the $\pi h_{9/2}\nu d_{5/2}$ strength will be fragmented but the states will retain characteristic $l=2$ angular distributions as observed (Fig. 5).

A detailed comparison of experimental excitation energies and spectroscopic strengths with the predictions of Refs. 2 and 7 is shown in Fig. 9. Both calculations predict exactly 10 strong $l=2$ states at almost exactly the observed excitation energies. Just as for the ground-state multiplet, the spreading width is slightly underestimated by Ref. 2 and overestimated by Ref. 7, but the overall agreement is extremely good. In addition, the weak $l=2$ transition to the state at 2280 keV can easily be accounted for by either shell-model calculation. Tentative spin assignments for these states which are consistent with observed spectro-

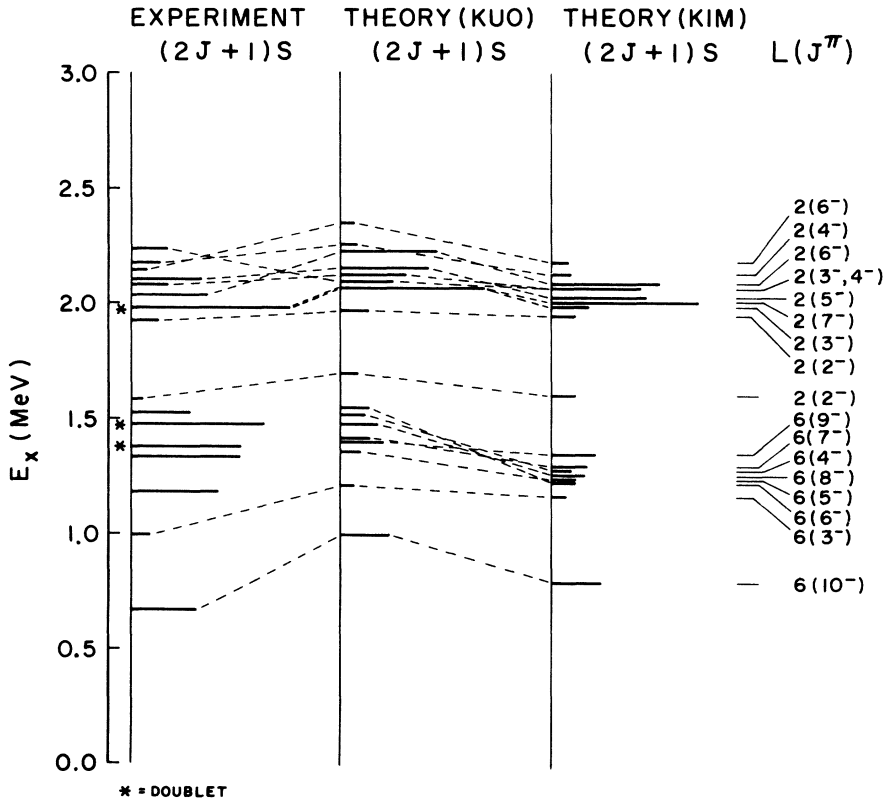

FIG. 9. Comparison of experiment with theory for the $\pi h_{9/2}\nu_{11/2}$ and $\pi h_{9/2}\nu d_{5/2}$ multiplets.

TABLE IV. Comparison of relative total predicted cross sections for the parameters of Table II as a function of l_j transfer. The first column ($\lambda = 0$) is included to demonstrate the importance of the $l \cdot s$ term in the bound-state well. Q values used correspond to the centroids of the l_j multiplets indicated.

l	Becchetti and Perey ($\lambda = 0$)	Becchetti and Perey ($\lambda = 25$)	Perey and Perey ($\lambda = 25$)	Bassel and Perey ($\lambda = 25$)
0 ($s_{1/2}$)	0.73	0.73	0.69	0.60
2 ($d_{5/2}$)	0.45	0.47	0.43	0.40
4 ($g_{9/2}$)	0.087	0.097	0.099	0.099
6 ($i_{11/2}$)	0.0120	0.0082	0.0088	0.0092
7 ($j_{15/2}$)	0.0062	0.0086	0.0122	0.0087

scopic strengths are indicated in Fig. 9. The weak 2280-keV state is most probably another 5^- state. It will be of interest to compare the present assignments, which are ambiguous to the extent that they depend on theoretical calculations, with the results of the more model-independent $(d, p\gamma)$ experiment.[6]

D. States Based on $s_{1/2}$, $d_{3/2}$, and $g_{7/2}$ Neutron Configurations

We observe two strong $l = 0$ transitions, which are assigned to the $\pi h_{9/2} \nu s_{1/2}$ doublet. The spectroscopic strengths and excitation energies are compared with theoretical predictions in Fig. 8, and the agreement is seen to be excellent. Kim and Rasmussen predict a nearly pure doublet, while Kuo and Herling's wave functions require a moderate (amplitude 0.4) admixture with the $\pi h_{9/2} \nu d_{5/2}$ multiplet. The total $l = 0$ strength (Table III) is reasonably consistent with either interpretation. The missing strength may indicate a somewhat greater fractionation than expected.

The $\pi h_{9/2} \nu d_{3/2}$ and $\pi h_{9/2} \nu g_{7/2}$ multiplets are expected to be strongly mixed, so that their angular distributions will be combinations of $l = 2$ and $l = 4$ transfer shapes. There are 15 states which can be fitted with this combination, but it is difficult to separate $l = 2$ and $l = 4$ contributions with any certainty, since the intrinsic strength of $l = 2$ transitions is almost an order of magnitude greater than that for $l = 4$ transfer. A comparison of the experimental results to the shell-model calculations is shown in Fig. 8. Unfortunately, a detailed comparison with Ref. 2 cannot be made because the complete wave functions for those states are not listed. The calculation of Kuo and Herling underestimates the fragmentation and mixing of the multiplets, although the centroid and spreading width are reproduced reasonably well. These discrepancies are not particularly surprising in view of the fact that the first excited state of the ^{208}Pb core, which is not accounted for in either calculation, occurs at an excitation energy of 2.6 MeV.

The $l = 2$ and $l = 4$ contributions to each of the 15 states was determined from the "best fit" to the experimental angular distributions (Fig. 7). This procedure is liable to be most uncertain for the $l = 4$ strength, as mentioned above. Nevertheless, the total spectroscopic strengths (Table III) are in reasonable agreement with expectations. It should be emphasized that this agreement was not forced but resulted directly from the fitting procedure.

IV. SUMMARY AND CONCLUSIONS

We have shown that the angular distributions predicted by the DWBA for the (d, p) reaction on heavy nuclei are very sensitive to the choice of optical-model parameters, and that the agreement between experimental and predicted shapes is markedly improved by a set of parameters which best satisfy the "well-matching" criterion. That is, it seems best to choose well depths and geometries for the entrance and exit channels which are as similar as possible. The dependence of the calculated spectroscopic factors on the spin-orbit term in the bound-state well and on the choice of optical-model parameters can be estimated from Table IV. The spin-orbit potential ($\lambda = 25$) produces a large change in the predicted cross section for high l values ($\Delta\sigma/\sigma = 0.39$ for $j_{15/2}$), as expected. The various optical-model parameter sets of Table II predict generally similar total cross sections and hence similar deduced spectroscopic factors for $l < 6$. But it is interesting to note that the successful Perey-Becchetti and Perey-Perey sets lead to spectroscopic factors that diverge in a systematic fashion. For $l = 7$ they disagree by a factor of 1.4, a number outside the commonly expected DWBA uncertainties. Although four of ten $j_{15/2}$ transitions have not been resolved from stronger groups it appears unlikely that the calculations with Set I would lead to a satisfactory value for $\Sigma_{15/2}$.

Shell-model calculations[2,7] of the structure of ^{210}Bi have been shown to be remarkably successful in accounting for nearly all of the 60 proton groups observed in the reaction ^{209}Bi$(d, p)^{210}$Bi up

to an excitation energy of 3.3 MeV in the residual nucleus. The detailed comparison between experiment and theory was best for the rather pure $g_{9/2}$, $s_{1/2}$, and $j_{15/2}$ neutron configurations, and for the $\pi h_{9/2}\nu d_{5/2}$ multiplet which is strongly mixed only with $\pi f_{7/2}\nu i_{11/2}$ states. Agreement was not as good for the $\pi h_{9/2}\nu i_{11/2}$ multiplet, because of the small intrinsic strength for $l=6$ transitions, or for the strongly mixed $\pi h_{9/2}\nu d_{3/2}$ and $\pi h_{9/2}\nu g_{7/2}$ multiplets which are at an excitation energy greater than that of the first excited state of the ^{208}Pb core.

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