[42]J. M. Laget, Commissariat á l'Energie Atomique, Saclay Report No. CEA-R-3572 (unpublished); J. M. Laget and J. Gastebois, Nucl. Phys. A122, 431 (1968); J. M. Laget, J. Vervier, G. Bruge, J.-M. Loiseaux, and L. Valentin, *ibid.* A125, 481 (1969).
[43]F. A. Rose, Nucl. Phys. A124, 305 (1969).

PHYSICAL REVIEW C VOLUME 5, NUMBER 2 FEBRUARY 1972

Phenomenological Analysis of the $^{12}C + ^{12}C$ Reaction

Georges J. Michaud*
Département de Physique, Université de Montréal, Montréal, Québec, Canada
and
Erich W. Vogt†
Physics Department, University of British Columbia, Vancouver, British Columbia, Canada
(Received 24 June 1971)

The total $^{12}C + ^{12}C$ low-energy reaction cross section has been reanalyzed. Three phenomena suggest an α-particle model for the reaction: (1) the giant resonances, (2) the intermediate-structure resonances, and (3) anomalous branching ratios. The energy dependence of the total cross section over the energy range 2.4 to 9.0 MeV cannot be explained solely by the energy dependence of the penetration of the Coulomb barrier. There is some nuclear structure. The structure can be explained by an optical potential based on the α model leading to absorption under the barrier. It could also be explained with two giant resonances in a Woods-Saxon potential. The intermediate structure is explained as due to special states of ^{24}Mg built of a ^{12}C core and three α particles, and so are the anomalous branching ratios.

Astrophysical reaction rates were calculated for a number of possible optical potentials. The giant resonances affect the extrapolation to the energies of astrophysical interest and lead to an uncertainty of a factor of 3 to 10 in the astrophysical reaction rate at $T_9 \simeq 0.6$, where $T_9 \equiv 10^{-9} T°K$.

I. INTRODUCTION

The low-energy reactions of the $^{12}C + ^{12}C$ system have long been a fertile field for new ideas on nuclear structure. This fact and the importance of these reactions for astrophysics have sparked a continued effort to understand these reactions experimentally and theoretically. Most recently some measurements of Patterson, Winkler, and Zaidins[1] of the total absorption cross sections at energies far below the Coulomb barrier and of some branching ratios at similar energies measured by Stephens and Mazarakis[2] have added to the data, but raised new problems in the interpretation of the reaction mechanisms. In this paper, we discuss some of the questions raised by the measurements which rule out a number of conventional interpretations, and we show, phenomenologically, how the recent measurements combined with old ones lead to a picture in which the reaction is described in terms of intermediate structure in an α-particle model. In a subsequent paper we shall derive the model for the reactions in terms of known α-α interactions.

The energy region of interest here is shown in Fig. 1. Throughout this paper, all energies are in the center of mass system, unless otherwise specified. It is at energies of the $^{12}C + ^{12}C$ system from 2.5 (well below the top of the Coulomb barrier) to 9 MeV (slightly above the top of the Coulomb barrier). This corresponds to excitation energies of the compound nucleus ^{24}Mg in the neighborhood of 20 MeV.

For a well-behaved heavy-ion interaction the dominant feature of the total reaction cross section would be a plummeting cross-section magnitude as the energy decreases and the Coulomb-barrier penetration takes effect; the dominant behavior of branching ratios would be given by the statistical averages of an evaporation model. But the $^{12}C + ^{12}C$ reaction is richer and more puzzling. The total cross section plummets, as it must; but it also exhibits an unusual resonance structure in most of its reaction channels. A structure first noted 10 years ago[3] as a few isolated resonances at moderate energies was analyzed as single-particle states of the heavy-ion system in a series of papers,[4-8] but, was recently[1,2] found to extend to many more resonances at very low energies. The resonances are easily shown, as demonstrated below, to be neither single compound-nucleus states nor statistical fluctuations: On the other hand, the

sheer number of states manifested in the recent data rules out the earlier interpretations[4-8] in terms of single-particle states of the heavy-ion system. One is led inexorably to some interpretation in terms of intermediate structure. Quite apart from the resonances, the total reaction cross section when averaged over such structure exhibits an energy dependence somewhat different from the one expected from barrier penetration alone. The cross section is anomalously large both at the lower energies ($E \simeq 2.5$ MeV) and higher energies ($\simeq$9 MeV) in the energy interval of interest to us – or alternatively it is too small in between. It is a natural expectation that the average cross section should tell us about the single-particle potential of the $^{12}C + ^{12}C$ system.

The branching ratios of the $^{12}C + ^{12}C$ system also misbehave. Although earlier calculations[9-12] showed that the $^{12}C + ^{12}C$ reactions above 9 MeV constituted one of the first compound-nucleus systems known, with all partial cross sections corresponding very well to the predictions of the statistical theory, at lower energies some strong deviations occur.

The questions raised by the data concern the simple single-particle picture and the more complex intermediate structure. What simple wave properties of the $^{12}C + ^{12}C$ system lead to the behavior of the total reaction cross section? How are these wave properties to be interpreted by a reasonable optical-model potential for the $^{12}C + ^{12}C$ system? How is the optical model related to the underlying intermediate structure? How does the intermediate structure arise from basic α-α or nucleus-nucleus interactions? Can the model for the intermediate structure describe both the intermediate resonances and the anomalous branching ratios? Our paper addresses these questions systematically but phenomenologically. In particular we want to understand the average total reaction cross section in terms of the $^{12}C + ^{12}C$ single-particle picture and the systematics of the average branching ratio in terms of an intermediate-structure model based on α-α interactions. That leaves for a later paper some secondary but equally interesting questions concerning the connections of the single-particle potential to the underlying intermediate structure, or concerning the spectroscopy – the widths and spacings – of the intermediate resonances.

The $^{23}Mg + n$ channels will not be analyzed. At low energy, they contain little information compared to the $^{20}Ne + \alpha$ and $^{23}Na + p$ channels.

2. OPTICAL POTENTIAL FOR THE $^{12}C + ^{12}C$ SYSTEM

The total absorption cross section of the $^{12}C + ^{12}C$ system is shown[1-2] in Fig. 2 for center-of-mass

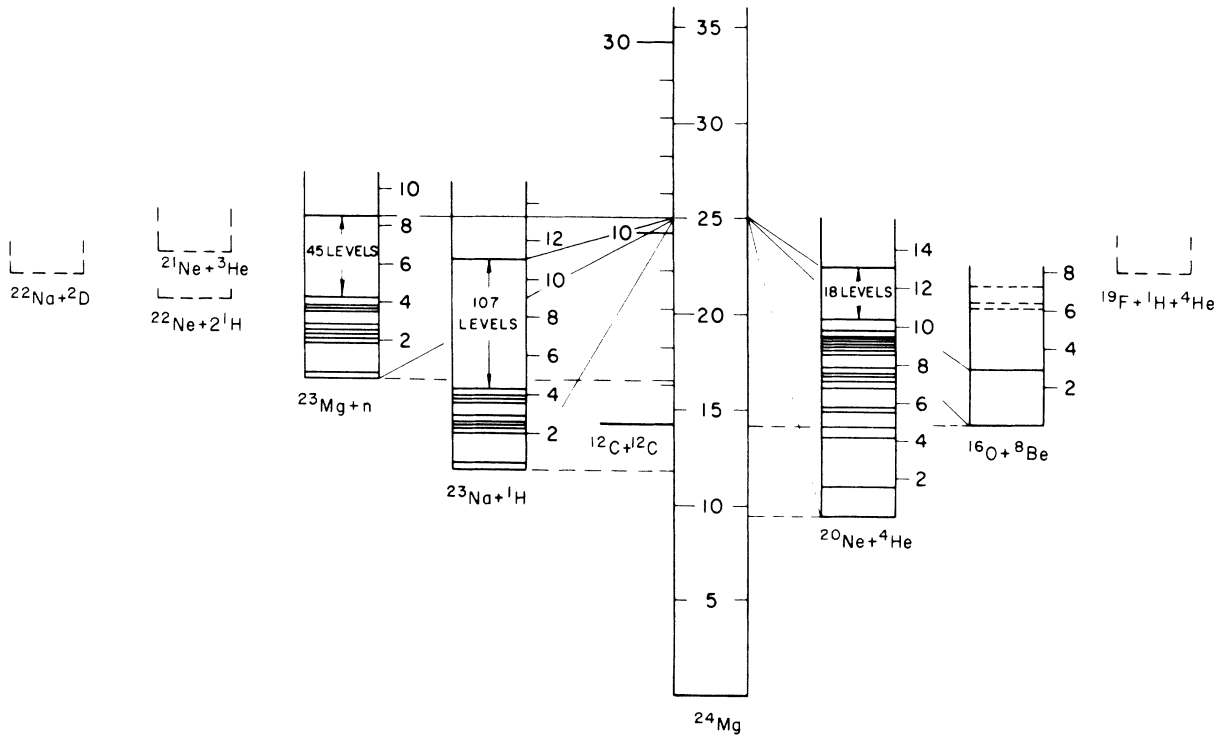

FIG. 1. The energy levels available for the decay of compound states of ^{24}Mg at an excitation energy of ~20 MeV.

energies between 2.4 and 9.0 MeV. To discuss the average potential well to be used for the absorption cross section we must clearly distinguish between single-particle, intermediate, and compound resonances, and then perform appropriate averages of the data in order to apply the single-particle picture.

The resonances attributed in Fig. 2 to intermediate structure are not statistical fluctuations. This possibility can be dismissed at once. The resonances appear in most channels at the same compound-nucleus excitation energy; whereas, statistical fluctuations are uncorrelated from channel to channel,[9] since they are related to an incoherent sum of Breit-Wigner resonance amplitudes of the form

$$\left|\sum_{\lambda}\frac{\Gamma_{\alpha\lambda}\Gamma_{\alpha'\lambda}}{(E-E_{\lambda})+i\Gamma_{\lambda}}\right|,$$

where E_λ is the resonance energy, $\Gamma_{\alpha\lambda}$ is a resonance amplitude, and α and α' refer to two different channels. There is no correlation in the signs of the $\Gamma_{\alpha\lambda}$ from channel to channel.

The distinction between the three kinds of resonances in Fig. 2 is empirical but straightforward because of the scale of the corresponding level densities. On the one hand, the single-particle resonances of the $^{12}C+^{12}C$ system are separated

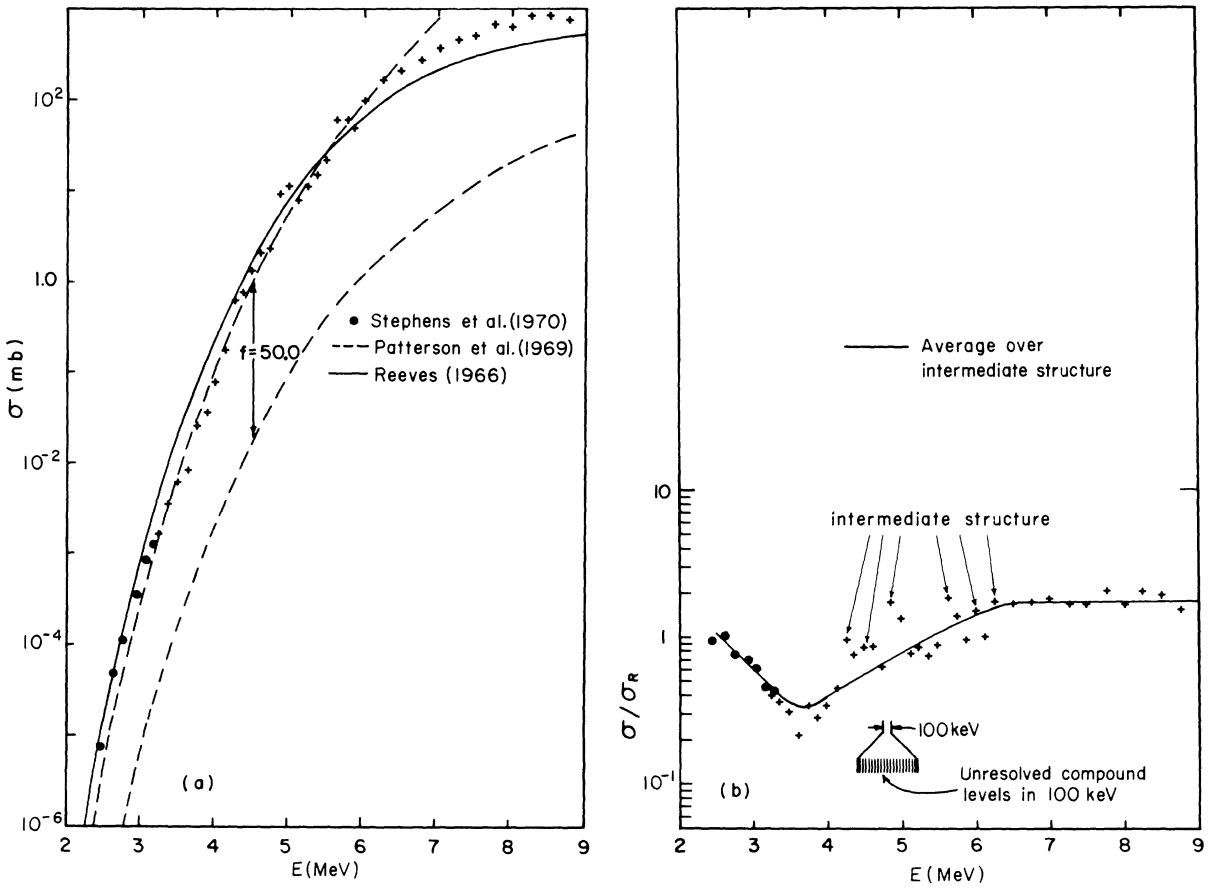

FIG. 2. In part (a) of the figure is seen the energy variation of the measured cross section and of a number of fits obtained. There are two curves attributed to PWZ (Ref. 1). The upper one corresponds to his "best fit" formula. The lower one has, at low energy, the same energy variation as the "best fit" formula. It comes from a square well with $V_0=50$ MeV, $W_0=10$ MeV, and $R_S=5.0$ fm. A factor $f=50$ is needed to reproduce the measurements. Such a large reflection factor is unphysical; see the text. There is no physically reasonable optical potential with a large W_0 that can reproduce the data between 3.0 and 6.0 MeV. Throughout this paper the +'s refer to the experimental results of PWZ (Ref. 1), while the circles refer to the results of Stephens and Mazarakis (Ref. 2). All energies are in the center-of-mass system. In part (b) all cross sections are divided by the cross section obtained with the Reeves (Ref. 21) potential ($V_0=50$ MeV, $W_0=10.0$ MeV, $R_0=5.7$ fm, $a=0.4$ fm). Because of the large imaginary potential he uses, there are no resonance effects in his calculated cross sections. His potential is equivalent to a black nucleus and a reflection factor. Two giant resonances then appear in the data: at $E\lesssim 2.5$ MeV and at $E=6.5$ MeV. Intermediate structure is easily identified and separated from the compound-nucleus resonances which are about 5.0 keV apart.

by at least 1 MeV, a scale determined by the reduced mass and radius of the interacting heavy ions, and to a lesser extent by the depth of their interaction potential. On the other hand, the compound-nucleus resonances which are not clearly manifest in the data of Fig. 2 because of inadequate resolution have a level separation of only a few keV,[9] a scale determined by extrapolation from the level spacing measured in ^{24}Mg in the vicinity of 12-MeV excitation energy. The level spacing of the intermediate structure, based on almost any model for the intermediate states, would fall in between these two extremes and have a value of a few hundred keV.

The recent measurements of Patterson, Winkler, and Zaidins[1] and Stephens and Mazarakis[2] at the low-energy end of Fig. 2 show that the most prominent resonances of the $^{12}C + ^{12}C$ system are clearly intermediate structure. The spacing of single-particle resonances is largely determined by the radius of the optical potential: For any reasonable radius the spacing between such levels is more than 1 MeV. The earlier Chalk River measurements[3] for three prominent resonances at 5.7, 6.0, and 6.3 MeV could have been due to three single-particle states accidentally close together, as Davis[7] suggested, but the continuation of such structure, with a spacing of less than half an MeV down to 4.0 MeV, clearly requires that the corresponding resonances be due to intermediate states. Consequently, it is appropriate to average over the individual intermediate states to obtain an average cross section for $^{12}C + ^{12}C$ which may be compared with optical-model calculations for the system.

The choice of the optical-model potential for the $^{12}C + ^{12}C$ system – including the shape of the real and imaginary terms of the potential and of its nonlocality – are subject to some physical constraints. The constraints are not as simple as for the optical model of the nucleon-nucleus system. In this latter case, the real potential has the same empirical and theoretical basis as the nuclear shell model, a subject which has dominated nuclear physics for the past 20 years. The empirical basis of the shell model is now comfortable; the theoretical foundation in the many-body treatment of nuclei interacting through known two-body forces is reasonably secure. In contrast, the interest in heavy-ion potentials is relatively recent, the empirical evidence for the potential parameters is meager, and the theoretical basis of the potentials is unsound. In this situation many kinds of optical-model potentials have been employed, not all of which are equally reasonable.

The central questions, concerning which some useful guidelines might be discussed, are the following:

(1) Is the real part of the potential several times as deep as the nucleon-nucleus optical potential ($\simeq$50 MeV) or is it much shallower?

(2) Does the real part of the $^{12}C + ^{12}C$ optical potential have a soft core?

(3) Is the real part of the potential strongly nonlocal?

(4) How strong is the absorptive term in the optical potential?

The first three questions concerning the depth, shape, and nonlocality of the real part of the interaction are dominated primarily by the strength of the two-body interactions and by the effects of the exclusion principle. In zero order, one might take the $^{12}C + ^{12}C$ collision to be viewed as 12 independent nucleons bombarding a ^{12}C nucleus and thus nelgect all effects of exchange among the 12 bombarding nucleons. In such a view the central depth of the potential would be perhaps 8 times (most of the 12 nucleons are in the surface of the target when the two nuclei fully overlap) the 50-

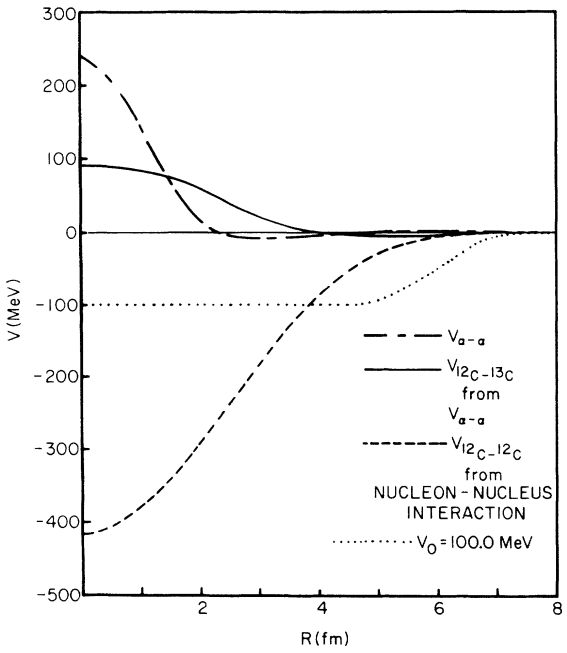

FIG. 3. Radial dependence of potentials of interest. The α-α potential was used to calculate a $^{12}C + ^{12}C$ potential (full line). A nucleon-nucleus potential with $V_0 = 50$ MeV, $R_0 = 1.25 \times 12^{1/3}$ fm, was used to calculate a $^{12}C + ^{12}C$ potential (dashed line). Another frequently used potential, with $V_0 = 100$ MeV, $\alpha = 0.5$ fm, and $R_0 = 6.0$ fm, is also shown. Because of the exclusion principle and exchange effects, we expect the $^{12}C + ^{12}C$ potential calculated with the α-α potential to be closest to reality (see the text). The α-α potential used fits the cluster model of $^{9}_{\Lambda}Be$, yields the S-wave resonance correctly, and also reproduces the experimental α-α phase shifts (Ref. 16).

MeV value of the nucleon-nucleus potential. This potential would not have a central plateau, but would drop rapidly from this maximum value for zero separation as shown in Fig. 3.

The question is, how much does exchange reduce the depth of this potential and how much does it make the potential nonlocal? A common empirical approach[13] has been to make the real interaction about 100 MeV deep and have a radius twice that of a single ^{12}C nucleus. Such a potential is shown in Fig. 3 as a "deep" potential. For it, the real depth $[\equiv 3\int_0^\infty V(r)r^2dr/R_0^3]$ is 20% larger than for the very deep potential which entirely neglects exchange. In contrast to this, Brueckner, Buckler, and Kelly[14] and Pruess and Greiner[15] have suggested that exchange effects not only make the interaction strongly nonlocal but effectively make it repulsive at small separations. The best local potential with at least a partial basis in nuclear theory might then be the soft-core potential also shown in Fig. 3.

Although we prefer the "soft-core" potentials to the "deep" potentials on physical grounds we apply both below to the $^{12}C+^{12}C$ system. One can also understand the effects of the exclusion principle (leading to the soft-core potential of Fig. 3) more easily from second principles than from first principles. Instead of considering the full effects of exchange on all 12 nucleons (first principles), we take the 12 nucleons 4 at a time as α particles (second principles). This view also underlies a discussion below of the whole reaction mechanism. The α-α interaction is understood from first principles.[16] It can be represented reasonably accurately by the α-α interaction of Fig. 3, in which the exclusion principle has introduced a soft core. We can adopt this α-α interaction or a similar one and proceed with it to the α-^{12}C interaction in which the target ^{12}C is viewed as consisting of the three α particles. The additional effects of the exclusion principle here are modest: Bhargava and Vogt[17] showed that a proper treatment of exchange leads to very nonlocal potentials obtained simply by averaging the three α-α interactions over the internal motion of the three α particles in the ^{12}C target. Because of the strong binding and closed-shell nature of the α particle, and the consequent soft core in the α-α interaction which keeps the α particles apart, it is not surprising that further effects of exchange are modest. It suggests that the $^{12}C+^{12}C$ interaction can be found from the α-α interaction simply by averaging over the internal motions of the α particles in each ^{12}C nucleus. Such an interaction is the soft-core potential of Fig. 3. The actual interaction will certainly be nonlocal, but not necessarily very different for elastic scattering and absorption. It will turn out below that a $^{12}C+^{12}C$ interaction derived in this way from the α-α interaction fits the $^{12}C+^{12}C$ absorption cross section surprisingly well without any adjustment in parameters.

The imaginary term of the $^{12}C+^{12}C$ interaction is less subject to physical guide lines than the real term. Conventional wisdom says that the ^{12}C system is easily broken up, so that one has a strong imaginary term. Correspondingly, any rough calculation of the effect of residual interactions in determining the strength of absorption leads to an order of magnitude of the imaginary term of at least 1 MeV when the two nuclei overlap appreciably. Exchange plays such a crucial role in any such calculation that no believable estimate can be made. It appears reasonable that the effects of exchange fall off with bombarding energy, so that the imaginary term may increase strongly with energy. Apart from this we shall take considerable freedom with the shape and strength of the imaginary term in fitting the absorption data.

Extrapolation to lower energies for astrophysical purposes will be seen, in a later section, to be sensitive on the model chosen. Uncertainties in the nuclear-reaction model to be used lead to uncertainties by factors of 5 or 10 in the astrophysical reaction rates.

To reproduce the measured cross sections and to extrapolate to energies lower than the energies at which measurements were made, use is generally made of a potential of the Woods-Saxon family:

$$V(r) = -\frac{V_0 + iW_0}{1 + e^{(r-R_0)/a}}, \tag{1}$$

where R_0 is the radius of the potential, and a the surface thickness. V_0, W_0, R_0, and a are constants to be determined for each reaction. For heavy-ion reactions, such as the $^{12}C+^{12}C$ reaction, the fashion until recently was to use

$$\begin{aligned} &V_0 \simeq 50\text{–}100 \text{ MeV}, \\ &W_0 \simeq 10 \text{ MeV}, \\ &R_0 \simeq 2.0 \times 1.2A^{1/3} \text{ fm}, \\ &a \simeq 0.4 \text{ to } 0.5 \text{ fm}. \end{aligned} \tag{2}$$

We will find it necessary to use a very different set of parameters. To vary the parameters in a way related to the physical phenomena, we will carry out a wave analysis of the cross section to see how the different parameters affect the cross section. Then we discuss different wells, including some with soft cores. Finally the data will tell us what potentials are appropriate.

The principal wave properties exhibited in the

cross sections of an optical-model well are resonance, absorption, barrier penetration, and reflection. Following Michaud, Scherk, and Vogt[18] we use the fact that the wave properties of an arbitrary Woods-Saxon potential can be discussed in terms of an equivalent square well (ESW), whose cross sections are known, analytical functions of the energy and orbital angular momentum. The radius R_S and depth $V_S + iW_S$ are related to the parameters V_0, W_0, R_0, and a of the Woods-Saxon potential in the following way, which is independent of energy, orbital angular momentum, or the presence of Coulomb barriers:

$$R_S = R_0 + \Delta R\,, \tag{3}$$

$$V_S R_S^2 = V_0 R_0^2\,, \tag{4}$$

$$W_S = W_0\,. \tag{5}$$

The value of ΔR as a function of $R_0^2 V_0$ and a/R_0 is given in Ref. 18.

Having established an ESW for the Woods-Saxon potential, we no longer need to rely on computer output for the evaluation of barrier transmission, giant resonance, or other wave properties of the potential. Instead we can use the familiar transmission functions, penetration factors, and strength functions of the square well, except that we must correct each square-well penetration function by a reflection factor, f, which accounts for the artificial wave reflection of the square edge. As for ΔR, the reflection factor f depends only on $R_0^2 V_0$ and on a/R_0 and is given in Ref. 18.

Hence for the absorption cross section of the channel c we can write

$$\sigma_c(\text{abs}) = \frac{2\pi}{k_c^2} \sum_l (2l+1) T_l(c)\,, \tag{6}$$

$$T_l(c) = \tau_l(c)/[1 + \tfrac{1}{4}\tau_l(c)]^2\,, \tag{7}$$

$$\tau_l(c) = 4\pi f_c P_l(c) s_l(c)\,, \tag{8}$$

$$P_l(c) = k_c R_c / [F_l^2(k_c R_c) + G_l^2(k_c R_c)]\,. \tag{9}$$

In Eqs. (6)–(9) $P_l(c)$ is the usual square-well penetration factor.[19] (F_l and G_l are the regular and irregular Coulomb wave functions, determined by the charge of the particles, their mass, the angular momentum, and the radius of the well). The penetration factor relates the intensity of the partial wave at R_c to the wave intensity at infinity. It is not affected by the depth of the nuclear potential, only by its radius. To the cross section it brings the effect of barrier penetration, and is thus responsible for the plummeting of the cross section at low energies.

The strength function $s_l(c)$ brings to the cross section the effect of the nuclear potential and its resonances. In a complex potential, close to a resonance, the strength function has the Breit-Wigner shape, with width $2W$. If W is as large as the resonance spacing for a given l in V_S, no resonance effect is seen. The well is then equivalent to a black nucleus. All the particles that get as far as R_c are absorbed. The whole cross section for the square well then depends only on the radius, through $P_l(c)$. If, however, W is smaller than the level spacing, giant-resonance structure appears in $s_l(c)$ and thus in the cross section.

The effect of the giant resonances on the cross section is enhanced both by degeneracy and by Bose statistics. Any purely attractive nuclear potential has degenerate even-parity ($l = 0, 2, 4$ etc.) levels alternating with degenerate odd-parity ($l = 1, 3, 5$ etc.) levels in the familiar sequence, which holds exactly for a harmonic oscillator and very nearly for a square well. The Woods-Saxon well is roughly intermediate between these two shapes. The introduction of a soft core can partially remove the degeneracy. Further, the fact that the two interacting ^{12}C nuclei are identical bosons removes all the odd-parity levels, while the transmission functions of the even-parity levels are multiplied by 2. Thus one should see only well-separated even states. The level spacing of these states is about 5–10 MeV, so that the giant-resonance effects become important if W_0 is less than 5 MeV.

There is one phenomenon which exists in a Woods-Saxon potential which does not exist in the ESW, and which has never been shown to exist or not to exist in nature: absorption under the barrier.[18] By an application of Green's theorem[20] it can easily be shown that

$$T_l = c \int_0^\infty W(r) \psi_l \psi_l^* dr\,, \tag{10}$$

where c is a constant, and ψ_l is the radial wave function (obtained by numerical integration of the wave equation with the optical potential). If $W(r)$ is not a square well but falls off at large radii like the Woods-Saxon potential, then at very low energies the product $\psi_l \psi_l^* W(r)$ has its maximum much further away than the nuclear radius. Most of the reactions then occur under the barrier. For the $^{12}C + ^{12}C$ reaction and $a = 0.5$ fm, absorption under the barrier is important below 3 MeV. It may be the explanation for the relative rise in the cross section below 3.2 MeV (see Fig. 2). We will come back to this phenomenon later. We will first exclude wells with large W by considering data between 3.0 and 6.0 MeV. If absorption under the barrier were important between 3.0 and 6.0 MeV it would make worse the disagreement that we will now describe using the ESW, since it would imply a smaller energy variation of the cross section

than obtained with the ESW. The ESW reproduces the cross section obtained with Woods-Saxon potentials *except* if absorption under the barrier is important.

Cross sections will be affected by the nuclear potentials in two ways: (a) The radius of the potential determines $P_l(c)$; (b) the details of the interior of the well determines $s_l(c)$, the strength function. There is a family of Woods-Saxon wells for which the details of the interior of the well are of no importance: They have W larger than the average level spacing. W is large enough to absorb all particles before they see the center. The whole cross section is determined by the radius. They are equivalent to black nuclei. Such wells have generally been used in the past to reproduce and extrapolate the average measured cross section for heavy ions such as $^{12}C+^{12}C$. For instance, Reeves[21] found a best fit (see Fig. 2) for the Chalk River data[3] for the $^{12}C+^{12}C$ absorption cross section with a Woods-Saxon well [Eq. (1)] with the parameters:

$$\begin{aligned} &V_0 = 50 \text{ MeV}, \\ &W_0 = 10 \text{ MeV}, \\ &R_0 = 5.77 \text{ fm}, \\ &a = 0.4 \text{ fm}. \end{aligned} \tag{11}$$

The ESW would have the parameters[18]:

$$\begin{aligned} &V_S = 41 \text{ MeV}, \\ &W_S = 10.0 \text{ MeV}, \\ &R_S = 6.7 \text{ fm}, \\ &f = 6.5. \end{aligned} \tag{12}$$

The lowest-energy data then available were at 5 MeV in the center-of-mass system. Figure 2 shows that the extrapolation based on these data exaggerated the size of the cross section at 3.5 MeV by about a factor of 4. The discrepancy at the higher energy comes from the neglect, in the earlier data, of the contribution of some excited states of ^{20}Ne.

Reeves's potential does not fit the new experimental data[1-2] very well. However, it has such a large imaginary term that the resonance effects are washed out. We will therefore use the cross sections of this potential as a basis for comparison for the cross sections of other potentials which exhibit more structure.

Realizing that the energy dependence had to be stronger, PWZ[1] searched for a square well that would have the proper energy dependence between 3.26 and 6.0 MeV. Such a procedure is frequently followed for astrophysical extrapolations. They found that the radius needed was 5.0 fm. Such a well is also plotted in Fig. 2 (with $V_S \simeq 50.0$ MeV). It is seen to lead to cross sections 50 times too small between 3.0 and 6.0 MeV. Between 3.0 and 6.0 MeV, the cross section is linearly proportional to τ, and so the square-well cross section must be multiplied by f to obtain the cross section of a physical Woods-Saxon well. Here the f_c needed for the Woods-Saxon well to reproduce the measured cross section is $f_c = 50$. Physically plausible Woods-Saxon wells with such large reflection factors do not exist. For the potentials of interest

$$K_0 R_0/\pi = 7. \tag{13}$$

From Fig. 6 of Ref. 18 for $a/R_0 \lesssim 0.18$, $f \lesssim 10$. Larger values of a/R_0 seem physically unrealistic when compared with the values $a/R_0 \lesssim 0.1$ obtained from experiment in a number of cases.

In Fig. 4 are shown similar calculations for square wells with different radii. The real part of the potential is chosen to be 56 MeV. The exact value of V_S is of little importance, since $s_l(c) \propto 1/\sqrt{V_S}$ and $f_c \propto \sqrt{V_S}$. The two cancel out. In Fig. 3 the calculated cross sections for the different radii were made to coincide with the measured cross section at $E = 6.0$ MeV by multiplying them by f. The required value for f was determined by

$$f = \frac{\sigma(\text{experimental}, E = 6.0 \text{ MeV})}{\sigma(\text{calculated}, E = 6.0 \text{ MeV})}. \tag{14}$$

The different values needed for f are shown in

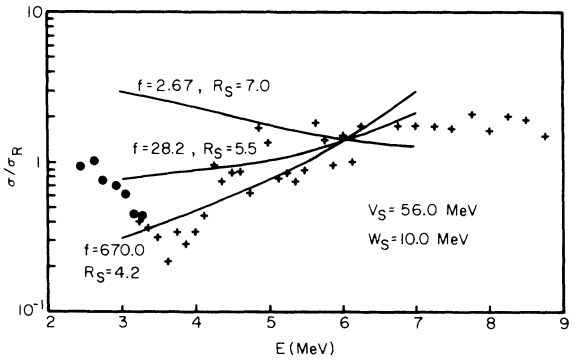

FIG. 4. Ratio of cross sections, calculated and measured, to cross sections calculated with the potential Reeves (Ref. 21) used in his extrapolation. The giant resonances and intermediate-structure resonances [see Fig. 2(b)] appear in the experimental points. The lines show the results obtained for square wells of different radii. The cross sections obtained with square wells were multiplied by the reflection factor f needed to make the calculated cross section equal to the measured cross section at $E = 6.0$ MeV. For those wells reproducing the energy variation of the cross section, the reflection factor needed is unphysically large.

Fig. 4. We feel that to obtain a fit the radius must be within the range

$$4.2 \leq R \leq 5.5 . \tag{15}$$

Then

$$670.0 \geq f \geq 30.0 . \tag{16}$$

As seen above, there are no physically plausible Woods-Saxon potentials with such large reflection factors. If $W_0 = 10$ MeV, it is impossible to fit the experimental results over the energy range $3.0 \leq E_{c.m.} \leq 6.0$ MeV with a Woods-Saxon potential. The sharp energy variation forces a small radius, but the relatively large measured cross sections cannot be obtained with such small radii. The small radii obtained by PWZ[1] are unphysical. The energy variation of $\sigma_l(c)$ must be increased through $s_l(c)$. Resonance effects are important in the well; the imaginary part of the potential must be small. We must study the behavior of wells with a variety of interior properties, since these are important if W is small.

In trying to fit the experimental $^{12}C + ^{12}C$ total reaction cross section with optical potentials, no computerized minimization of deviation from the experimental points will be attempted, nor is it necessary. We instead first carry out a qualitative wave analysis of a number of wells, to find out which ones differ from the Woods-Saxon potentials with large W in such a way as to fit the data better. Quantitative comparison between cross sections calculated using those wells and the experimental data will then be presented.

We will study the behavior of two main types of wells. Woods-Saxon wells and wells with a soft core. For most purposes Woods-Saxon wells are sufficient to reproduce the energy variation of reactions. We will confirm this for $^{12}C + ^{12}C$ total reaction cross sections. Actually both Woods-Saxon wells with $V_0 \simeq 25$ MeV and with $V_0 \simeq 85$ MeV give a satisfactory approximation at low energy. But the exclusion principle for the $^{12}C + ^{12}C$ reaction leads to wells with repulsive cores. What effects will such cores have if added to the usual potentials? In Woods-Saxon wells, the $l = 0, 2, 4, \ldots$, resonances lie on top of each other. Will soft cores separate them sufficiently from each other to spoil the fit? Could we exclude the α-particle model in this way? Comparing extrapolations obtained with the different models fitting the available data will then give a measure of the uncertainties involved in the astrophysical extrapolation.

From the preceding discussion, the strength function must have a maximum somewhat above 6.0 MeV. It must decrease down to 3.0 MeV and increase below that. The most naive assumption is that there is one giant resonance slightly above 6.0 MeV and one somewhat below 3.0 MeV. Close to the resonance the strength function then rises as observed. Potentials of the type given by Eq. (1) that have properly placed resonances will be looked for. However, the low-energy rise may be due to absorption under the barrier. To study this possibility, in addition to potentials of the type given by Eq. (1), we will also study potentials of the type

$$V = -V_0/[1 + e^{(r-R_0)/a_R}] - iW_0/[1 + e^{(r-R_0)/a_I}] . \tag{17}$$

The surface thickness of the real and imaginary parts of the potential need not be the same. If $a_I > a_R$, absorption under the barrier is increased. The rise in the cross section at low energy would then be a measure of a_I. If there is absorption under the barrier at low energy, there is a need only for the resonance above 6.0 MeV. There may be no low-energy resonance.

The imaginary part of the potential, W_0, is frequently taken to be energy-dependent: As the energy increases, the number of unoccupied levels increases, and any effect the exclusion principle has in reducing the cross section is diminished. Woods-Saxon potentials will be studied with $W_0 = cE$.

The α-particle model for the $^{12}C + ^{12}C$ reaction does not lead to Woods-Saxon potentials. If, as the two ^{12}C nuclei come closer together, the average potential is taken to be the sum of the α-α interaction between the appropriate α particles, the resulting average potential will have a soft core and will be very shallow. The soft cores that exist in the α-α interaction lead to a soft core in the $^{12}C + ^{12}C$ interaction. Details of the calculations of $^{12}C + ^{12}C$ potentials from an α-α interaction will be presented in a subsequent paper. We will then show to what extent the parameters of the wells used in reproducing the $^{12}C + ^{12}C$ interaction can be obtained from the α-α interaction. Such potentials may reproduce the drop of the strength function between 6.0 and 3.0 MeV, if there is no resonance in the well below 6.0 MeV. The rise in the strength function below 3.0 MeV could then be due to absorption under the barrier.

Comparing calculated total cross sections with measured ones will now show what parameters such wells must have. The parameters of the wells used are listed in Table I. A Woods-Saxon well must have $V_0 \simeq 20$ MeV [Fig. 5(a)] if it is to explain the variation in the strength function as due to two resonances, one at $E \simeq 2.0$ MeV and the other at $E \simeq 6.0$ MeV. To obtain this value, wells

were searched having properly placed resonances and a cross section of the right order of magnitude. This imposed $V_0 \simeq 20$ MeV and $R_0 = 6.0$ (± 0.2) fm. If $R_0 = 5.8$ fm, one must use a somewhat larger imaginary potential. The search for the parameters V_0 and R_0 is made much easier by using the properties of square wells; the average size of the cross section is determined by the radius (the penetrability of the square well). The distance between two resonances is then mainly determined by the product $V_0 R_0^2$, which affects the cross section through the strength function s_l when W_0 is small enough. The surface thickness was not varied. It was found to have small effects in making the fit better or worse except for the surface thickness of the imaginary part of the potential, if it is taken larger than the real part, as we shall see below.

The cross section above 7.5 MeV would have been closer to the experimental cross section if $W = cE^2$. At $E \leq 6.0$ MeV, a small imaginary part is needed to allow for energy variation of the strength function. But this is not so above 6.0 MeV. The resonance structure below 6.0 MeV imposes a small imaginary part on the potential there, but above 6.0 MeV the absence of structure imposes a large W.

In Fig. 5(b) is shown the cross section obtained with the well found[22] to reproduce the elastic scattering data in the $^{12}C + ^{12}C$ system. It has a resonance at 3.5 MeV; whereas, it should have one at 2.0 MeV, and the next resonance is at 7.0 MeV, only 3.5 MeV above the first one. This potential is very similar to that leading to a good fit in Fig. 5(a); however, it is not deep enough (the distance between two resonances is too small) and the resonances happen to fall in the wrong places. Using $V_0 = 16.0$ MeV would bring the low-energy resonance to the right place but the next resonance would be too close to give as good a fit as obtained in Fig. 5(a). The fit, however, would still be reasonably good.

Compare Figs. 5(a) and 5(c). In both wells there are resonances between 1.0 and 2.0 MeV. The next resonance is at 6.0 MeV for the shallow and at 8 MeV for the deeper wells.[23,24] In the deeper well the next resonance is at 8.0 MeV; the bottom of the valley between the resonances then occurs around 4.2 MeV, about 0.7 MeV too high. The fit has, we feel, deteriorated appreciably as compared with that obtained with the shallow well.

A similar fit is shown in Fig. 5(d). The reduced value of R_0 is compensated for mainly by an increase in W. V_0 had to be taken larger to allow a great enough distance between resonances and thus a sufficient decrease in the cross section around 3.5 MeV. The larger imaginary part of the potential implies wider resonances and not so shallow valleys. Figure 5(e) shows the same potential as Fig. 5(d) except that it is 1.0 MeV deeper. The fit between 4.0 and 6.0 MeV is greatly improved, but here there is no rise in the cross section below 3.0 MeV due to the resonance. It is too far away. Figure 5(e) shows, however, that a fit nearly as good as that obtained with a shallow potential can be obtained with a much larger potential if there is absorption under the barrier (see Fig. 6). This was achieved by taking $a_I = 1.2 a_R$. It does indicate how sensitive the extrapolation to lower energies is to the surface thickness of the imaginary part of the potential.

Finally, Fig. 5(f) shows a fit obtained with a well even shallower than the first one. It was obtained from the α-α interaction without any parameter variation and has a soft repulsive core. In it, the *first* resonance occurs at $E = 6.0$ MeV. The rise in the strength function as E is decreased below 3.0 MeV is again due to absorption under the barrier. The fit is as good as obtained with any Woods-Saxon well. It is improved if W increases (dashed line) above 6.0 MeV, but W may be constant below that.

Considering the energy fluctuations due to intermediate structure and the approximation im-

TABLE I. Potentials used for the $^{12}C + ^{12}C$ channel. All potentials whose parameters are given here are of the Woods-Saxon shape [Eq. (1)]. For a few potentials different radii or surface thicknesses were used for the real and imaginary parts of the potentials. R and I are used as indices for the real and imaginary parts, respectively.

Identification	V_0 (MeV)	W_0 (MeV)	R_R (fm)	R_I (fm)	a_R (fm)	a_I (fm)
Reeves (Ref. 21)	50.0	10.0	5.77	···	0.4	···
Potential I	23.0	$0.2E$	6.0	···	0.5	···
Reilly *et al.* (Ref. 22)	14.0	$0.4 + 0.1E$	6.18	6.41	0.35	0.35
Potential II	63.0	$0.1E$	6.0	···	0.5	···
Potential III	84.8	$0.2E$	5.7	···	0.5	···
Potential IV	85.8	$0.2E$	5.7	···	0.5	···
Potential V	From α-α interaction	1.0	···	6.0	···	0.55

plied by using a local potential for heavy ions, it is felt that at least three of these wells [Figs. 5(a), 5(e), and 5(f)] give very good fits to the data. The large wells [Figs. 5(c) and 5(d)] with two resonances appearing give slightly poorer fits but should probably not be rejected completely. The total absorption cross section does not allow us to determine uniquely one well with which we could extrapolate to lower energies. The elastic scattering data suggest that we should use a shallow well and give an argument for our well with $V_0 = 23$ MeV. However, as we shall see below, the α model allows us to explain the branching ratios. It also suggests an intermediate structure consistent with the measured resonances. Physically, the well obtained from the α model seems more reasonable. All the other wells suffer from a serious drawback: because of the need for resonances, the imaginary part of the potential has to be so small that when one ^{12}C nucleus is superimposed on the other it is not destroyed very quickly. This is hard to reconcile with the exclusion principle, among other things. The well based on the α model is much less subject to these objections for two reasons:

(a) It can accomodate a large imaginary term in the region of strong overlap because of the repulsive central core: The two ^{12}C very rarely overlap strongly, and whatever happens when they do does not affect the average cross section (i.e., W could be very large at $r = 0$, and it would not affect the cross section).

(b) Because the real attractive part of the potential is much shallower than for the other wells, the well is less transparent in its outer regions ($r = 5.5$ fm) than the other wells are. For those reasons we prefer the extrapolation based on the α model. We do not feel, however, that the argument is binding and we will consider extrapolations with most of the wells described above. The

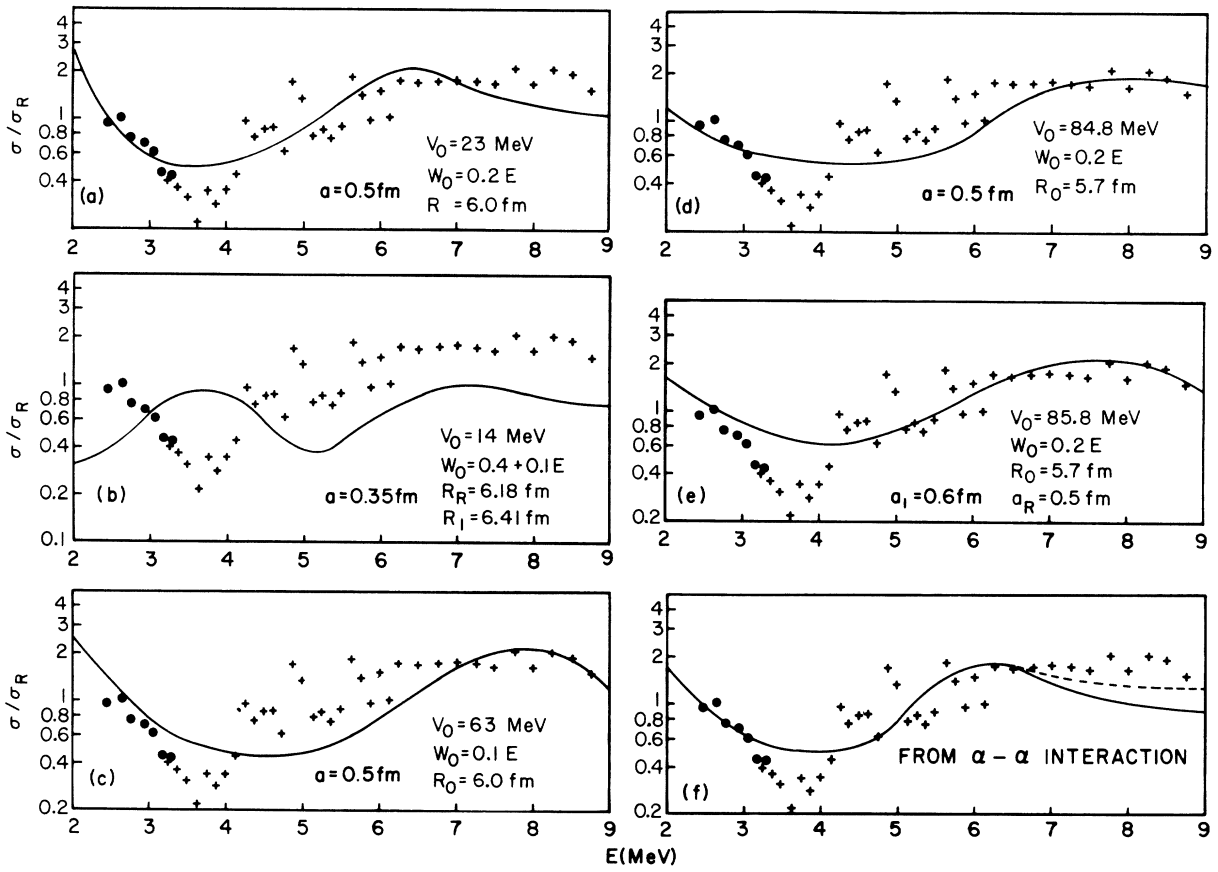

FIG. 5. Ratio of cross sections, calculated and measured, to cross sections calculated with the potential Reeves (1966) used in his extrapolation. It appears that the wells of (a), (e), and (f) fit the data best. Those in (c) and (d) should probably not be rejected off-hand. These are the best fits obtained after a thorough search based on our wave analysis. The surface thickness was generally not varied except the surface thickness of the imaginary parts of the potential for (e) and (f). In (f) the imaginary potential was $W_0 = 1$ MeV except for the dashed line, above 6 MeV, where $W_0 = 1 + (E - 6.0) \times 0.2$ MeV.

potentials which fit the total absorption cross section of the entrance channel make it possible for us to discuss quantitatively the branching ratios of the compound nucleus to individual exit channels (Sec. 3).

3. BRANCHING RATIOS, STATISTICAL THEORY, AND α-PARTICLE MODEL

The evaporation model and the statistical theory have been found to explain successfully the reaction cross section to a number of exit channels in the reaction $^{12}C+{}^{12}C \rightarrow {}^{20}Ne+\alpha$ at $E=12$ MeV.[9] But at lower energy ($E \simeq 3.0$ MeV) major problems appear. The statistical theory predicts the plummeting of the cross section as the threshold is approached. But some of the measured cross sections remain more than 1 order of magnitude larger than predicted. The α-particle model for the underlying intermediate structure suggests an explanation.

Calculations of average cross sections were made using the evaporation theory for two similar spinless incoming bosons[13,25]:

$$\bar{\sigma}_{cc'}=\frac{2\pi}{k_c^2}\sum_{J^\pi}(2J+1)T_l(c)\frac{\sum_{s'l'}T_{l'}(c')}{\sum_{\alpha''s''l''}T_{l''}(c'')}, \qquad (18)$$

where c represents a pair of particles and their state of excitation, s is the channel spin, l the orbital angular momentum of the pair, and J^π the angular momentum and parity of the compound nucleus. The vector sum $\vec{J}=\vec{l}+\vec{s}$ must be consistent with the conservation laws for angular momentum, parity, and statistics. Here, in the incoming channel $J^\pi=l^+$, where l is limited to even values because we have identical bosons. The unprimed quantities refer to the incoming channel, the primed quantities refer to the outgoing channel, and the double-primed quantities should be summed over all open channels in the compound nucleus. It is assumed that all $T_{sl}(c)=T_l(c)$, and similarly in the primed and double-primed cases. The total reaction cross section [Eq. (6)] may be obtained from Eq. (18) by adding up the contributions of all the outgoing channels. In Sec. 2, the behavior of $T_l(^{12}C+{}^{12}C)$ [or $T_l(c)$ in Eq. (18)] was studied by the analysis of the total cross section. Although the entrance channel transmission functions are then reasonably well known, we shall remove any small influence of the properties of the $^{12}C+{}^{12}C$ system by taking ratios of outgoing channels. For example, the value of the cross section $^{12}C+{}^{12}C \rightarrow {}^{20}Ne^*+\alpha$ to a given level of ^{20}Ne will not be given here; only the ratios of this cross section to the total reaction cross section or to the total $^{20}Ne+\alpha$ cross section are given. In all the calculations we have used potential I (Table I) for the entrance channel.

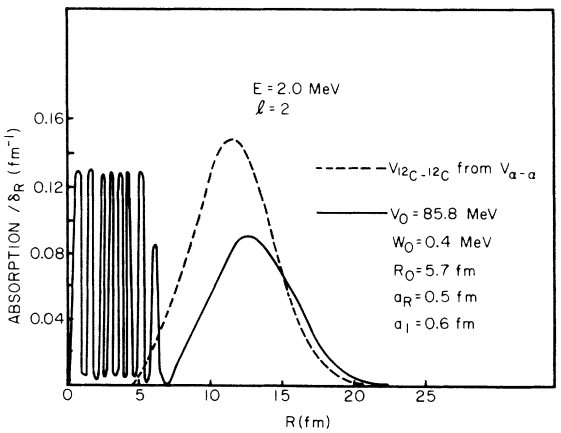

FIG. 6. Absorption per unit radius as a function of the radius. In the very shallow well, at $E=2.0$ MeV, most of the reactions would occur at $r=13$ fm. In the deeper well, half the reactions would occur within the well and half outside. The rise in the cross section at low energy may well be the first good evidence of the existence of absorption under the barrier.

The parameters appropriate for the optical models of $^{20}Ne+\alpha$ and $^{23}Na+p$ have been discussed elsewhere.[13,18] The effect of varying the parameters will be discussed with the presentation of the results. All transmission functions were calculated numerically[26] for the wells of interest. Some of the transmission functions used are shown in Fig. 7.

In Fig. 8 is shown the ratio of the total $^{12}C+{}^{12}C \rightarrow {}^{20}Ne+\alpha$ cross sections to the total cross sections for $^{12}C+{}^{12}C \rightarrow {}^{23}Na+p$. The total cross sections are obtained by adding up cross sections to each of the states of ^{20}Ne or ^{23}Na. Using the smaller radius for the $^{20}Ne+\alpha$ channel changes the ratio by 10%. The ratios of the corresponding measured cross sections are also shown. No significant disagreement appears.

In Fig. 9 is shown the ratio of the cross section for $^{12}C+{}^{12}C \rightarrow {}^{23}Na+p$, where ^{23}Na is left in the 9th excited state, to the total cross section to all ^{23}Na levels. Both the measured and calculated cross sections are shown. No significant disagreement appears. For the 9th excited level of ^{23}Na we assumed $J^\pi=\frac{3}{2}^+$. Its spin and parity are unknown. If it had the spin and parity $J^\pi=\frac{3}{2}^-$, the calculated cross section would have been given by the dashed line, showing strong disagreement. It is important for a proper understanding of the $^{12}C+{}^{12}C$ reaction that the spin and parity of more excited states of ^{23}Na be determined. With our present degree of ignorance, there is no disagreement between calculated and measured branching ratios

in the proton channels.

In Fig. 10 is shown the ratio of the cross section where ^{20}Ne is left in a definite excited state to the cross section obtained by summing cross sections to all individual levels of ^{20}Ne. There is no strong dependence of the cross sections on the choice of radius, but there is one major effect: At the lowest energy the observed branching ratios of the $2^-, 3^-$, and 4^+ levels grow much larger than the theory predicts. The 3^- and 1^- levels were not separated experimentally.

We have also varied parameters other than the radius for the $^{20}Ne + \alpha$ channel, but we were not able to find any set of parameters that could explain the measured branching ratios. In particular, we tried reducing the imaginary part of the potential of the $^{20}Ne + \alpha$ channel. We hoped to introduce a giant resonance in that channel and so explain the rise of the cross section near threshold. The effort proved unsuccessful unless we introduced a different potential (especially a different V_0 and W_0) for each of the excited states of ^{20}Ne.

An intermediate-structure model based on α particles does explain the major discrepancies. In the α model, one of the ^{12}C nuclei is taken to be made of three α particles. With the remaining ^{12}C core the three α particles can form directly $^{16}O + ^{8}Be$, $^{20}Ne + \alpha$, or ^{24}Mg. The ^{16}O, ^{20}Ne, and ^{24}Mg states formed must be made of a ^{12}C core and one, two, and three α particles, respectively. The ground states of ^{16}O, ^{20}Ne, and ^{24}Mg are not expected to be formed: Because of their relatively strong binding it is hard to imagine that they would be made of a ^{12}C core and some α particles; the α particles would be crushed if they were so strongly bound. However, the excited states may be constructed in this manner. Direct reactions are then expected to some excited states of ^{16}O and ^{20}Ne. Intermediate-structure resonances are due to the states of ^{24}Mg built of three α particles around a ^{12}C core. The direct reaction giving $^{16}O + ^{8}Be$ was found by Kozlovsky[27] to be relatively unimportant. The direct reaction giving $^{20}Ne + \alpha$ might be important. If it is, it would, in the α-particle model, lead to an anomaly in the compari-

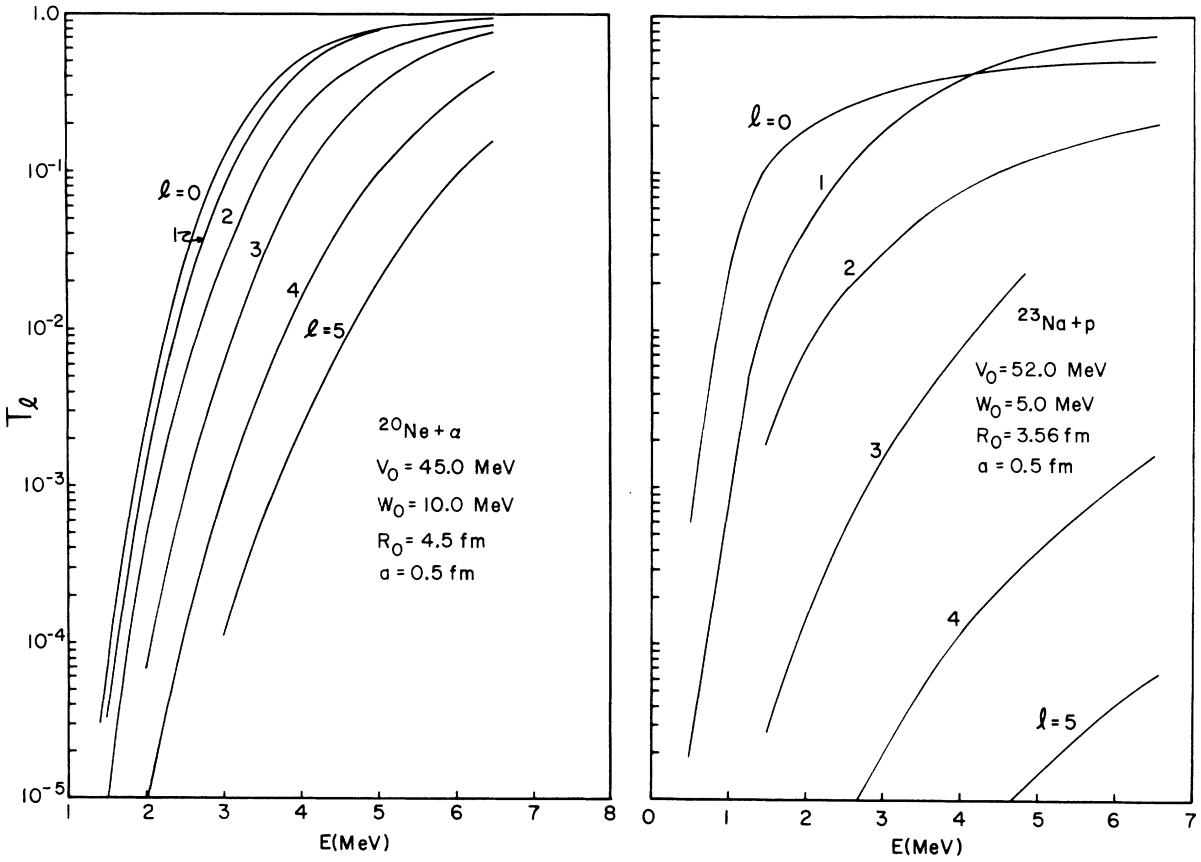

FIG. 7. Transmission functions as a function of energy for a number of angular momenta. Because W is larger than the level spacing in the $^{20}Ne + \alpha$ channels, the transmission functions depend only on the radius of the well. In the $^{23}Na + p$ channel, however, there are resonance effects, and R, W_0, and V_0 are all important parameters.

son of the branching ratios of the 3^- and 1^- levels. It would be interesting to distinguish experimentally between those two levels. Some of the ^{24}Mg states built around ^{12}C will be similar to the ^{20}Ne states built around ^{12}C; they will lead to the anomalous rise in the branching ratio as the threshold is approached. These resonances should be close to the threshold of the ^{20}Ne states involved, since the added α particle will be barely bound or slightly unbound.[28]

This picture is supported by the experimental evidence gathered on the θ_α^2 reduced width in the ^{16}O + α reaction.[29] The θ_α^2 reduced width is close to 1 for the 5.78-MeV, 1^- and the 6.72-MeV, 0^+ levels but is only 0.006 for the 5.62-MeV, 3^- level. This shows that the 1^- level has a large overlap with the ^{16}O ground state plus one α particle, but the 3^- level has a small overlap with it. Consequently, the overlap of the 1^- level with a ^{12}C core and two α particles is expected to be small, while that of the 3^- level may be large. Similarly, the low-energy rise in the branching ratio does not exist for the 0^+, 6.75-MeV level.

Thus, the α-particle model can explain qualitatively the anomalous branching ratios measured in the ^{12}C + ^{12}C reaction. In a forthcoming paper, we shall see, using the formalism developed below, how quantitative the model can be made.

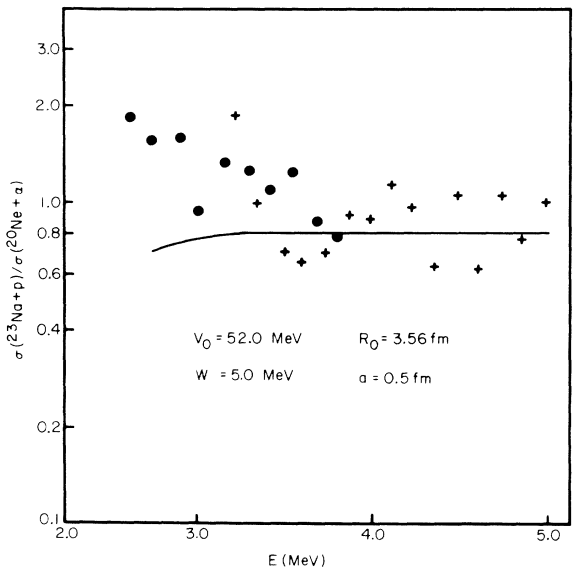

FIG. 8. Ratio of the cross section for $^{12}C + ^{12}C \rightarrow ^{23}Na + p$ to the cross section for $^{12}C + ^{12}C \rightarrow ^{20}Ne + \alpha$ as a function of energy. For the $^{20}Ne + \alpha$ well, we used $R_0 = 5.0$ fm. The parameters in the figure are for the $^{23}Na + p$ channel. The solid line comes from our statistical-theory calculations. It agrees reasonably well with the experimental results.

4. FRAMEWORK FOR AN α-PARTICLE INTERMEDIATE-STRUCTURE MODEL

Phenomenologically, we have seen that an α-particle model could lead to an optical potential reproducing the total cross section for the $^{12}C + ^{12}C$ reaction, and that an α-particle model leads to intermediate structure explaining the anomalous branching ratios observed to some levels, and the 70-keV-wide resonances. We must now tie together these various aspects of the $^{12}C + ^{12}C$ reaction into a comprehensive framework.[30] We must relate them to the existence of intermediate structure and we must relate the α states to the normal states of ^{24}Mg. Finally, we will discuss briefly the difficulties met by other intermediate-structure models, in particular the quite successful and interesting model proposed by Imanishi.[8]

Let us assume that the 24 nucleons of the problem interact only through two-body interactions v_{ij}, so that the total Hamiltonian of the system is

$$H = -\frac{\hbar^2}{2m_n}\sum_{i=1}^{24}\nabla_i^2 + \sum_{i<j}^{24} v_{ij}. \tag{19}$$

The compound states χ_λ of the ^{24}Mg system are then eigenstates of the total Hamiltonian in the region of configuration space in which all 24 nucleons are close together:

$$H\chi_\lambda = E_\lambda \chi_\lambda . \tag{20}$$

Next we can consider the 24 nucleons arranged as six α particles.[31] The wave function then is a product of the internal wave functions φ_α of the six α particles (which can each be in the α-parti-

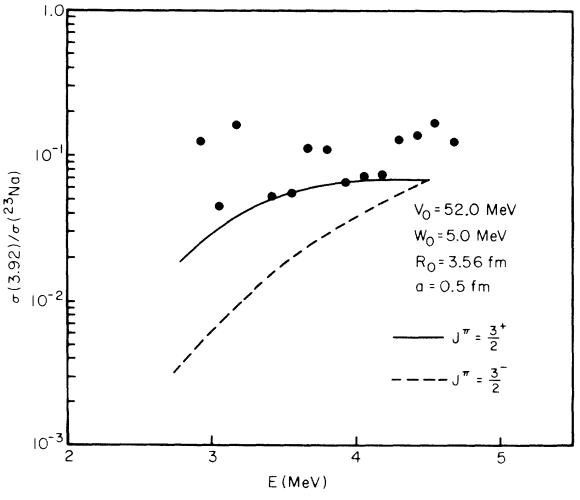

FIG. 9. Ratio of the reaction cross section to the 9th excited level of ^{23}Na to the summed cross section to all ^{23}Na levels. Our calculations depend on the unknown spin and parity of the level. No well-established disagreement exists yet.

cle ground state or in an arbitrary excited state) and of the wave function Φ_i describing the relative motion of the six α particles. In the decomposition we have chosen to regard Φ_i as the intermediate state of the problem. It satisfies the approximate Hamiltonian

$$\mathcal{H}^{(\mathrm{int})} = -\frac{\hbar^2}{2m_\alpha}\sum_{i=1}^{6}\nabla_i^2 + \sum_{i<j}^{6} v_{ij}, \tag{21}$$

where m_α is the α-particle reduced mass and v_{ij} the α-α interaction[16] based on the two-nucleon forces v_{ij}. That is,

$$v_{12} = \langle \varphi_{\alpha_1}\varphi_{\alpha_2} | \sum_{i=1}^{4}\sum_{j=5}^{8} v_{ij} | \varphi_{\alpha_1}\varphi_{\alpha_2}\rangle, \tag{22}$$

where φ_{α_1} is the ground-state wave function of the first α particle (with nucleons $1-4$) and φ_{α_2} the ground-state wave function of the second α particle (with nucleons $5-8$). The two α particles are separated by a distance which is the argument of v_{12}. Antisymmetrization is left implicit.

Just as the average α-α interaction is based on the two-nucleon interaction given by Eq. (22) it is clear how the intermediate and compound states are related by means of the two-nucleon potentials. We can write

$$H = \mathcal{H}^{(\mathrm{int})} + \mathcal{H}_\alpha + \mathcal{H}', \tag{23}$$

where $\mathcal{H}_\alpha$ is the exact Hamiltonian for the internal motion of the six α particles and $\mathcal{H}'$ is a configuration interaction

$$\mathcal{H}' = \sum{}' v_{ij} - \sum_{i<j}^{6} v_{ij}, \tag{24}$$

where the primed sum runs over the values $(i = 1-4,\ j = 5-24)$, $(i = 5-8,\ j = 9-24)$, etc., that is, over all nucleon-nucleon interactions between different α groups but not within each group. Alternatively we could write Eq. (24) as

$$\mathcal{H}' = \sum{}' v_{ij} - \langle \sum{}' v_{ij}\rangle_{\mathrm{av}}, \tag{25}$$

where the average is over the internal coordinates of the α particles as in Eq. (22). The configuration interaction mixes intermediate states into compound states. However, both $\mathcal{H}$ itself and

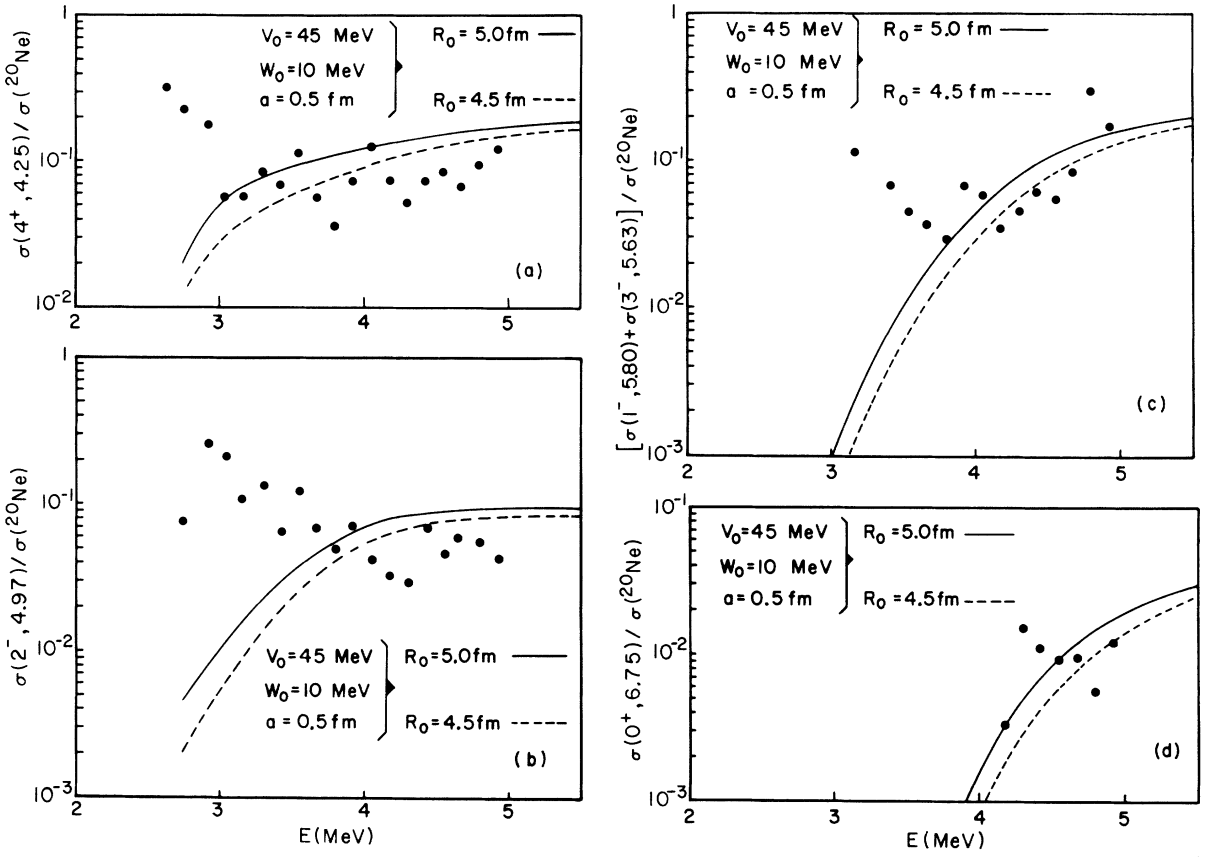

FIG. 10. (a)–(d) Ratio of the reaction cross section to a given excited state of ^{20}Ne to the summed total cross section to all ^{20}Ne levels. Calculations are shown for two values of the ^{20}Ne + α optical potential. The relative rise in the cross section close to threshold can be explained, at least qualitatively, by the α model.

$\mathcal{H}^{(\mathrm{int})}+\mathcal{H}_\alpha$ constitute operators whose eigenstates span the complete space of all 24 nucleons.

Next, we go further and group the six α particles into two carbon nuclei. We proceed as for the intermediate state and write the wave function as a product of the internal wave function ψ_C of the two carbon nuclei and of the single-particle wave function Ψ_C describing the relative motion of the ^{12}C, ^{12}C pair. The corresponding single-particle Hamiltonian is

$$h^{(\mathrm{sp})}=-\frac{\hbar^2}{2m_c}\nabla_C^2+V_C\,. \tag{26}$$

Here V_C is the single-particle (or optical-model) potential

$$V_C=\langle\Phi_0|\sum_{i=1}^{3}\sum_{j=4}^{6}v_{ij}|\Phi_0\rangle, \tag{27}$$

where Φ_0 is that intermediate state in which the two ^{12}C nuclei are in the α-particle configuration (triangular) of the ^{12}C ground state with the ^{12}C, ^{12}C pair separated by a distance which is the argument of V_C.

Again, we can relate $h^{(\mathrm{sp})}$ to the intermediate Hamiltonian in the following way:

$$\mathcal{H}^{(\mathrm{int})}=h^{(\mathrm{sp})}+h_C+h'\,, \tag{28}$$

where h_C is the internal Hamiltonian of each ^{12}C nucleus and h' is a configuration interaction

$$\begin{aligned}h'&=\sum_{i=1}^{3}\sum_{j=4}^{6}v_{ij}-V_C\\&=\sum_{i=1}^{3}\sum_{j=4}^{6}v_{ij}-\langle\sum_{i=1}^{3}\sum_{j=4}^{6}v_{ij}\rangle_{\mathrm{av}}\,,\end{aligned} \tag{29}$$

where the average is over the intermediate states as in Eq. (27). The configuration interaction h' mixes single-particle states into intermediate states. The single-particle states of the ^{12}C + ^{12}C system along with *all* the excited states of the ^{12}C nuclei would constitute a complete set of states for the 24 nucleons. However, in forcing, as we do, each α particle to be internally in its ground state in defining h' we implicitly accept that a ^{12}C nucleus, in the first step of the reaction, can only be excited to a state of ^{12}C completely made up of three α particles internally in their ground state. All the states of ^{12}C would be included only if the α particles were allowed to be excited internally. The approximation of our model then essentially consists in assuming that in the first step of the reaction the α particles do not break up. They will be allowed to break up only in the second step. The main justification of the approximation is the width of the intermediate resonances, $\Gamma \simeq 100$ keV, implying that the intermediate structure lasts much longer than the single-particle structure.

The average configuration interaction of the two operators h' and $\mathcal{H}'$ can be incorporated by adding appropriate imaginary terms to V_C and each v_{ij}. In turn, the imaginary part of the potential can be related to the observed width of the single-particle states or of the intermediate states ($\simeq$100 keV). To make such a calculation for the width would carry us beyond the scope of the present paper. Perhaps the greatest miracle required of the whole model is that the effects of $\mathcal{H}'$ should be so small that the intermediate states have their observed long lifetime. We hope to show that this arises in part from the soft repulsive cores of the v_{ij}, in part from the threshold nature of the ^{12}C + ^{12}C states, and in part from the difficulty of exciting an α particle internally.

Next, a word about the alternative decomposition in which we would retain one of the ^{12}C nuclei as being an eigenstate of the exact ^{12}C Hamiltonian, rather than of its approximation through the α-particle model. Then the intermediate states are those of three α particles and a carbon nucleus. Half of the configuration interaction $\mathcal{H}'$ also disappears. The prescription for the single-particle motion remains unchanged – except for adopting the change already noted for Φ_0 in Eq. (27) and by corresponding changes in Eq. (29). Again, one has the same basic relations between compound, intermediate, and single-particle states. We will use the first decomposition to do all our calculations, for example those of the real and imaginary parts of average potentials for the ^{12}C + ^{12}C interaction. However, we will use the second decomposition in our discussion of the intermediate structure in ^{24}Mg. The contradiction is not so profound as it appears. The use of the α-α interaction to calculate the ^{12}C + ^{12}C interaction could be justified even if the ^{12}C nuclei were not thought of as made of three α particles: In the α-α interaction the exchange and exclusion-principle effects are already taken into account. The use of α-α interaction in the calculation of the interaction between heavier nucleons then allows us to neglect the effects of exchange and of the exclusion principle. Our hypothesis is that even though ^{12}C nuclei can be represented perhaps only half the time as made of three α particles, the contribution of the nucleon-nucleon, or nucleon-α force would not be very different from an α-α force once the exchange and exclusion-principle effects are taken into account. In principle, once the nucleon-nucleon forces are specified then the real and imaginary parts of optical potential for both the intermediate and single-particle motion are fixed without arbitrariness.

The discussion of the experimental results pre-

sented in Secs. 1–3 then argues for the existence of the following phenomena in the $^{12}C + ^{12}C$ interactions:

(1) At energies lower than 6.0 MeV, as one ^{12}C nucleus approaches another, either can be considered to be made of three α particles. Presumably it is only a fraction of the time in the form of three α particles, but the prominence of intermediate structure shows that the α-α interaction mechanism is the dominant feature. Also, an optical potential obtained assuming only α-α interactions leads to a cross section very similar to the total $^{12}C + ^{12}C$ cross section.

(2) The reaction proceeds through the internal excitation of one ^{12}C nucleus. α particles that are its building blocks are excited by the residual interaction from the other ^{12}C nucleus. Once excited, they can easily form a state of ^{24}Mg made of a ^{12}C core and three α particles around it. These states lead to the intermediate structure observed in the total reaction cross section and to the relative rise in the branching ratio to excited states of ^{20}Ne as the threshold to these states is approached.

(3) Instead of all three α particles from one ^{12}C nucleus transferring to the other ^{12}C nucleus, it is possible that only two transfer, causing a direct reaction to some states of ^{20}Ne. This will be observed to states of ^{20}Ne made of a ^{12}C core surrounded by two α particles.

(4) The residual interaction between the α particles causes the states formed of a ^{12}C core and three α particles to disintegrate into the ^{24}Mg "normal" states. This takes place 10 to 20 times more slowly than the breakup of the ^{12}C nucleus: The imaginary part of the potential needed to reproduce the $^{12}C + ^{12}C$ total reaction cross section is of the order of 1 MeV, but the width of the intermediate-structure levels restricts to less than 50 keV the imaginary part of the potential due to the residual interaction between the α particles.

These last remarks lead us to discuss the intermediate-structure model of Ref. 8. Other such models[4–7] have been proposed and most of our remarks here would also apply to them.

Imanishi[8] carried out a calculation of resonances in elastic scattering with the 2^+ level of ^{12}C at 4.43 MeV coupled to the ground state of ^{12}C. This carefully done coupled-channel calculation gave very promising results: It was possible to reproduce the energy, level width, and spin of the first three levels measured.[3] Because of the number of arbitrary constants necessarily involved, however, the study was not conclusive. It reproduced the position of three levels, but there were two related arbitrary constants: the depth of the potential and the coupling constant. The width of the levels can be related to the arbitrary size of the imaginary potential and to the arbitrary radius used for the potentials.

We have two main objections to Imanishi's model: the small size of the imaginary potential and the small number of intermediate-structure resonances it is possible to get from such a model. Since the intermediate-structure resonances are only 100 keV wide (the data of Ref. 1 actually reveal 50-keV-wide resonances), Imanishi was forced to use optical potentials with an imaginary part of 100 keV to reproduce the width of the measured resonances. It is hard to imagine how the system formed by such loosely bound nuclei as two ^{12}C nuclei, one of which is in its ground state, can last as long as indicated by a resonance width of 100 keV (or equivalently an imaginary potential of 100 keV). A nucleon interacting with a complex nucleus lasts only $\frac{1}{30}$ of that time. Furthermore, as seen in Sec. 2, the imaginary part of the $^{12}C + ^{12}C$ potential must be 5 to 10 times larger than what was used by Imanishi to reproduce reasonably well the total cross section. It is true that reactions will also occur through the coupled channel, but, even multiplied by 2, the imaginary part of his potential is still far too small. We have

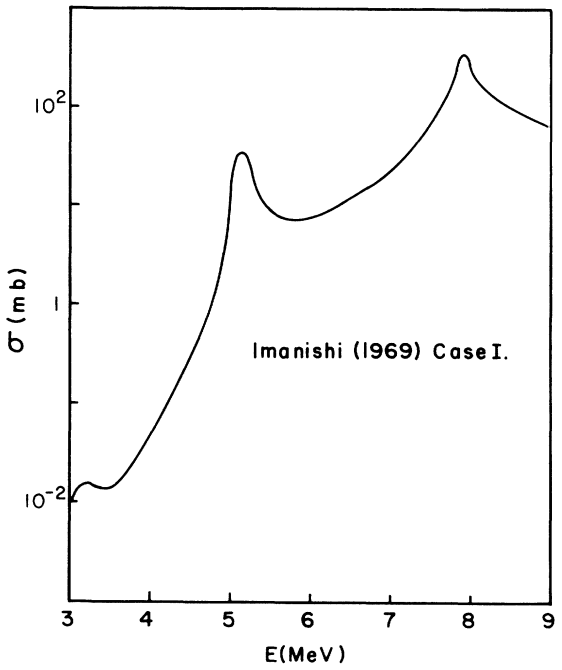

FIG. 11. Total absorption cross section calculated with the optical potential for the ground state of ^{20}Ne used by Imanishi (Ref. 8) in his coupled-channel calculations. Adding the coupled channel will change the position of the resonances, but is not expected to change the averaged energy variation of the total cross section. Comparison with Fig. 2 shows how far it is from a realistic potential.

also calculated what kind of total cross section to expect from the $^{12}C + ^{12}C$ optical potential he uses in the ground state, but neglecting the coupled channels. The result does not look anything like the measured cross section, but is in itself not too significant, since the resonance structure of the entrance channel is altered by the coupled one. Above 5.5 MeV, however, the average cross section (averaged over resonances in the well) is far too small, as can be seen from Fig. 11. This is a rather naive approach, and a systematic study of the properties of coupled-channel calculations should be undertaken to shed light on such questions.

In his calculations Imanishi reproduces the three resonances measured in Ref. 3. He does not predict any other close by. At least three more have been measured by PWZ.[1] It appears that wells similar to Imanishi's cannot support as many resonances as are measured. Wells reasonable for the $^{12}C + ^{12}C$ system have a resonance density of 1 MeV^{-1}; whereas, at least six resonances are measured within 2 MeV.

Both because of the narrowness of the resonances and because of their density it is hard to see how they could arise in distorted-wave calculations. A more complex intermediate structure appears necessary. The α model offers such a structure. However, others may exist.

5. APPLICATION TO ASTROPHYSICS

Previous attempts to extrapolate the measured cross sections to the energy region of astrophysical interest[1,21,32] have assumed that all the energy variation was due to the Coulomb barrier. The extrapolation is then straightforward. However, Fig. 3 clearly shows that there are strong resonance effects in the $^{12}C + ^{12}C$ potential well. It is then much more difficult to extrapolate with confidence. What is the rise in the strength function below 3 MeV due to? A resonance at 1.0 MeV, at 2.0 MeV, or absorption under the barrier? The extrapolation depends sensitively on the well chosen. To see how sensitive the astrophysical calculations are on the extrapolation, we have calculated the thermonuclear-reaction rate for a number of the wells presented in Sec. 2.

The thermonuclear-reaction rate per second per pair of interacting particles is[33]

$$\langle \sigma v \rangle = \left(\frac{8kT}{\pi M}\right)^{1/2} \int_0^\infty e^{-E/kT} \left(\frac{E}{kT}\right) \sigma d\left(\frac{E}{kT}\right), \tag{30}$$

where E is the energy in the center-of-mass system, T is the temperature, M is the reduced mass

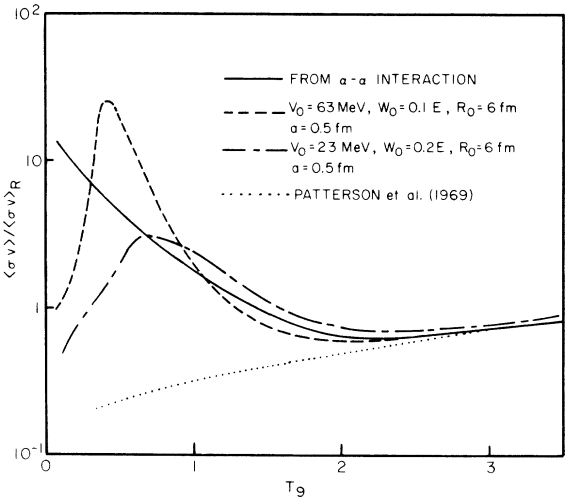

FIG. 12. Ratio of astrophysical reaction rates calculated with some of our potentials ($\langle \sigma v \rangle$) to those calculated with the potential Reeves (Ref. 21) used in his extrapolation ($\langle \sigma v \rangle_R$). This gives an estimate of the uncertainty of the low-temperature reaction rate. It does not represent lower or upper limits except perhaps the PWZ (Ref. 1) results; see text.

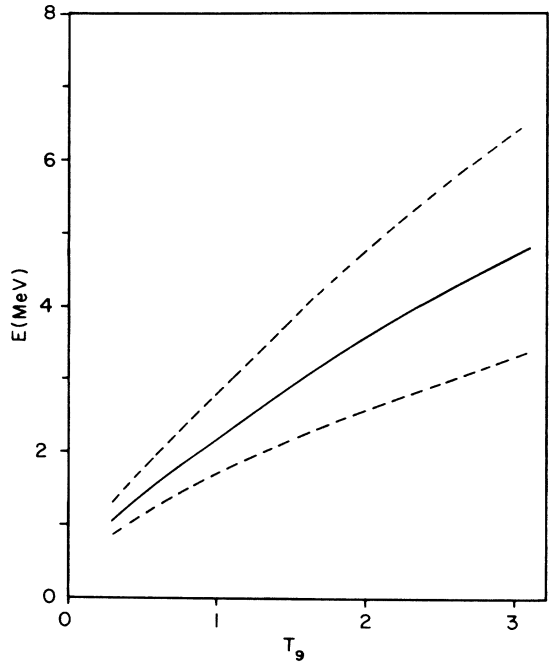

FIG. 13. Energy range of importance in the calculations of the astrophysical reaction rates as a function of temperature. The full line gives the maximum of the integrand. Within the dashed lines lies 90% of the integral at a given temperature; 5% of the integral lies at lower energies and 5% at higher energies.

of the system, and k is Boltzman's constant. The reaction rate can then be calculated once the cross section is known as a function of energy. The cross section was calculated numerically over the energy range

$$0.3 \leq E \leq 10 \text{ MeV}$$

by solving numerically Schrödinger's equation for the wells presented in Sec. 2. The reaction rates were calculated numerically to a 1% accuracy using Eq. (30). The results obtained for four wells were then compared with those of Reeves[21] (Fig. 12).

The temperature range of interest for hydrostatic carbon burning is[34] ($T_9 \equiv 10^{-9}T^\circ\text{K}$)

$$0.4 \leq T_9 \leq 1.0 .$$

This is exactly the range where the effect of a giant resonance would be felt. Especially for $T_9 \leq 0.7$ the uncertainty in the extrapolation leads to an uncertainty of at least a factor of 3, and more conservatively of a factor of 10 in the stellar reaction rate.

To understand how the extrapolation in energy affects the reaction rate, one may refer to Fig. 13. The full line indicates for each T_9 the maximum of the integrand in Eq. (30). Within the dashed lines lies 90% of the total integral; 5% lies outside at each end. For $T_0 \geq 1.6$ the stellar reaction rate is determined by the measurements. They are available above 2.6 MeV. At lower temperatures, the reaction rate is mainly determined by extrapolated values of the cross section. Comparing our different extrapolations gives an order of magnitude estimate of the uncertainty, but it does *not* give the maximum range of uncertainties. Wells can easily be imagined that give either larger or smaller extrapolated values of the cross sections. For instance, the low-energy resonance might be at $E = 2.6$ MeV and the cross section might go back close to the PWZ[1] value below 2.0 MeV. From our studies of a number of wells, this appears unlikely and is a lower limit, and so is the reaction rate calculated with the PWZ[1] values of the extrapolations. What is the upper limit? The potential well with $V_0 = 63.0$ MeV, $R_0 = 6.0$ fm leads to a resonance at $T_9 = 0.45$. By adjusting slightly the parameters of this well the resonance could appear anywhere between $T_9 = 0.0$ and $T_9 = 1.0$. For $T_9 \leq 0.7$, the upper limit is then approximately

$$\langle \sigma v \rangle / \langle \sigma v \rangle_R = 50 . \tag{31}$$

Above $T_9 = 0.7$, the upper limit is lower, since the range of energy values where the integrand is large is getting close to the measured values.

Assuming that there exists an optical potential fitting the measured cross section between 0.0 and 6.0 MeV, the above discussion is somewhat conservative. A reasonable uncertainty is a factor of 3 either way from the α-model extrapolation. A conservative one is a factor of 10 either way.

Up to now we assumed that the mechanism dominating the reaction of the $^{12}C + ^{12}C$ reaction was the same below and above 2.6 MeV. Such an hypothesis made at $E = 3.2$ MeV leads to Patterson's extrapolation.[1,32] A new mechanism probably appeared below 3.2 MeV: absorption under the barrier. There is no guarantee that no new surprise awaits us below 2.6 MeV. In view of this, an uncertainty factor of 10 in $\langle \sigma v \rangle$ for $T_9 \leq 0.7$ MeV might actually be merely reasonable.

In numerical stellar-evolution calculations, the nuclear reaction rates are required. We fitted our calculated rates to the formula:

$$\log_{10}(N_A \langle \sigma v \rangle) = A_1 - \tfrac{2}{3} \log_{10} T_9 - \frac{A_2}{T_9^{1/3}} , \tag{32}$$

TABLE II. Thermonuclear reaction rates.

Potential	Equation used	A_1	A_2	Temperature range	Accuracy (%)
Reeves (Ref. 21)	(32)	25.428	35.352	$0.4 \leq T_9 \leq 1.3$	15
	(32)	23.309	33.026	$1.3 < T_9 \leq 3.5$	15
I	(32)	27.584	36.834	$0.08 \leq T_9 \leq 0.80$	20
	(32)	22.654	32.235	$0.8 < T_9 \leq 3.5$	10
II	(32)	24.829	34.928	$0.4 \leq T_9 < 1.4$	15
	(32)	23.166	33.027	$1.4 \leq T_9 \leq 3.5$	10
V	(32)	24.954	34.513	$0.3 \leq T_9 \leq 0.9$	20
	(32)	22.838	32.558	$0.9 < T_9 \leq 3.5$	20
PWZ (Ref. 1)	(32)	25.639	36.023	$0.3 \leq T_9 < 1.0$	15
	(32)	24.318	34.676	$1.0 \leq T_9 \leq 3.5$	15

or to the formula

$$\log_{10}(N_A\langle\sigma v\rangle) = A_1 - \log_{10}\{T_9^{2/3}[(T_9 - 0.45)^2 + (0.2)^2]\} - \frac{A_2}{T_9^{1/3}}, \quad (33)$$

where N_A is Avogadro's number. The range of validity of the different formulas used and the accuracy with which they represent the calculated rates are shown in Table II. We do not feel that we can say with confidence which of these approximations to use. However, insofar as the α-particle model explains the intermediate structure *and* the anomalous branching ratio, the optical potential obtained from the α-particle model should be preferred for the extrapolation.

ACKNOWLEDGMENT

We are indebted to Dr. H. Winkler and Dr. W. Stephens for communicating to us experimental results before publication. We are especially thankful to Dr. Stephens for allowing us to use his results ahead of publication. We would like to thank Dr. Len Scherk and Tom Schnackenberg for numerous useful discussions. We would also like to thank Dr. W. A. Fowler and Dr. W. D. Arnett for their comments.

---

*Work supported by Grant No. A-6193 from the National Research Council of Canada.

†Work supported by Grant No. A-3207 from the National Research Council of Canada.

[1]J. R. Patterson, H. Winkler, and C. S. Zaidins, Astrophys. J. 157, 367 (1969).

[2]W. E. Stephens and M. G. Mazarakis, private communication; M. G. Mazarakis and W. E. Stephens, Bull. Am. Phys. Soc. 15, 629 (1970); M. G. Mazarakis, G. O. Debolt, Jr., and W. E. Stephens, *ibid.* 16, 600 (1971).

[3]E. Almqvist, D. A. Bromley, and J. A. Kuehner, Phys. Rev. Letters 4, 515 (1960); E. Almqvist, D. A. Bromley, J. A. Kuehner, and B. Whalen, Phys. Rev. 130, 1140 (1963).

[4]E. Vogt and H. McManus, Phys. Rev. Letters 4, 518 (1960).

[5]K. Wildermuth and R. L. Caravillano, Nucl. Phys. 28, 636 (1961).

[6]A. S. Kompaneits, Zh. Eksperim. i Teor. Fiz. 39, 1713 (1960) [transl.: Soviet Phys.-JETP 12, 1196 (1961)].

[7]R. H. Davis, Phys. Rev. Letters 4, 521 (1960).

[8]B. Imanishi, Nucl. Phys. A125, 33 (1969).

[9]E. W. Vogt, D. McPherson, J. Kuehner, and E. Almqvist, Phys. Rev. 136, B99 (1964).

[10]E. Almqvist, J. Kuehner, D. McPherson, and E. W. Vogt, Phys. Rev. 136, B84 (1964).

[11]J. Borggreen, B. Elbek, and R. B. Leachman, Kgl. Danske Videnskab. Selskab, Mat.-Fys. Medd. 34, No. 9 (1964).

[12]J. P. Bondorf and R. B. Leachman, Kgl. Danske Videnskab. Selskab, Mat.-Fys. Medd. 34, No. 10 (1964).

[13]E. W. Vogt, in *Advances in Nuclear Physics*, edited by M. Baranger and E. W. Vogt (Plenum, New York, 1968), Vol. 1, p. 261.

[14]K. A. Brueckner, J. R. Buckler, and M. M. Kelly, Phys. Rev. 173, 944 (1968).

[15]K. Pruess and W. Greiner, Phys. Letters 33B, 197 (1970).

[16]S. A. Afzal, A. A. Z. Ahmad, and S. Ali, Rev. Mod. Phys. 41, 247 (1969).

[17]P. Bhargava and E. W. Vogt, to be published.

[18]G. Michaud, L. Scherk, and E. W. Vogt, Phys. Rev. C 1, 864 (1970).

[19]E. W. Vogt, Rev. Mod. Phys. 34, 723 (1962).

[20]M. A. Preston, *Physics of the Nucleus* (Addison-Wesley, Reading, Mass., 1962).

[21]H. Reeves, Astrophys. J. 146, 447 (1966).

[22]W. Reilly, A. Gobbi, R. Wieland, L. Chua, D. Shapira, M. Sachs, and D. A. Bromley, Bull. Am. Phys. Soc. 16, 100 (1971).

[23]In a square well without barriers the level spacing is proportional to the square root of the well depth. The presence of barriers and the accompanying Thomas-Ehrmann (Ref. 24) level shifts make the spacing depend less strongly on well depth for our case; still the deeper wells in our problem inexorably have the larger level spacing shown which makes it difficult for them to fit the data.

[24]R. G. Thomas, Phys. Rev. 81, 148 (1951).

[25]W. Hauser and H. Feshbach, Phys. Rev. 87, 366 (1952).

[26]Note that we correct an error of Ref. 13 where parameters for the real part of the potentials of protons and neutrons had been inverted in the calculations.

[27]B. Z. Kozlovsky, Astrophys. Space Sci. 8, 114 (1970).

[28]H. Horiuchi and K. Ikeda, Progr. Theoret. Phys. (Kyoto) 40, 277 (1968).

[29]C. Van der Leun, D. M. Sheppard, and P. J. M. Smulders, Phys. Letters 18, 134 (1965).

[30]It would be possible, alternatively, to consider other decompositions at this stage. For example, we could retain one ^{12}C nucleus as a whole nucleus and decompose the other into three α particles. Although such an alternative view is closer to our view of the actual reaction we prefer, for heuristic reasons, to take both ^{12}C nuclei as being decomposed into α particles.

[31]G. Michaud and E. W. Vogt, Phys. Letters 30B, 85 (1969); E. Vogt, in *Proceedings of the International Conference on Properties of Nuclear States, Montréal, Canada, 1969,* edited by M. Harvey, R. Y. Cusson, J. S. Geiger, and J. M. Pearson (Presses de l'Université de Montréal, Montréal, Canada, 1969), p. 5.

[32]W. D. Arnett, Astrophys. J. 162, 349 (1970).

[33]W. A. Fowler and F. Hoyle, Astrophys. J. Suppl. 9, 201 (1964).

[34]G. Beaudet, E. E. Salpeter, Astrophys. J. 155, 203 (1969); W. D. Arnett, Astrophys. Space Sci. 5, 180 (1969); W. D. Arnett and J. W. Truran, Astrophys. J. 157, 339 (1969).