Separation Method for O^{18} Using One-Boson-Exchange Potentials

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Realistic, generalized, and regularized velocity-dependent one-boson-exchange-potential models of the nucleon-nucleon interaction are applied to calculate the low-lying energy levels of the nucleus O^{18} in the harmonic-oscillator shell model. The singlet even states are treated by the Moszkowski-Scott separation method technique. The energy levels so obtained are comparable to those obtained from the phenomenological Hamada-Johnston hard-core potential. We do not attempt to calculate core-polarization effects explicitly, but rather we attempt to simulate them by varying the coupling constant of the light scalar meson.

I. INTRODUCTION

In a previous paper[1] we calculated the low-lying energy levels of O^{18} in the harmonic-oscillator shell model, using recent models of the one-boson-exchange potential. It was found that the shell-model matrix elements exhibited too much repulsion, and we concluded that it was necessary to use reaction-matrix theory, where the potential V is replaced by the Brueckner reaction matrix G for the scattering of two nucleons. In this paper we calculate the shell-model reaction matrix elements for O^{18} using several velocity-dependent one-boson-exchange models. The main part of the calculation is the determination of the Moszkowski-Scott separation distances[2] for the singlet even states. We consider potential models I and III of Ueda and Green,[3] referred to as UG I and UG III; the two-parameter model of Green and Sawada,[4] referred to as GS II; and the most recent model of Bryan and Scott,[5] referred to as BS III. All these models have p^2 velocity-dependent forces and no hard cores. They are discussed in Sec. 2 of Ref. 1.

II. SEPARATION METHOD

The G matrix is defined by the integral equation

$$G = V - V(Q/e)G\,, \tag{1}$$

where Q is the Pauli operator and e is the energy denominator. G also satisfies

$$G\phi = V\psi\,, \tag{2}$$

where ϕ is the unperturbed wave function, which is taken to be a harmonic-oscillator wave function in the case of finite nuclei, and ψ is the correlated wave function. From Eqs. (1) and (2) it follows that

$$\psi = \phi - (Q/e)V\psi\,. \tag{3}$$

The application of the Brueckner theory to the nuclear many-body problem is facilitated by the Moszkowski-Scott separation method technique.[2] This technique has been applied to the shell model by a number of authors[6,7] using phenomenological potentials. Basically it involves a separation of the potential (for a given state and relative momentum) into a short-range part V_s and a long-range part V_l in such a way that V_s gives no phase shift. Then, to a first approximation, V_l is the effective interaction between nucleons. In calculating the energy levels of O^{18} we are only concerned with the singlet even and triplet odd parts of the potential. Since the triplet odd potential is entirely repulsive, it is not susceptible to treatment by the separation method. The reaction matrix associated with V_s is given by

$$G_s = V_s - V_s(1/e_0)G_s\,. \tag{4}$$

Substituting Eq. (4) into Eq. (1) we find that

$$G \approx G_s + V_l \tag{5}$$

to first order in G_s and V_l. The higher-order terms, such as the Pauli and dispersion corrections, are expected to be much smaller than the contribution from V_l and will not be computed in this work.

The energy denominator e_0 is chosen as

$$\begin{aligned} e_0 &= H_0(r) - E_{nl} + H_0(R) - E_{N\mathcal{L}} \\ &= \frac{p^2}{2M} + \tfrac{1}{2}M\omega^2 r^2 - E_{nl} + \frac{P^2}{2M} + \tfrac{1}{2}M\omega^2 R^2 - E_{N\mathcal{L}}\,, \end{aligned} \tag{6}$$

where $H_0(r)$ and $H_0(R)$ are the relative and center-of-mass oscillator Hamiltonian whose eigenfunctions are, correspondingly, $\phi_{nl}(\vec{r})$ and $\phi_{N\mathcal{L}}(\vec{R})$ with eigenvalues E_{nl} and $E_{N\mathcal{L}}$, and M is the nucleon mass. It is important to note that Moshinsky's coordinates[8] are used throughout the discussion.

TABLE I. Separation distances.

Model	n	l	d_n (F)
HJ	0	0	1.015
	1	0	1.061
	2	0	1.134
	Average		1.070
UG I	0	0	1.047
	1	0	1.102
	2	0	1.182
	Average		1.110
UG III	0	0	1.047
	1	0	1.103
	2	0	1.180
	Average		1.110
GS II	0	0	1.018
	1	0	1.078
	2	0	1.160
	Average		1.085
BS III	0	0	1.056
	1	0	1.115
	2	0	1.200
	Average		1.124

They are defined by

$$\vec{r}=(\vec{r}_1-\vec{r}_2)/\sqrt{2}\,,\quad \vec{R}=(\vec{r}_1+\vec{r}_2)/\sqrt{2}\,, \tag{7a}$$

$$\vec{P}=(\vec{p}_1-\vec{p}_2)/\sqrt{2}\,,\quad \vec{P}=(\vec{p}_1+\vec{p}_2)/\sqrt{2}\,. \tag{7b}$$

We have

$$E_{nl}=(2n+l+\tfrac{3}{2})\hbar\omega\,,\qquad n=0,1,2,\ldots, \tag{8a}$$

$$E_{N\mathcal{L}}=(2N+\mathcal{L}+\tfrac{3}{2})\hbar\omega\,,\qquad N=0,1,2,\ldots, \tag{8b}$$

$$H_0(r)\phi_{nl}=E_{nl}\phi_{nl}\,. \tag{8c}$$

The correlated wave function may be expressed as $\phi_{N\mathcal{L}}(\vec{R})\psi_s(\vec{r})$, where $\psi_s(\vec{r})$, which is the correlated wave function for the relative motion, satisfies an equation similar to Eq. (3)[9]:

$$\psi_s(\vec{r})=\phi_{nl}(\vec{r})-\frac{1}{H_0(r)-E_{nl}}V_s\psi_s(\vec{r})\,. \tag{9}$$

Equation (9) may be rewritten as

$$(H_0+V_s)\psi_s=E_{nl}\psi_s\,. \tag{10}$$

TABLE II. Talmi integrals of the static singlet even potential. Units are MeV.

p	HJ	UG I	UG III	GS II	BS III
0	−5.464	−4.587	−4.266	−3.610	−3.778
1	−1.783	−1.629	−1.514	−1.388	−1.333
2	−0.585	−0.554	−0.525	−0.514	−0.448
3	−0.242	−0.229	−0.225	−0.226	−0.187
4	−0.124	−0.116	−0.118	−0.118	−0.097

The separation distance d is chosen so that the diagonal matrix element of G_s between oscillator states vanishes:

$$(\phi_{nl}|G_s|\phi_{nl})=0\,,\quad \text{at } r=d\,. \tag{11}$$

By manipulating Eqs. (8c) and (10), and using Green's theorem, it may be shown that the condition (11) is satisfied if the logarithmic derivative of the radial part of the correlated wave function matches the logarithmic derivative of the radial part of the oscillator wave function at the separation point:

$$\frac{R_s'(r)}{R_s(r)}=\frac{R_{nl}'(r)}{R_{nl}(r)}\,,\quad \text{at } r=d\,. \tag{12}$$

The singlet even components of the one-boson-exchange potentials under consideration have p^2 velocity-dependent forces and no hard cores. Accordingly, we consider

$$V_s(r)=V_c(r)+\tfrac{1}{4}a^2[p^2J(r)+J(r)p^2]\,, \tag{13a}$$

where

$$a=M^{-1}\,, \tag{13b}$$

and

$$p^2=-\hbar^2\nabla^2\,. \tag{13c}$$

On account of the relationship

$$\frac{1}{2\hbar^2}(p^2J+Jp^2)=-J\nabla^2-\langle\nabla J\rangle\cdot\nabla-\tfrac{1}{2}\langle\nabla^2J\rangle\,, \tag{14}$$

Eq. (10) becomes

$$(1+aJ)\nabla^2\psi_s+\left(\epsilon_{nl}-\frac{2M}{\hbar^2}V_c-\nu^2r^2+a\langle\nabla J\rangle\cdot\nabla+\tfrac{1}{2}a\langle\nabla^2J\rangle\right)\psi_s=0\,, \tag{15}$$

where

$$\nu=M\omega/\hbar=\lambda^{-2}\,, \tag{16a}$$

and

$$\epsilon_{nl}=2\nu(2n+l+\tfrac{3}{2})\,. \tag{16b}$$

The quantity λ is the size parameter of the oscillator and has the value 1.71 F for O^{18}. The first-order derivative in Eq. (15) may be eliminated by

TABLE III. Decomposition of $\langle 1d_{5/2}{}^2J=0|G|1d_{5/2}{}^2J=0\rangle$ for the singlet even potential. Units are MeV. The static singlet even and velocity-dependent singlet even contributions are denoted by CSE and VSE, respectively.

	HJ	UG I	UG III	GS II	BS III
CSE	−2.319	−1.837	−1.722	−1.398	−1.513
VSE		0.071	0.082	0.099	0.100
Total	−2.319	−1.767	−1.641	−1.299	−1.413

the transformation

$$\psi_s = \chi(1+aJ)^{-1/2}, \tag{17}$$

which leads to the differential equation

$$\nabla^2\chi + \left[\frac{\epsilon_{nl} - (2M/\hbar^2)V_c - \nu^2 r^2}{1+aJ} + \frac{1}{4}\left(\frac{a\langle\nabla J\rangle}{1+aJ}\right)^2\right]\chi = 0. \tag{18}$$

If the radial part of χ is expressed in the form $u(r)/r$, then u satisfies

$$\frac{d^2u}{dr^2} + \left[\epsilon_{nl} - \frac{l(l+1)}{r^2} - V(r)\right]u = 0, \tag{19}$$

where

$$V(r) = \frac{aJ\epsilon_{nl} + (2M/\hbar^2)V_c + \nu^2 r^2}{1+aJ} - \frac{1}{4}\left(\frac{a\langle\nabla J\rangle}{1+aJ}\right)^2. \tag{20}$$

Equation (19) is the standard form of Schrödinger's equation and may be integrated easily to give the function $u(r)$. The radial part of the correlated wave function is then determined from

$$R_s(r) = \frac{1}{(1+aJ)^{1/2}}\frac{u(r)}{r}. \tag{21}$$

The radial part of the oscillator wave function is given by

$$R_{nl}(r) = N_{nl}e^{-\nu r^2/2}(\nu r^2)^{l/2}\eta_{nl}(\nu r^2), \tag{22a}$$

where

$$\eta_{nl}(x) = \sum_{k=0}^{n}(-1)^k \frac{n!\,2^k}{(n-k)!\,k!}\frac{(2l+1)!!}{(2l+2k+1)!!}x^k, \tag{22b}$$

$$N_{nl}^2 = \frac{2^{l-n+2}}{\sqrt{\pi}}\frac{(2l+2n+1)!!}{[(2l+1)!!]^2 n!}\nu^{3/2}. \tag{22c}$$

The separation distance is therefore determined by integrating Eq. (19) outwards from the origin, where $u = 0$, until a distance d is reached at which the boundary condition (12) is satisfied. In addition, the correlated wave function is made to heal to the oscillator wave function for $r \geq d$ by letting $R_s(d) = R_{nl}(d)$. The shell-model G matrix elements of the singlet even potential may be calculated using

$$(\phi_{nl}|G|\phi_{nl}) \approx (\phi_{nl}|V_l|\phi_{nl}). \tag{23}$$

The right-hand side of Eq. (23) is evaluated by expanding it in a series of Talmi integrals, as discussed in Ref. 1. The Talmi integral of $V_l(r)$ is

TABLE IV. Matrix elements $\langle abJ|G|cdJ\rangle$ for O^{18}. Units are MeV. The column labeled HJ is derived from the Hamada-Johnston singlet even potential in combination with the UGI triplet odd potential.

a	b	c	d	J	HJ	UG I	UG III	GS II	BS III
$1d_{5/2}$	$1d_{5/2}$	$1d_{5/2}$	$1d_{5/2}$	0	−1.021	−0.469	−0.421	+0.271	+0.329
				2	−0.849	−0.692	−0.702	−0.522	−0.510
				4	−0.333	−0.256	−0.247	−0.157	−0.140
$1d_{5/2}$	$1d_{5/2}$	$1d_{5/2}$	$2s_{1/2}$	2	−0.500	−0.387	−0.346	−0.290	−0.343
$1d_{5/2}$	$1d_{5/2}$	$2s_{1/2}$	$2s_{1/2}$	0	−0.710	−0.577	−0.533	−0.432	−0.470
$1d_{5/2}$	$1d_{5/2}$	$1d_{5/2}$	$1d_{3/2}$	2	−0.547	−0.436	−0.352	−0.351	−0.354
				4	−0.954	−0.800	−0.740	−0.657	−0.685
$1d_{5/2}$	$1d_{5/2}$	$2s_{1/2}$	$1d_{3/2}$	2	−0.487	−0.395	−0.377	−0.289	−0.285
$1d_{5/2}$	$1d_{5/2}$	$1d_{3/2}$	$1d_{3/2}$	0	−3.511	−3.060	−2.861	−3.010	−3.310
				2	−0.663	−0.543	−0.483	−0.461	−0.494
$1d_{5/2}$	$2s_{1/2}$	$1d_{5/2}$	$2s_{1/2}$	2	−0.947	−0.747	−0.693	−0.473	−0.443
				3	−0.195	−0.195	−0.199	−0.090	−0.076
$1d_{5/2}$	$2s_{1/2}$	$1d_{5/2}$	$1d_{3/2}$	2	−0.085	−0.005	+0.010	+0.138	0.164
				3	−0.062	−0.062	−0.114	−0.253	−0.295
$1d_{5/2}$	$2s_{1/2}$	$2s_{1/2}$	$1d_{3/2}$	2	−1.477	−1.314	−1.222	−1.219	−1.267
$1d_{5/2}$	$2s_{1/2}$	$1d_{3/2}$	$1d_{3/2}$	2	−0.737	−0.650	−0.626	−0.610	−0.644
$2s_{1/2}$	$2s_{1/2}$	$2s_{1/2}$	$2s_{1/2}$	0	−2.231	−1.823	−1.699	−1.431	−1.472
$2s_{1/2}$	$2s_{1/2}$	$1d_{3/2}$	$1d_{3/2}$	0	−0.580	−0.471	−0.436	−0.353	−0.384
$1d_{5/2}$	$1d_{3/2}$	$1d_{5/2}$	$1d_{3/2}$	2	−0.124	−0.045	−0.087	+0.084	+0.101
				3	−0.433	−0.433	−0.362	−0.260	−0.256
				4	−1.764	−1.456	−1.357	−1.143	−1.167
$1d_{5/2}$	$1d_{3/2}$	$2s_{1/2}$	$1d_{3/2}$	2	−0.674	−0.609	−0.578	−0.625	−0.699
$1d_{5/2}$	$1d_{3/2}$	$1d_{3/2}$	$1d_{3/2}$	2	−0.782	−0.697	−0.680	−0.657	−0.683
$2s_{1/2}$	$1d_{3/2}$	$2s_{1/2}$	$1d_{3/2}$	2	−0.344	−0.211	−0.194	+0.025	+0.074
$2s_{1/2}$	$1d_{3/2}$	$1d_{3/2}$	$1d_{3/2}$	2	0.062	0.132	0.155	0.254	0.251
$1d_{3/2}$	$1d_{3/2}$	$1d_{3/2}$	$1d_{3/2}$	0	0.412	0.781	0.747	1.500	1.680
				2	−0.150	−0.058	−0.058	0.090	0.142

defined by

$$I_p[V_l] = \frac{2}{(p+\frac{1}{2})!}\int_{t_0}^{\infty} e^{-t^2} t^{2p+2} V_l(\lambda t)dt\,, \tag{24a}$$

where

$$t_0 = d/\lambda\,. \tag{24b}$$

This is exactly the definition given in Ref. 1, except that the lower limit of integration is no longer zero, but depends on the separation distance. A factor $\sqrt{2}$ has been dropped because distances are in $\sqrt{2}$ F.

For the triplet odd potential we make the approximation

$$(\phi_{nl}|G|\phi_{nl}) \approx (\phi_{nl}|V_{TO}|\phi_{nl})\,, \tag{25}$$

where V_{TO} is the total triplet odd potential. We consider the approximation (25) to be adequate for the following reasons: (i) The one-boson-exchange models under consideration are nonsingular, and therefore the matrix elements of V_{TO} do not diverge; (ii) the triplet odd matrix elements are independent of the $p=0$ Talmi integral. Since the short-range repulsion of the potential manifests itself mostly in the $p=0$ Talmi integral, then the matrix elements of V_{TO} are not expected to exhibit excessive repulsion. It turns out that the computed values from the one-boson-exchange models, based on the approximation (25), are close to the values computed from the hard-core Hamada-Johnston (HJ) potential using the reference-spectrum method.[1,7]

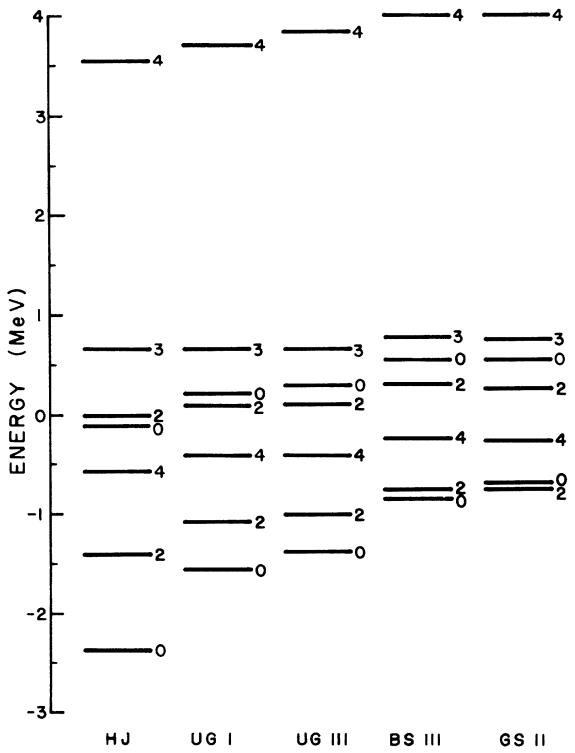

FIG. 1. Energy levels for O^{18}. The column labeled HJ is calculated using the Hamada-Johnston singlet even potential in combination with the UG I triplet odd potential.

III. RESULTS AND DISCUSSION

We have computed the separation distances for the s states for the UG I, UG III, GS II, BS III, and HJ potentials; the results are presented in Table I. The separation distances are, of course, state-dependent. We denote by d_n the separation distance for the s state characterized by radial quantum number n. It is evident from Table I that d_n varies rather slowly with n; therefore it is not unreasonable to define an average separation distance d for each model,

$$d = \tfrac{1}{3}(d_0 + d_1 + d_2)\,. \tag{26}$$

We see that corresponding separation distances for the various models are very close to one another, differing by at most 5% in every case. This is remarkable agreement considering how different are the meson-theoretic, velocity-dependent one-boson-exchange potentials from the phenomenological hard-core potential.

In Table II we present the Talmi integrals of the static singlet even potential. We see that the various models have similar Talmi integrals; in particular, the $p=0$ Talmi integrals are in good agreement with one another. This represents a considerable improvement over the results obtained with V matrices.[1] The UG I model gives the closest results to the phenomenological potential.

TABLE V. Contributions, in MeV, to the $p=0$ Talmi integral of the static singlet even potential.

Model	Meson	Contribution	Model	Meson	Contribution
UG I	π	−1.219	UG III	π	−1.203
	η	−0.153		ρ	−1.092
	ρ	−1.179		ω	1.978
	ω	1.864		σ_1	−0.196
	π_V	−0.310		σ_0	−1.280
	η_V	−0.276		σ_C	−2.472
	σ_C	−3.314			
GS II	π	−1.034	BS III	π	−1.077
	ρ	−0.360		η	−0.123
	ω	2.346		ρ	0.055
	σ	−1.246		ω	2.101
	σ_1	−0.064		σ_1	−0.584
	σ_C	−3.253		σ_0	−4.152

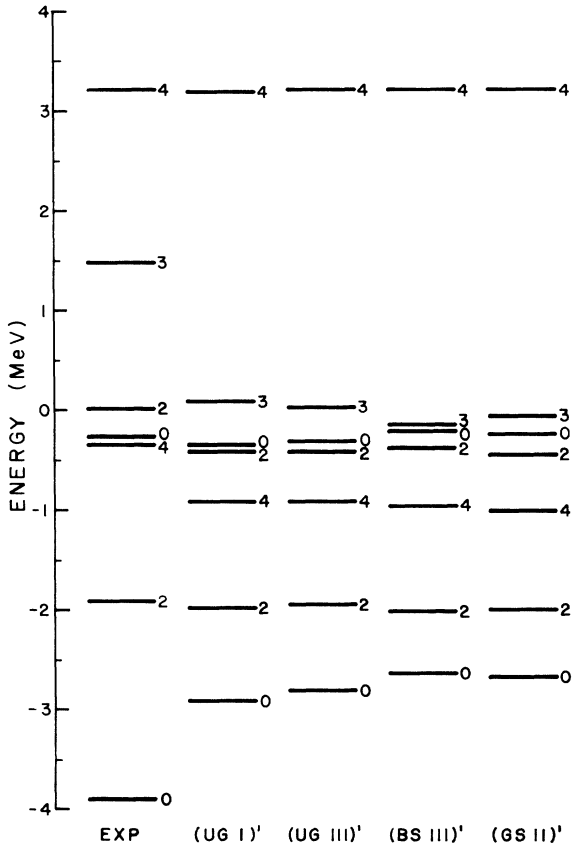

FIG. 2. Energy levels for O^{18}. The notation (UGI)', for example, means that the levels are derived from the UGI model which has been adjusted by varying the appropriate coupling constant.

In Table III we decompose the matrix element

$$\langle 1d_{5/2}{}^2 J=0 \,|\, G \,|\, 1d_{5/2}{}^2 J=0 \rangle$$

for the singlet even potential. The results for the various models are reasonably close to one another. We note that the contributions from the p^2 velocity-dependent components are almost negligible. This is consistent with the well-known effect of the Brueckner G matrix – that it suppresses the repulsive core of the potential.

In Table IV we list the G matrix elements for O^{18} computed from the various potential models. Corresponding matrix elements from the various models are similar for the most part. The resulting energy levels are shown in Fig. 1. The UG I model gives the closest agreement with the phenomenological model. The models differ most from one another with respect to the $J=0$ ground state, which is a consequence of the fact that this level is particularly sensitive to the $p=0$ Talmi integral of the static singlet even potential.

Finally, we attempt to achieve agreement with experiment by varying the appropriate potential parameters. Of course, as is well known, core-polarization effects play an important role in establishing agreement with experiment. However, the calculation of core-polarization effects would greatly lengthen our calculations. Since the $J=0$ ground state is controlled mainly by the $p=0$ Talmi integral of the static singlet even potential, we compute the contributions to this quantity from the individual mesons. The contributions are listed in Table V. We see that the light scalar meson gives the dominant contribution in each model. Accordingly, we vary the coupling constant of the light scalar meson in order to obtain the best fit to the experimental levels. The final values of this coupling constant for the UG I, UG III, GS II, and BS III models are found to be, respectively, 3.06, 2.72, 4.25, and 12.79, which represent increases on the order of 60–80%. The corresponding energy levels are shown in Fig. 2. They agree reasonably well with experiment.

IV. CONCLUSION

The present calculations confirm the conclusion of Ref. 1 that it is necessary to use reaction-matrix theory when performing shell-model calculations with one-boson-exchange potentials. We found that the shell-model reaction matrix elements exhibited insufficient attraction to give agreement with experiment. It has been found[7] that the inclusion of core-polarization increases the attraction of the matrix elements and depresses the energy levels. Since the role of the light scalar meson in the one-boson-exchange model of the nucleon-nucleon interaction is to provide a long-range attraction, it is perhaps not surprising that we have found it necessary to strengthen the contribution from this meson in order to fit the experimental energy levels more closely. The adjustments which we have made to the potential may be regarded as simulating core-polarization effects. It is extremely unlikely that these adjustments are consistent with the nucleon-nucleon scattering data.

[1] P. S. Ganas, Phys. Rev. C 2, 93 (1970).
[2] S. A. Moszkowski and B. L. Scott, Ann. Phys. (N.Y.) 11, 65 (1960).
[3] T. Ueda and A. E. S. Green, Phys. Rev. 174, 1304 (1968).
[4] A. E. S. Green and T. Sawada, J. Phys. Soc. Japan Suppl. 24, 616 (1968).
[5] R. A. Bryan and B. L. Scott, Phys. Rev. 177, 1435 (1969).

[6]A. Kallio, Phys. Letters 18, 51 (1965).
[7]T. T. S. Kuo and G. E. Brown, Nucl. Phys. 85, 40 (1966).
[8]T. A. Brody and M. Moshinsky, *Tables of Transformation Brackets for Nuclear Shell-Model Calculations* (Dirección General de Publicaçiones, Universidad Naçional de México, Çiudad Universitaria, Mexico, D. F., 1960).
[9]Equation (9) may be regarded as a highly symbolic representation of the Green's function.

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Brueckner-Hartree-Fock Calculations Using Density-Dependent Effective Interactions. Application to O^{16}†

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The effects of allowing variations in the radial orbital wave functions, or major shell mixing, have been investigated within the framework of Brueckner-Hartree-Fock (BHF) theory for O^{16}. The calculation is carried out in the matrix representation using harmonic-oscillator wave functions as a basis. Effective G matrix elements are calculated through the use of a density-dependent, two-body operator following a prescription of Bethe. Two possible parametrizations, the local-density approximation and an average-density approximation, are compared. Consideration of the rearrangement energy shows that it is an important contribution to the binding energy. The problem is formulated in a way which facilitates the application of BHF theory to deformed as well as spherical nuclei.

1. INTRODUCTION

Recent results have shown the feasibility of applying Brueckner-Hartree-Fock (BHF) theory to finite nuclei.[1-22] This theory has emerged from the efforts to incorporate Brueckner's treatment of the nuclear two-body interaction into the framework of a Hartree-Fock-like variational calculation. Using a suitable procedure for calculating effective G matrix elements, the result is a problem somewhat more tractable than the conventional approach of solving the Bethe-Goldstone equation.

The Hartree-Fock (HF) part of the calculation allows variations in the radial orbital wave functions, a situation which has become known as major shell mixing.[23-27] We use the matrix representation with harmonic-oscillator wave functions as the basis states. The states from the higher oscillator shells which may mix into the ground-state wave functions are determined from symmetry considerations.

In the calculation of the effective G matrix elements for a realistic interaction, we follow a prescription of Bethe[20] which utilizes a density-dependent two-body operator. The density enters through a parametrization of the tensor force and also as a factor in the short-range part of G, the so-called core correction. Two possible alternatives are considered: the local-density approximation (LDA) in which the effective interaction is a function of the density at the center of mass of the two interacting particles, and an average-density approximation (ADA) in which the density parameter is the average single-particle density.

The principal objective of this work is to establish a viable calculational procedure which can be used for the application of the BHF theory to deformed nuclei. It is for this reason that two possible parametrizations of G are considered. We compare in this paper the uses of the LDA and the ADA in calculations of O^{16}. A treatment of deformed nuclei will be given in a subsequent paper,[28] designated by II. Our LDA calculation of O^{16} resembles work done by Negele[19] who, using the LDA, did calculations in configuration space for several closed-shell nuclei and obtained excellent fits to the electron scattering data with his density distributions.

In Sec. 2 we outline the theory which underlies this work. We include a consideration of the so-called rearrangement energy and discuss the relevance of Brandow's work. In Sec. 3 we describe the method of calculating effective G matrix elements. The application to $4N$, even-even nuclei, in particular O^{16}, is presented in Sec. 4. Section 5 contains the results of the calculations, while the conclusions are summarized in Sec. 6.