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Level Structure of ¹⁷⁸Hf

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The level structure of ¹⁷⁸Hf has been studied via γ singles, $\gamma - \gamma$ coincidences in the 2048 ×4096 channel mode, and $\gamma - \gamma$ directional-correlation measurements with Ge(Li) detectors from the decay of ¹⁷⁸Ta. These data yielded 7 new transitions and the addition of 13 transitions to the decay scheme, which includes 5 new levels. The following levels were observed (energies in keV followed by the spin and parity): 93.13, 2⁺; 306.52, 4⁺; 1174.64, 2⁺; 1199.24, 0⁺; 1276.54, 2⁺; 1309.91, 1[±], 2⁺; 1362.36, 2⁽⁻⁾; 1433.97, 0⁺; 1443.86, 0⁺; 1496.02, 2⁺; 1513.64, 1[±], 2⁺; 1561.27, 2⁺; 1566.45, 1[±], 2[±]; and 1771.95, 0⁺. $\gamma - \gamma$ directional correlations were measured with the 2⁺ \rightarrow 0⁺, 93.13-keV transition and eight transitions that feed the first excited state. From the $\gamma - \gamma(\theta)$ data, spins of 2 and 0 are established for the 1362.36-and 1771.95-keV levels and M1 admixtures of 85.6 $\frac{+1}{2}$, $\frac{2}{2}$ and 65.2±3.2% for the 1183.40- and 1402.87-keV level cannot be brought into agreement with theory for a single Z_β bandmixing parameter; while for the $K^{\pi}I = 0^+2$, 1496.02-keV level they can be.

I. INTRODUCTION

The Bohr-Mottelson model,¹ even with perturbational corrections for admixing of the β , γ , and ground-state bands,² appears to be unable at present to explain the branching ratios³⁻⁵ from the β vibrational states in ¹⁵²Sm, ¹⁵⁴Gd, and ¹⁵⁶Gd. The earlier proposal^{4,6} that M1 admixtures in the $\Delta I = 0$ transitions between the β and ground bands may prove the way to bring theory and experiment into agreement has not been found true^{7,8} for ¹⁵²Sm and ¹⁵⁴Gd. These two nuclei are often referred to as transitional nuclei, since they are just at the beginning of the region where permanent deformation is observed and are not highly deformed. When the effects of band mixing are included, the Bohr-Mottelson model has adequately described⁶ the transitions from γ bands in highly deformed nuclei and is in near, though not exact, agreement with experiments³ on the branching ratios from the γ bands in ¹⁵²Sm and ¹⁵⁴Gd. It seems most important that in strongly deformed nuclei we check very carefully the properties of β vibrations, as well as of all near-lying excited states which may present any perturbations to them. The nucleus

¹⁷⁸Hf is an excellent candidate for a detailed study of this problem.

⁴⁷R. L. Becker and M. R. Patterson, to be published.

⁴⁸A. E. S. Green, Science <u>169</u>, 3949 (1970). ⁴⁹F. Riewe, private communication.

The energy levels of ¹⁷⁸Hf populated by the decay of ¹⁷⁸Ta have been studied by Nielsen *et al.*⁹ who observed nine excited states. Of particular interest in our present work is a study of the properties of the 0⁺ and 2⁺ levels of ¹⁷⁸Hf at 1199 and 1276 keV, respectively. The strong E0 component observed⁹ in the $2^+ \rightarrow 2^+$, 1183-keV transition from the 1276-keV level to the 93-keV first excited state indicates a K=0 assignment for the 1276-keV level. Guided by this and the energy spacings, Nielsen et al.⁹ proposed that the 1199- and 1276keV levels are the 0^+ and 2^+ members of the β vibrational band. The K=0 character of these levels also has been established by comparison of the experimental branching ratio of the β feed to these levels with the ratio predicted by the intensity rules of Alaga et al.¹⁰ To add further support to their argument, Nielsen et al.⁹ pointed out that the moment of inertia calculated from these two levels differed by only 20% from that calculated for the ground-state rotational band - in keeping with the observed trends.¹¹ A 4^+ member of this proposed β band has been reported in (n, e) work.¹²

To test the possibility that a large M1 component in the 1183-keV $(2^+ \rightarrow 2^+)$ transition in ¹⁷⁸Hf from the proposed β -vibrational state could be giving rise to the observed anomalies in the branching ratios, Nielsen, Nielsen, and Rudd (NNR)¹³ performed a $\gamma - \gamma(\theta)$ experiment to measure the admixture in this transition. Their results indicated an $(86 \pm 10)\%$ M1 component (as calculated from their δ) in the $2^+ \rightarrow 2^+$ transition. For an *M*1 component of the order of 80%, they found that their experimental branching ratios agreed with theory for a $Z_{\beta} = 0.010$. Hence, it would appear that the use of the Bohr-Mottleson theory adequately describes this proposed $2^+ \beta$ -vibrational level and confirms Mottleson's expectation⁶ of the large M1 components.

The $\gamma - \gamma(\theta)$ work of NNR¹³ was carried out with NaI(Tl) detectors, and the earlier decay-scheme work⁹ with NaI(Tl) and small Ge(Li) detectors. Because of the importance of precise information on the properties of β bands in strongly deformed nuclei and on the general level structure of nuclei in this region where quasiparticle excitations can occur in the same energy region as the β and γ vibration, we have carried out a thorough reinvestigation of the decay scheme of ¹⁷⁸Ta in which γ -ray singles, coincidence, and directional-correlation measurements were made with largevolume Ge(Li) detectors. These measurements provided significant new information on the level structure of ¹⁷⁸Hf. Seven new transitions were observed and five levels added to the level scheme, which now includes the 2^+ member of the γ -vibrational band. From more precise measurements of the γ -ray intensities and the E2/M1 admixtures in $\Delta I = 0$ transitions, it is shown that the branching ratios of the earlier proposed β -vibrational band in ¹⁷⁸Hf do not agree with theory even when a large M1 component is included. This latter conclusion was presented in a recent letter.¹⁴ The present paper presents the complete results of our directional-correlation and decay-scheme studies which provide a detailed picture of the level structure in ¹⁷⁸Hf, where the vibrational and quasiparticle excitation are intermixed.

II. EXPERIMENTAL PROCEDURES AND RESULTS

A. Source Preparation

The half-life of ¹⁷⁸Ta is only 9.4 min. However, we have used the parent activity ¹⁷⁸W, which has a 21.5-day half-life and decays by electron capture to the ground state of ¹⁷⁸Ta. Sources were prepared by proton bombardment of 10-mil tantalum foils in the Oak Ridge isochronous cyclotron. The ¹⁷⁸W resulted from the reaction ¹⁸¹Ta(p, 4n)- 178 W. After chemical purification the activity was precipitated as H₂WO₄, and then dissolved in a minimum of NaOH. Aliquots of this solution were used for the sources.

B. γ-Ray Energies

Figures 1 and 2 show the γ -ray singles spectrum of ¹⁷⁸Hf. A large-volume ORTEC detector which had 2.4-keV full width at half maximum (FWHM) system resolution and an efficiency of 10.7% relative to a 3-in.×3-in. NaI detector (at 25 cm) for 1.3-MeV γ rays was used in conjuction with the ND-3300, 4096-channel analyzer system for these measurements.

No transitions were observed in the energy region 511-970 keV. The method of internal standards was used to obtain the energies of the transitions in ¹⁷⁸Hf. Two separate calibration runs were taken: one to calibrate the relatively intense high-energy transitions, and one to calibrate the relatively intense low-energy transitions. After calibrating the intense lines, these in turn were used to obtain the energies of the weaker transitions in two additional runs.

Computer programs¹⁵ were used in the calibration procedure. The energies so obtained are given in Table I. Also, given for comparison, are the energies of Ref. 9.

C. y-Ray Intensities

The intensities were obtained from measurements taken with the same detector previously described and are given in Table I. Corrections were made for the room-background spectrum which was measured for an equal length of time. Periodic half-life checks over 60 days were made to ensure that the newly reported transitions of Table I were not the result of source contaminants. It was observed that, within experimental error limits, (5-15%), the relative decay rates of these new lines were the same as that of the stronger transitions.

Careful checks were made to ensure that the newly observed 1174.63-keV transition was not a miscalibrated ⁶⁰Co line. In addition the region surrounding a possible 1332-keV transition was studied to ensure a ⁶⁰Co contaminant-free source.

Three methods were used to determine the relative intensities of these transitions as carefully as possible. The data were hand-plotted and analyzed with a planimeter, then the data were analyzed by computer programs,¹⁵ and finally a channel-bychannel summation technique with background subtraction was used. These methods all gave internally consistent results.

Special emphasis was placed on the intensity



FIG. 1. The low-energy γ -ray spectrum in ¹⁷⁸Hf.



FIG. 2. The high-energy γ -ray spectrum in ¹⁷⁸Hf. The parentheses on an energy indicates where a transition expected on the basis of the decay scheme, such as the $2^+_{\gamma} \rightarrow 4^+$, 868.12-keV transition, would occur.

determination of the 1183.40-, 1189.50-, and 1276.56-keV transitions. It is crucial to obtain these intensities with small error limits in order to obtain good branching ratios for comparison with theory. First consider the 1183.40-keV transition. The Compton edge of the 1402.87-keV transition occurs in the region of 1186 keV. The occurrence of a 1189.50-keV transition masks this Compton edge, so that one might construct a linear background under the 1183-1189-keV complex and hence increase the 1183.40-keV photopeak area. Such an increase in intensity would necessitate a larger M1 component to obtain consistent branching ratios. Figure 3 shows this Compton edge under the 1183.40-keV transition. This Compton edge was constructed by projecting the background from either side of the peak to the channel which corresponds to the energy of the Compton edge. Also shown in Fig. 3 is the newly reported 1174.63keV transition.

In Fig. 4 is shown the Compton edge of the 1496.05-keV transition at 1277.7 keV as it lies under the 1276.56-keV photopeak. This back-ground was constructed in the same way as was that for the 1183.40-keV peak.

D. Coincidence Measurements

A 2048 × 4096-channel, γ - γ coincidence experi-

Energies (keV)			Relative inten	sities	
Present work	(Ref. 9) ^a	Spins and parities	Present work	(Ref. 9) ^b	Multipolarities
93.13 ± 0.08	93.2	$2^+ \rightarrow 0^+$	с	1372	E2
151.28 ± 0.14 ^{d,e}	• • •	$2^+ \rightarrow 2^+$	1.0 ± 0.2	• • •	E2 + M1 + E0
203.73 ± 0.12 ^{d,e}	204	$1^{\pm}, 2^{+} \rightarrow 1^{\pm}, 2^{+}$	1.4 ± 0.7		
204.09 ± 0.18 ^{d,e}	204	$1^{\pm}, 2^{\pm} \rightarrow 2^{(-)}$	0.8 ± 0.4	1.86	
$\textbf{213.39} \pm \textbf{0.06}$	213.7	$4^+ \rightarrow 2^+$	20.68 ± 0.77	18.60	E2
$\textbf{256.54} \pm \textbf{0.16}$	•••	$1^{\pm}, 2^{\pm} \rightarrow 1^{\pm}, 2^{+}$	0.7 ± 0.14	•••	
269.41 ± 0.12	270	$0^+ \rightarrow 2^+$	4.66 ± 0.21	3.49	E2
511	511	Annihilation radiation	436 ±13	488	•••
970.02 ± 0.05	969	$2^+ \rightarrow 4^+$	11.10 ± 0.47	11.63	E2
$\boldsymbol{1081.52 \pm 0.07}$	•••	$2^+ \rightarrow 2^+$	3.73 ± 0.18	•••	E2
1106.11 ± 0.07	1106.2	$0^+ \rightarrow 2^+$	110.7 ± 3.5	102.3	E2
$\boldsymbol{1174.63 \pm 0.07}$	•••	$2^+ \rightarrow 0^+$	3.27 ± 0.18	•••	E2
1183.40 ± 0.07	1182.6	$2^+ \rightarrow 2^+$	34.50 ± 1.4	25.58	E2 + M1 + E0
$\textbf{1189.50} \pm \textbf{0.11}$	1189	$2^{+-} 4^{+}$	5.66 ± 0.15	4.65	E2
$\textbf{1199.24} \pm \textbf{0.11}^{\text{ d}}$	1199	$0^+ \rightarrow 0^+$		<3.02	E0
$\boldsymbol{1216.79 \pm 0.12}$	• • •	$1^{\pm}, 2^{+} \rightarrow 2^{+}$	$\textbf{1.38} \pm \textbf{0.08}$	• • •	
1254.72 ± 0.13	1255	$2^+ \rightarrow 4^+$	7.21 ± 0.27	6.98	E2
1269.23 ± 0.08	1270	$2^+ \rightarrow 2^+$	6.08 ± 0.24	4.65	
1276.56 ± 0.08	1276	$2^+ \rightarrow 0^+$	6.96 ± 0.25	4.65	E2
$\textbf{1309.90} \pm \textbf{0.08}$	1310	$1^{\pm}, 2^{+} \rightarrow 0^{+}$	8.58 ± 0.33	9.30	
1340.84 ± 0.09	1341.0	$0^+ \rightarrow 2^+$	212.6 ± 6.8	221	E2
1350.55 ± 0.09	1350.7	$0^+ \rightarrow 2^+$	244.1 ± 7.6	256	E2
1402.87 ± 0.09	1403	$2^+ \rightarrow 2^+$	100 ± 3.1	100	E2 + M1 + E0
1420.52 ± 0.11	1422	$2^+ \rightarrow 2^+$	0.88 ± 0.06	0.93	E2 + M1 + E0
1433.97 ± 0.12 d	1433	$0^+ \rightarrow 0^+$	с	<0.35	E0
1443.86 ± 0.09 d	1444	$0^+ \rightarrow 0^+$	с	<0.35	E0
1468.13 ± 0.12	1468	$2^+ \rightarrow 2^+$	2.69 ± 0.10	3.49	$(E2 + M1 + E0)^{f}$
$\boldsymbol{1473.32 \pm 0.12}$	•••	$1^{\pm}, 2^{\pm} \rightarrow 2^{+}$	0.65 ± 0.05	•••	. ,
1496.05 ± 0.11	1496	$2^+ \rightarrow 0^+$	56.3 ±1.8	55.8	E2
$\boldsymbol{1513.63 \pm 0.12}$	1515	$2^+ \rightarrow 0^+$	0.94 ± 0.05	1.16	E2
1561.30 ± 0.13	1561	$2^+ \rightarrow 0^+$	2.18 ± 0.10	2.09	E2
1678.82 ± 0.18	1677	$0^+ \rightarrow 2^+$	0.70 ± 0.07	0.70	E2

TABLE I. Energies and intensities of transitions in ¹⁷⁸Hf.

^a Error limits were quoted as ± 0.5 keV for the stronger lines.

^b Relative uncertainty was given as $\pm 20\%$.

^c Not measured in this work.

^d These energies were obtained from differences in level energies.

^e These transitions were placed between their respective levels from coincidence data. Their intensities were calculated from coincidence gates.

^f The E0 and M1 admixtures are not definitely established.

ment was taken on a ND-3300 system with buffertape storage. The Nuclear Diodes and ORTEC (coaxial) Ge(Li) detectors used for this experiment had efficiencies of 3.5 and 10.7%, respectively, (for a 1.3-MeV γ ray) compared with that of a 3-in.×3-in. NaI detector at 25 cm. Each detector was surrounded by a heavy lead shield lined with cadmium and copper. Each shield was milled in such a geometry as to allow detection of only those γ rays emitted into the solid angle subtended by the detector at a point 5 cm from the detector. The two detectors were placed in a 90° geometry and the source placed at the intersection of the extended axes of the two crystals.

The resolving time for the coincidence unit and the source strength were chosen to give a true-tochance ratio of about 15:1. The coincidence efficiency of the system was essentially unity for a 270-keV transition in coincidence with a 1310-keV one, while it was about 0.9 for a 150-keV transition in coincidence with a 1270-keV one. Data collected for seven days were stored on three 2400-ft magnetic tapes. From the directionalcorrelation experiments to be described next, the summed three-angle data also provided a gate on the 93.13-keV transition, but with much better statistics.

Note that the lowest-energy transition above 270 keV is at 970.02 keV. With a Q value of 1912 keV, this indicates that any transition above 970 keV, and not in coincidence with the 93.13-keV γ ray, must feed the ground state.

The spectra coincident with the following gating energies were extracted from the coincidence experiment: 93.13, 203.73, 204.09, 213.39, 256.54, 269.41, 970.02, 1081.52, 1106.11, 1174.63, 1183.40, 1189.50, 1216.79, 1254.72, 1276.56, 1309.90, 1340.84, 1350.55, 1402.87, 1496.05, and 1561.30 keV. Selected gates used to establish new levels and new transitions and to establish and resolve the 204-keV doublet are shown in Figs. 5-8. A convenient summary of the coincidence information from all the gates is given in Table II.

E. γ-γ Directional-Correlation Measurements

1. $\gamma - \gamma(\theta)$ System and Procedures

The system consists of a fixed Ge(Li) detector with a movable NaI(Tl) detector as used in previous studies.⁷ Two different Ge(Li) detectors



FIG. 3. An expanded view of the 1183.40-keV transition and the Compton edge at 1186 keV.

were used in two different experiments carried out several months apart. The detector used in the first experiment had a peak-to-Compton ratio of 16:1 and a resolution of 2.8 keV FWHM at 1.3 MeV. Its efficiency was 3.8% relative to a 3-in. \times 3-in. NaI(Tl) detector at 1.3 MeV. The detector used in the second experiment had a 28:1 peakto-Compton ratio, a resolution of 2.5 keV FWHM, and an efficiency of 10% under the same conditions. Further details of the setup are described in the literature.¹⁶ With dual analog-to-digital converters (ADC) in the 4096-channel analyzer system, a routing circuit allowed two gates to be selected from the NaI detector and data in coincidence with each gate to be stored in each half of the 4096channel memory. A biased amplifier was used to eliminate events below 500 keV from the Ge(Li) detector. A graded lead-cadmium-copper absorber was placed over the Ge(Li) detector to cut out low-energy γ rays (70% absorption at 125 keV and 4% at 700 keV). The 2-in.×2-in. NaI(Tl) detector had a 0.05-cm cadmium absorber in front of it to reduce the γ rays and a lead cone to reduce crystal-to-crystal scattering. Figure 9 shows the spectrum from the NaI detector. The arrows indicate the gate settings on the transition of interest at 93 keV $(2_{e}^{+} \rightarrow 0_{e}^{+})$. A second gate of the same width was set with its lower edge about 120 keV to obtain the contribution from Compton events under the 93-keV peak. This was a very small correction in this experiment. The resolving time of the coincidence circuit in the first and second experiments was 58 and 71 nsec, respectively.

The correlations with gates set on the 93-keV peak and on the Compton background just above it were measured simultaneously. The first of the two separate experiments was carried out for 14 cycles and the second for 7 cycles. Each threeday cycle consisted of a 23-h running time at 90, 135, and 180° for a total of 42 days in the first measurement, and for a total of 21 days with the larger detector. Alternation of the sequence of angles through 90, 135, 180; 180, 135, and 90° compensated for the source decay to better than 1%. The NaI gate counts recorded for each 23-hperiod were constant to better than 1%. These counts were used to normalize the coincidence results and coupled with the measured 2τ , and the Ge(Li) singles spectra taken between each run were also used to calculate chance corrections for both the 93-keV and background correlations. The true-to-chance ratio varied from 9 to 25.

Since the 93-keV state has a relatively long halflife ($T_{1/2} = 1.50$ nsec), correlations involving it are attenuated. The attenuation corrections Q_2G_2 and Q_4G_4 for the Ge(Li) and NaI detectors were obtained from known correlations in the decay. There are three 0⁺ states⁹⁻¹¹ with prominent transitions to the 2⁺ level of the ground-state rotational band. Cascades with 0-2-0 spins have unique correla-

								γ	tra	nsit	ions	(ke	V)											
Gates (keV)	93.13	151.28	203.73	204.09	213.39	256.54	269.41	970.02	1081.52	1106.11	1174.63	1183.40	1189.50	1216.79	1254.72	1269.23	1309.90	1340.84	1350.55	1402.87	1420.52	1468.13	1473.32	1678.82
$\begin{array}{c} 93.13\\ 203.73\\ 204.09\\ 213.39\\ 256.54\\ 269.41\\ 970.02\\ 1081.52\\ 1106.11\\ 1174.63\\ 1183.40\\ 1189.50\\ 1216.79\\ 1254.72\\ 1269.23\\ 1309.90\\ 1340.84\\ \end{array}$	x x x x x x x x x x x x x x x x x	x x	x x x	x	x x x x	x x x	x x x	x	x	x	x	x	x	x x x	x	x x	x	x	x	x	x	x	x	x
1350.55	x x																							

TABLE II. Summary of the coincidence experiments on ¹⁷⁸Hf. No transitions were observed in coincidence with the 1276.56-, 1496.05-, and 1561.30-keV transitions. The x indicates a coincidence relationship.

tion coefficients $A_2 = 0.357$ and $A_4 = 1.14$. Therefore, from the experimental $Q_2G_2A_2$ and $Q_4G_4A_4$ coefficients of these cascades, we obtain Q_2G_2 and Q_4G_4 for each run.

2. $\gamma - \gamma(\theta)$ Results

(a) First experiment. The results of the first directional-correlation experiment are summarized Table III. Note the excellent consistency of the uncorrected coefficients for the three strong 0-2-0 cascades. From a straight average of these results we obtain, $G_2Q_2 = 0.510 \pm 0.014$ and $G_4Q_4 = 0.442 \pm 0.004$. An example of the data in the first experiment is given in Fig. 10, which shows the summed data at each angle for the 1183.40-93.13-keV, $2^+-2^+-0^+$ cascade of particular interest. The Compton edge was fitted by the same procedure as described in the singles work. Note that the Compton edge is rather pronounced in these data taken with the smaller detector. The A_2 and A_4 values of NNR¹³ are given for comparison.

Our data yield for the 1183.40-keV transition a mixing ratio of $\delta = 0.38 \pm 0.08$, which corresponds to an *M*1 component of $(87.4^{+4.4}_{-5.4})\%$. These results are to be compared with those of NNR¹³ $\delta = 0.41 \pm 0.17$ and an *M*1 admixture of $(86 \pm 10)\%$. From

our γ -ray singles work, a $(73.0 \pm 1.3)\%$ M1 component is required in the 1183.4-keV transition to bring the branching ratios into agreement for a single value of the mixing parameter as discussed in the last section. It is particularly important to know if the M1 admixtures obtained in the two ways are not the same. Thus the experiment was repeated under improved conditions as described below.

In addition to the 2⁺ level at 1276.54 keV there are also 2⁺ levels at 1496.02 and 1561.27 keV. These levels decay to the 2⁺ member of the groundstate band via 1402.87- and 1468.13-keV transitions, respectively. While inadequate statistics prevented a determination of the admixtures in the 1468.13-keV transition, the A_K coefficients of the 1402.87-keV transition were reliably measured and these are shown in Table III. A discussion of the results of this correlation is deferred until the results of the second experiment are presented.

(b) Second experiment. With the measured value of $(87.4^{+4.4}_{-5.4})\%$ for the 1183.40-keV, $2^+ - 2^+$ transition, we are within 2.4 standard deviations of the $(73.0 \pm 1.3)\%$ M1 component required in this transition to obtain consistent branching ratios with a single band-mixing parameter. Since NNR¹³ reported finding agreement between theory and ex-



FIG. 4. An expanded view of the 1276.56-keV transition and the Compton edge underneath it.

periment, one might raise the possibility that our measurement was indeed merely a statistical deviation from the actual value. Still, under the assumption that the 1276.54-keV, 2⁺ level is a β -vibrational level, and since Mottelson⁶ has emphasized the importance of measuring the admixtures in such transitions, it was felt that a repeat of our γ - $\gamma(\theta)$ experiment would be worthwhile. The second experiment was set up identically (source-to-detector distance, shield, and other features) to the first with the exception of the substitution of a larger-volume Ge(Li) detector.

Again, after correction for chance and Compton background events, the combined data at each angle were used to calculate the raw A_K coefficients. As with the first experiment, excellent agreement was obtained for the A_K coefficients of three $0 \rightarrow 2 \rightarrow 0$ cascades as is shown in Table IV. From the average of these A_K values we obtain correction coefficients: $G_2Q_2 = 0.700 \pm 0.011$ and $G_4Q_4 = 0.505 \pm 0.004$.

In Fig. 11 we have the three-angle data from the second experiment for the 1183.40-93.13-keV $2^+ \rightarrow 2^+ \rightarrow 0^+$ correlation. The background shown is that calculated by the computer for a least-squares fit of a first-degree polynomial to the background points at the left of the peak. The background is lower, and the 1186-keV Compton edge is smaller than in the first experiment. The corrected coefficients one obtains from these data are: A_2 $= -0.064 \pm 0.028$ and $A_4 = 0.063 \pm 0.043$. Little deviation is observed now in these coefficients when this reduced Compton edge is not taken into account, since the peak-to-Compton ratio for the second detector is so much better than the first. These yield a $\delta = (0.425^{+0.060}_{-0.070})$ and an *M*1 admixture of $(84.5^{+3.5}_{-4.0})\%$.

A weighted average of the results of the two experiments yielded $A_2 = -0.054 \pm 0.023$, $A_4 = 0.041 \pm 0.033$. From a comparison with theory, Fig. 12,



FIG. 5. Spectra in coincidence with the 213.39- and 256.54-keV transitions. The low points followed by high points around channels 880, 1120, and 1160 arise from Compton scattering between the crystals and the fact that the Compton background is subtracted by using points just below the peak.

In addition, the results of our analysis of the 1402.87-keV transition are given in Table IV. The averages of the A_2 and A_4 coefficients from the two experiments are given in Table V for the 1402.87-93.13-keV cascade. These yield a $\delta = -(0.73 \pm 0.05)$ and an *M*1 admixture of $(65.2 \pm 3.2)\%$ for the 1402.87-keV transition. This admixture is slightly

larger than that earlier reported¹⁴ (61.0%) which was obtained from the second experiment only.

The results of all of the correlations measured in ¹⁷⁸Hf are given in Table V, where the δ values are in the notation of Ref. 17. The results of the first and second experiments were averaged for the two stronger cascades, but for the three weak cascades the statistical accuracy was not sufficient for meaningful results in the first experiment with the small detector. Thus for the weak cas-



FIG. 6. Spectra in coincidence with the 269.41-, 1081.52-, and 1174.63-keV transitions.

cades only the results with the second detector are reported.

III. ¹⁷⁸Hf LEVELS POPULATED BY ¹⁷⁸Ta

A. Introduction

The coincidence relations of Table II were used to establish the levels in ¹⁷⁸Hf populated by ¹⁷⁸Ta. The placement of every transition and every level is based on coincidence information. Table VI gives the energy-sum relations of the level observed. The excellent agreement between the sum and crossover energies indicates that the errors assigned to the γ -ray energies are conservative. The ¹⁷⁸Ta decay scheme is shown in Fig. 13, and each of the levels is discussed below. For four of the levels only spins and parities can be assigned, so a *K* assignment is not included for these in Fig. 13.

B. Ground-State Rotational Band

The energy levels at 93.13 ± 0.08 and 306.52 ± 0.10 keV are the known⁹ respective 2^+ and 4^+ members of the ground-state rotational band, and are depopulated by the 93.13-keV $(2^+ \rightarrow 0^+)$ and 213.39-keV $(4^+ \rightarrow 2^+)$ transitions, respectively.

C. 1174.64 \pm 0.06-keV Level, $K^{\pi}I = 2^{+}2$

The level at 1174.64-keV is observed for the first time in radioactive decay. It deexcites by the emission of two newly observed transitions to the ground-state rotational band. From coincidence data, the 1081.52-keV transition feeds the 2^+ member of the ground-state rotational band, while the 1174.63-keV one is a ground-state transition. To add further support to this level, a





FIG. 7. Spectra in coincidence with the 203.73- and 204.09-keV gates.

Energies (keV)	Spin sequence	$Q_2G_2A_2$	$Q_4G_4A_4$	A_2	A_4
1106-93	0-2-0	0.180 ± 0.011	0.496 ± 0.011		
1341-93	0-2-0	0.178 ± 0.008	0.508 ± 0.009		
1351-93	0-2-0	0.187 ± 0.007	0.508 ± 0.007		
Average above	e of three	0.182 ± 0.005^{a}	0.504 ± 0.005 ^a		
1183-93	2-2-0	-0.018 ± 0.028	0.023 ± 0.029	-0.031 ± 0.041	0.011 ± 0.051
1403-93	2-2-0	0.283 ± 0.011	0.038 ± 0.010	-0.056 ± 0.051^{b} 0.55 ± 0.03 0.415 ± 0.035^{b}	-0.025 ± 0.057^{b} 0.087 \pm 0.023 0.122 \pm 0.040^{b}

TABLE III. Directional-correlation coefficients of cascades in "Hi (results of first experiment	TABLE III.	Directional-correlation	coefficients of	f cascades f	in ¹⁷⁸ Hf	(results of	first ex	periment)
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 $^{a}G_{2}Q_{2} = 0.510 \pm 0.014$ and $G_{4}Q_{4} = 0.442 \pm 0.004$.

^b Data from Ref. 13. The errors include statistical ones only. When systematic effects are taken into account, the errors on δ are increased a factor of 2 over that obtained from the statistical errors.

269.41-keV transition, which depopulates a level at 1443.85 keV, is found in coincidence with the 1174.63- and 1081.62-keV gating transitions. Likewise, the 1174.63- and 1081.52-keV transitions are found to be coincident with the 269.41-keV γ ray as the gating transition.

A level at 1175.0 keV has been reported from Coulomb-excitation studies,¹⁸ (n, e) work,¹² (n, γ) work,¹⁹ and deuteron-induced reactions.²⁰ On the basis of the branching ratios and B(E2) value, this level is assigned¹⁸ as the 2⁺ band head of the γ vibrational band. Our more precise data yield an energy of 1174.64 keV for the 2⁺ level. The branching ratio calculated from the intensities of the 1081.52-keV, 2⁺ \rightarrow 2⁺ and the 1174.63-keV, 2⁺ \rightarrow 0⁺ transitions agree with the result of Varnell, Hamilton, and Robinson (VHR).¹⁸

For the 1081.52-keV transition VHR¹⁸ found a δ value of 0.6 > 32, or < -11. Our directionalcorrelation experiment allows only the values δ > 32 or δ < -11 of VHR.¹⁸ Hence the *E*2 component in the 1081.52-keV, 2⁺ + 2⁺ transition is >99%.

An upper limit on the intensity of a possible 868.12-keV, $2^+ \rightarrow 4^+$ transition from the 1174.64-keV level is placed at 0.16 relative to 100 for the 1402.87-keV transition.

D. 1199.24 ± 0.11-keV Level, $K^{\pi}/=0^{+}0$

The level at 1199.24 keV has been proposed⁹ as the 0^+ band head of a β -vibrational band. It is de-

populated by a strong $0^+ \rightarrow 2^+$ (pure *E*2) 1106.11-keV transition and a pure *E*0 ground-state transition. Nielsen *et al.*⁹ have calculated the branching ratio of the β rays feeding this state and the 2^+ state at 1276.54 keV, and found agreement with a $K^{\pi} = 0^+$ assignment. The parity assignment comes from the observation of an *E*0 transition to the ground state. The *E*0 transition also confirms the K=0 assignment.

E. 1276.54 ± 0.06 -keV Level, $K^{\pi}I = 0^{+}2$

This 2⁺ level at 1276.54 keV is depopulated by the 970.02-keV $(2^+ \rightarrow 4^+)$, 1183.40-keV $(2^+ \rightarrow 2^+)$, and 1276.56-keV $(2^+ \rightarrow 0^+)$ transitions. The coincidence spectra confirm that these transitions are not doublets and are correctly placed. The strong electron conversion of the 1183.40-keV transition is explained by a large E0 admixture.⁹ This E0 component and the energy spacing of this and the 1199.24-keV level are the basis on which Nielsen et al.⁹ proposed that the 1199.24- and 1276.54-keV levels are the members of a onephonon β -vibrational band. As discussed later, our reduced B(E2) ratios from the 1276.54-keV level are consistent with a single parameter Z_{β} fit to the data when a $(73.0 \pm 1.3)\%$ M1 component is assumed in the 1183.40-keV transition. Our $\gamma - \gamma(\theta)$ measurements indicate a $(85.6^{+1.1}_{-2.1})\% M1$ component. Thus, there is a clear discrepancy on this point.



FIG. 8. Spectra in coincidence with the 1269.23-, 1217.79-, and 1309.90-keV transitions.

Since β -vibrational states arise from collective excitations, one would expect to see the 1276.54keV state strongly populated in Coulomb-excitation studies with α particles. The $B(E2)_{exc}$ is less than 0.07 single-particle units, ¹⁸ which fact indicates that this level is not collective in nature. The lack of strong population of this level in Coulomb excitation might be explained by outof-phase admixing of these levels with an equal component from the ground-state band. But such large admixing would require a Z_{β} value too large to be treated by perturbation theory. The conclusion, then, is that the proposal⁹ that the 1199.24and 1276.54-keV levels are the members of a rotational band based on a β vibration is probably incorrect, though the $K^{\pi} = 0^+$ assignment still holds.

F. 1309.91 ± 0.08-keV Level, $I^{\pi} = 1^{\pm}, 2^{+}$

Although a transition of this energy had been reported previously, it had not been placed in the decay scheme. By summing together the threeangle data from the directional-correlation experiment, we find that the 1309.90-keV transition must go to the ground state. To add further support to this level, we observe a 1216.79-keV transition, which is seen from the coincidence spectra to feed the 93.13-keV level.

In addition, we observe the 1309.90- and 1216.79keV transitions in the 256.54-keV gate. The newly observed 256.54-keV transition is proposed to depopulate a new level at 1566.45 keV. Also, this new transition is seen in the 1309.90- and 1216.79keV gates, as is a 203.73-keV transition; so both of these new levels are confirmed. The spectrum coincident with the 203.73-keV gate also shows the 1309.90- and 1216.79-keV transitions. This 203.73-keV γ ray depopulates the level at 1513.64 keV. The accurate energies of the 256.54- and 203.73-keV transitions were obtained from differences in level energies.

Since the intensity of the 1309.90-keV ground-



FIG. 9. The NaI(Tl) γ -ray spectrum to show the gate in the γ - $\gamma(\theta)$ experiments. The arrows indicate the window settings.

state transition is a factor of 6.2 larger than that of the 1216.79-keV one, the spin and parity of the 1309.91-keV level are limited to 1^{\pm} , 2^{\pm} . The branching ratio from this level was calculated but did not compare well with any of the predictions made by Alaga's intensity rules, ¹⁰ even with oneparameter band mixing. The log *ft* value of 6.8 to this does not further restrict the spin.

If the spin and parity of the 1309.91-keV level is 2^+ , then one might expect to see a 1003.38-keV, $2^+ - 4^+$ transition from this level. An upper limit of 0.11 units (relative to 100 for the 1402.87-keV

Energies (keV)	Spin sequence	$Q_2G_2A_2$	$Q_4G_4A_4$	A_2	A_4
1106-93	0-2-0	0.245 ± 0.011	0.577 ± 0.010		
1341-93	0-2-0	0.253 ± 0.006	0.572 ± 0.007		
1351-93	0-2-0	$\textbf{0.252} \pm \textbf{0.006}$	0.580 ± 0.006		
Averag above	e of e three	0.250 ± 0.004 ^a	0.577 ± 0.005^{a}		
1183-93	2-2-0	-0.045 ± 0.020	0.032 ± 0.022	-0.064 ± 0.028	0.063 ± 0.043
1403-93	2-2-0	0.346 ± 0.009	0.072 ± 0.010	0.494 ± 0.015	0.143 ± 0.020

TABLE IV. Directional-correlation coefficients of cascades in ¹⁷⁸Hf (results of second experiment).

^a Coupled with the theoretical values of 0.357 and 1.14 for A_2 and A_4 these numbers give $Q_2G_2 = 0.700 \pm 0.011$ and $Q_4G_4 = 0.505 \pm 0.004$.

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transition) is placed on the intensity of such a transition.

G. 1362.36 ± 0.09 -keV Level, $I^{\pi} = 2^{(-)}$

Evidence for a level at 1362.36 keV is found when one observes that the 1269.23-keV transition is in coincidence with the 93.13-keV transition, but is not in coincidence with the 213.39-keV one. In further support of this level, we find 151.28- and 204.09-keV transitions in the spectrum coincident with the 1269.23-keV gate. The 151.28-keV γ ray depopulates a level at 1513.64 keV. Thus, the 1362.36-keV level is firmly established.

Other possible transitions out of this level, if existent, have extremely small intensities. The possible 1362.36-keV ground-state transition is at least 50 times weaker than the 1269.23-keV one; while a 1055.84-keV transition to the 4⁺ state is at least 15 times weaker than the 1269.23-keV one. A 0⁺ or 2⁻ state would exhibit the properties mentioned above of decaying only to the 93.13-keV 2⁺ level. From our $\gamma - \gamma(\theta)$ work on the 1269.23-93.13-keV correlation, we were able to determine the spin of the 1362.36-keV level to be 2. With 2 standard deviations allowed on the A_K coefficients (96% confidence limits), the corresponding limits placed on the mixing ratio are $0 \le \delta \le 2.25$, which yield only an upper limit of 83.5% for the quadrupole radiation. Hence, our directional-correlation data do not help in determining the parity of the 1362.36-keV level. An odd-parity assignment for the 1362.36-keV level would indicate E1 radiation for the 1269.23-keV transition, and require the 1362.36- and 1055.84-keV transitions to be M2radiations. Since these multipolarities could explain the observed characteristics of the branching from this level, a negative-parity assignment is tentatively given to the 1362.36-keV level.

H. 1433.97 ± 0.12 -keV Level, $K^{\pi}I = 0^{+}0$

The 1433.97-keV level is a well-established 0^+ level which deexcites via the intense *E*2, 1340.84-keV transition to the 2^+ member of the ground-state band, and by the emission of pure *E*0 conversion electrons to the ground state.

I. 1443.86 ± 0.09-keV Level, $K^{\pi}I = 0^{+}2$

Like the 1433.97-keV level, the 1443.86-keV level is a known 0⁺ state. It depopulates by an intense *E*2, 1350.55-keV transition and by *E*0 decay to the ground state. Although the β -ray feeding to these two 0⁺ levels is almost equal in strength, the observed *E*0 intensities from them differ considerably.⁹ This is indicated by the *X* values,²¹ where *X* is a measure of the branching ratio of



FIG. 10. The sum of the three-angle data for the 1183-93-keV γ - γ correlation from the first experiment. This measurement was made with the 3.8%-efficient Ge(Li) detector.

E0 to E2 depopulation of a given level. For the 1433.99-keV level, X is reported⁹ as 0.10 ± 0.02 , while that of the 1443.86-keV level is 0.38 ± 0.08 .

J. 1496.02 \pm 0.07-keV Level, $K^{\pi}I = 0^{+}2$

The level at 1496.02-keV is well established.⁹ It decays to the 4⁺, 2⁺, and 0⁺ members of the ground-state rotational band via the 1189.50-, 1402.87-, and 1496.05-keV transitions, respectively. The 1402.87-keV transition to the 2⁺, 93.13-keV state has a large E0 admixture. Thus, the spin and parity of this level are 2⁺. The measured M1/E2 mixing ratio in the 2⁺ \rightarrow 2⁺, 1402.87keV transition from this level gives a (65.2 ± 3.2)% M1 component.

This level is only very weakly populated in Coulomb excitation¹⁸ by 15-MeV ⁴He particles. The $B(E2)_{\rm exc}$ value is less than 0.34 single-particle units.¹⁸ This small value suggests that this level does not have the collective character expected for a β -vibrational state.

K. 1513.64 ± 0.09-keV Level, $I^{\pi} = 1^{\pm}, 2^{\pm}$

The 1513.64-keV level has been reported previously.⁹ It is based on the 1513.63- and 1420.52- keV transitions that feed the 0^+ and 2^+ members of the ground-state band. From the coincidence data the 151.28-keV transition is determined as a transition from this level to the 2^+ level at 1362.36 keV; and, similarly, the 203.73-keV transition feeds the 1309.91-keV level.

From the log ft value for the electron capture to this level, Nielsen *et al.*⁹ have given a tentative spin-parity assignment of 2⁺ to it. However, a 1[±] assignment is not ruled out. Unfortunately, the poor statistics for the 1420.52-keV peak in the γ - $\gamma(\theta)$ experiment prevented any further restriction of the spin by this technique. An upper limit for an energetically possible 1207.12-keV transition to the 4^+ level at 306.52 keV is placed at 100 times weaker than the 1183.40-keV transition.

L. 1561.27 ± 0.13-keV Level, $K^{\pi}I = 0^{+}2$

The 1561.27-keV level is another known $I^{\pi} = 2^+$ level⁹ and depopulates via the emission of three γ transitions to the ground-state rotational band. The energies of these transitions to the 0^+ , 2^+ , and 4^+ members of the ground-state band are 1561.30, 1468.13, and 1254.72 keV, respectively. That the establishment of this level is correct is seen when one examines the coincidence spectra for the 93.13- and 213.39-keV gates. The 1561.27keV transition is not in coincidence with the 93.13keV γ ray, while the other two transitions are. Hence, the 1561.27-keV γ ray is a ground-state transition. Also, the 1254.72-keV transition is in coincidence with the 213.39-keV γ ray, while the 1468.13-keV one is not. Hence, the 1561.27keV level is clearly established in agreement with Nielsen et al.9

Since all three levels of the ground-state band are fed from this level, it is obvious that the spin and parity are 2^+ . Nielsen *et al.*⁹ have placed a K = 0 assignment on this level by observing an *E*0 admixture in the 1468.13-keV transition, however, this *E*0 admixture is questionable.²²

M. 1566.45 ± 0.14-keV Level, $I^{\pi} = I^{\pm}$, 2^{\pm}

When the three-angle data from the $\gamma - \gamma(\theta)$ experiment are summed, clear evidence is found that the 1473.32-keV transition is in coincidence with the 93.13-keV ground-state transition. In the $\gamma - \gamma$ data, there is no evidence for this transition in the 213.39-keV gate, but the statistics are so poor that one cannot definitely exclude it.

TABLE V. A summary of the $\gamma - \gamma(\theta)$ studies in ¹⁷⁸Hf. The results of correlations that involve the 1183.40- and 1402.87-keV transitions are averages of the two experiments. The statistics were too poor for the correlations with the 1081.52-, 1269.23-, and 1678.82-keV transitions in the first experiment.

Energy transition (keV)	Spin sequence $(x \rightarrow 2^+ \rightarrow 0^+)$ x	A_2	A_4	δ	Admixture (%)
1081.52	2_{v}^{+}	-0.11 ± 0.020	0.75 ± 0.28	>32 or <-11 ^a	E2 > 99
1106.11	0+	0.357	1.14	•••	
1183.40	2+	-0.054 ± 0.023 ^b	0.041 ± 0.033 ^b	$0.410^{+0.036}$	M1 85 6+1.1
1269.23	2 [±]	0.55 ± 0.15	0.25 ± 0.15	$0 \ge \delta \ge -2.25$	$an = 00.02_{2.1}$ Quad <83.5%
1340.84	0+	0.357	1.14	•••	Quuu: _00.0%
1350.55	0+	0.357	1.14	• • •	
1402.87	2+	0.505 ± 0.014 ^b	0.118 ± 0.016 b	$-(0.73 \pm 0.05)$	M1 65 2 + 3 2
1678.82	0+	0.04 ± 0.26	1.77 ± 0.37	Used to establish the level	spin of the 1771.95-keV

^a Based on present work and that of Ref. 18.

^b An average of the first and second experiments.

However, the 1309.90- and 1216.79-keV gates show that a 256.54-keV transition feeds the 1309.91-keV level, and the 1269.23-keV gate shows that a 204.09-keV transition feeds the level at 1362.36 keV. As a cross-check, the spectrum in coincidence with the 204.09-keV gate contains a 1269.23-keV peak, and that in coincidence with the 256.54-keV gate contains peaks at 1309.90 and 1216.79 keV. These coincidence data, thus, establish a new level at 1566.45 keV, which is depopulated by the 204.09-, 256.54-, and 1473.32keV transitions. An upper limit on an energetically favored 1566.45-keV transition is placed at 30 times weaker than the 1473.32-keV transition.

Since the 1473.32-keV peak in the $\gamma - \gamma(\theta)$ experiment has extremely poor statistics, the data are sufficient only to exclude an I=0 spin assignment for the 1566.45-keV level. The log ft of 7.0 allows 1^{\pm} and 2^{\pm} assignments and excludes 3^{\pm} .

N. 1771.95 \pm 0.20-keV Level, $K^{\pi}I = 0^{+}0$

Again, from a sum of the three-angle data, the 1678.82-keV transition is seen to be in coincidence with the 93.13-keV transition. Since the Q value for the ¹⁷⁸Ta decay is 1912 keV, we know that the 1678.82-keV transition cannot feed the 306.52keV level. Hence, a new level is established at 1771.95 keV. The only spin sequence that is consistent with the large A_4 term for the 1678.82-93.13-keV correlation (Table V) is $0 \rightarrow 2 \rightarrow 0$. Hence, the spin of this level is I=0.

Recent conversion-electron data²² show a new transition at 1771.95 keV with a very large K conversion coefficient. The α_K is consistent only with an E0 transition, so the parity of the 1771.95-keV level is positive.

O. Summary of ¹⁷⁸Hf Level Scheme

This work on the energies and intensities of the γ rays and on the γ - γ coincidence relationships has resulted in the discovery of seven new transitions, the establishment of five new levels, and the addition of thirteen transitions to the decay scheme.

IV. BRANCHING RATIOS FROM THE 2⁺ STATES

As discussed in the Introduction, it is of considerable importance to determine the degree to which the rotational model can predict the branching ratios of the β - and γ -type vibrational states. On the basis of results in the ¹⁵²Sm and ¹⁵⁴Gd nuclei, the pure rotational model¹ was modified to include the mixing of the β and γ ground-state band mixing and also β - γ ground-state band mixing.^{2, 6} This mixing changes the simple Alaga rules¹⁰ so that ratios of B(E2) values from a given level depend on the parameters Z_{β} , $\zeta_{\beta\gamma}$ for the β level and Z_{γ} , $Z_{\beta\gamma}$ for the γ level. These parameters characterize the first- and second-order mixing of these bands. Such parameters should be a constant for a given level in a β or γ band. The exact expressions for the B(E2) ratios are found in Ref. 3.

There are four 2⁺ levels in ¹⁷⁸Hf for which it is of interest to calculate branching ratios and bandmixing parameters. In the calculations that follow, only a single band-mixing parameter can be considered for the γ -vibrational band, since we have only one branching ratio there. Without knowledge of $Z_{\beta\gamma}$ from the γ band, it is very difficult to extract any meaningful information on β - γ band mixing for the β band. Thus, only a one-parameter fit is considered there, too.

The $2^+ \gamma$ -vibrational band head depopulates via the $2^+ \rightarrow 2^+$, 1081.52-keV and the $2^+ \rightarrow 0^+$, 1174.63keV transitions. The $2^+ \rightarrow 4^+$ transition is too weak



FIG. 11. The sum of the three-angle γ - γ correlation data in the 1183-93-keV cascade from the second experiment. The 10.7%-efficient Ge(Li) detector was used here.

to be seen in the present experiments. The B(E2) $(2^+ \rightarrow 0^+)/(2^+ \rightarrow 2^+)$ branching ratio agrees with theory for a one-parameter band-mixing value of $Z\gamma = 0.032 \pm 0.010$. This compares favorably with the value of 0.023 ± 0.010 obtained by Varnell, Hamilton, and Robinson.¹⁸

Since the 2⁺ level at 1276.54 keV had been proposed⁹ as a β -vibrational excitation, one of the primary objectives of the present work was to obtain further information about this level. Table VII shows the branching-ratio data from this level. When a $(73.0 \pm 1.3)\%$ M1 component is assumed in the $2^+ \rightarrow 2^+$ transition, we find consistent branching ratios with a Z_{β} value of $0.026^{+0.002}_{-0.003}$. This is in good agreement with the $(70.5 \pm 1.5)\%$ M1 reported by Siddiqi, Carlson, and Emery.²³ An average of these two results shows that a (71.8 $\pm 1.0)\%$ M1 component in the $2^+ - 2^+$ transition would bring the branching ratios into agreement. This lower value and smaller error limits compared to those of NNR¹³ are significant, since along with the low error limits on the M1 admixture from our $\gamma - \gamma(\theta)$ work, they allow the interpretation that the 1276.54-keV level is inadequately described by the Bohr-Mottelson model extended to include first-order perturbational effects. That is, when one corrects for the measured M1component in the 1183.40-keV transition, as shown in Table VII, one sees that, although the



FIG. 12. The experimental A_2 and A_4 coefficients in the 1183-93-keV cascade are compared with the theoretical curves for a $2 \rightarrow 2 \rightarrow 0$ cascade as a function of δ (notation Ref. 17). The dashed lines indicate the values of δ allowed within the experimental errors.

TABLE VI. Energy-sum relations of transitions in ¹⁷⁸Hf. The first entry in column 1 is a transition energy and the second a level energy which is added to yield the higher level energy. In the first two columns, the error in the γ energy is found in Table I and in the level energy in the last column of this table.

Energy sum	Adopted level energies	
(keV)	(keV)	
$93.13 = 93.13 \pm 0.08$	93.13±0.08	
$213.39 + 93.13 = 306.52 \pm 0.10$	306.52 ± 0.10	
1174.63 ± 0.07 1081.52 + 93.13 = 1174.65 \pm 0.11	1174.64 ± 0.06	
$1106.11 + 93.13 = 1199.24 \pm 0.11$	1199.24 ± 0.11	
1276.50 ± 0.08		
$1183.40 + 93.13 = 1276.53 \pm 0.11$	1276.54 ± 0.06	
$970.02 + 306.52 = 1276.54 \pm 0.12$		
1309.90 ± 0.08	1309.91 ± 0.08	
$1216.79 + 93.13 = 1309.92 \pm 0.14$		
$1269.23 + 93.13 = 1362.36 \pm 0.11$	1362.36 ± 0.11	
$1340.84 + 93.13 = 1433.97 \pm 0.12$	1433.97 ± 0.12	
$1350.55 + 93.13 = 1443.68 \pm 0.12$	1443 86+0 09	
$269.41 + 1174.64 = 1444.05 \pm 0.13$	1110.00 - 0.00	
1496.05 ± 0.11		
$1402.87 + 93.13 = 1496.00 \pm 0.12$	1496.02+0.07	
$1189.50 + 306.52 = 1496.02 \pm 0.15$		
1513.63 ± 0.12		
$1420.52 + 93.13 = 1513.65 \pm 0.14$	1513.64 ± 0.09	
1561.30 ± 0.13		
$1468.13 + 93.13 = 1561.26 \pm 0.14$	1561.27 ± 0.13	
$1254.72 + 306.52 = 1561.24 \pm 0.16$		
$1473.32 + 93.13 = 1566.45 \pm 0.14$	1566.45 ± 0.14	
$1678.82 + 93.13 = 1771.95 \pm 0.20$	1771.95 ± 0.20	



FIG. 13. The decay scheme of ¹⁷⁸Ta to ¹⁷⁸Hf as established in this work. Only spin (total angular momentum) and parity assignments are given for the 1309.91-, 1362.36-, 1513.64-, and 1566.45-keV levels.

Energy	τπτπ	Def 99	γ -ray intens	ity This work	Pres Experimental	sent worl l relative	K = B(E2) values	The $B(E)$	oretia 2) val valu	cal rel lues fo	lative or Z_{β} f:
(Kev)	$I_i \rightarrow I_f$	Ref. 22	Rel. 13		MI = 0 MI	. = 7.3%	M1 = (85.6- <u>2</u>	1)% 0 0	0.020	0.026	0.040
970.0	$2^+ \rightarrow 4^+$	3.66 ± 0.23	3.57 ± 1.09	3.54 ± 0.18	85.9±4.4 32	2 ±16	603_{32}^{+35}	180	295	323	437
1183.4	$2^+ \rightarrow 2^+$	11.00	11.00	11.00	100 10	0	100	100	100	100	100
1276.6	$2^+ \rightarrow 0^+$	2.52 ± 0.15	2.07 ± 0.64	2.22 ± 0.10	13.8±0.6 5	1.1 ± 2.2	$95.8_{-4.3}^{+4.7}$	70	54	51.2	40
		Ref. 9	Ref. 13	This work	M1=0 M1	= 62.5%		0 -0,0	20 -0	.040 -	-0.060
1189.5	$2^+ \rightarrow 4^+$	4.65		5.66 ± 0.28	12.9 ± 0.6 34	4.4 ± 1.6		180	93.3	34.8	4.6
1402.9	$2^+ \rightarrow 2^+$	100		100.0	100 100	0		100	100	100	100
1496.1	$2^+ \rightarrow 0^+$	65.9		56.3 ±1.9	40.8 ±1.4	109±3.7	7	70	87.4	108	129

 TABLE VII. Relative intensities and branching ratios from the 1276.54- and 1496.02-keV 2⁺ levels in ¹⁷⁸Hf populated in the decay of ¹⁷⁸Ta.

^a These values are for various M1 components in the 1183.4- and 1402.9-keV $2^+ \rightarrow 2^+$ transitions as indicated.

^b The notation here is the same as Ref. 3. The lower half of the table is for the 1496.1-keV level and the Z_{β} values are negative.

presence of a large M1 component is a significant factor in altering the B(E2) branching ratios, the new values still cannot be accounted for by a simple mixing of the wave functions of the groundstate band and the proposed β -vibrational band. If the 1276.54-keV level is a $2^+ \beta$ -vibrational level, then the first case for which it was thought that a large M1 admixture in the $2^+ \rightarrow 2^+$ transition could account for a consistent Z_β parameter is removed, as pointed out in our earlier letter.¹⁴ So, as in the case of ¹⁵²Sm and ¹⁵⁴Gd, the Bohr-Mottelson model, corrected to first order for band mixing, does not provide an adequate de-

scription of this $K^{\pi}I = 0^+2$ state in the highly deformed ¹⁷⁸Hf nucleus. However, we again point out that since this work was completed it has been shown¹⁸ that the 1276.54-keV state is not highly collective, and hence, is presumably not a β vibrational level.

One may ask if there are other levels to which it is permissible to apply the predictions of this model. As mentioned, there are three other 2^+ states which present themselves as possible candidates for the $2^+ \beta$ -vibrational level. The next lowest in energy, and the strongest populated in radioactive decay, is the 1496.02-keV level. Also shown in Table VII are the branching ratios from this level.

If it is assumed that the 1496.02-keV level is the result of a β vibration, then consistent branching ratios are obtained for $Z_{\beta} = -0.041 \pm 0.004$ when a $(62.5 \pm 1.0)\% M1$ component is assumed present in the $2^+ \rightarrow 2^+$ transition. This *M*1 component is in close agreement with the $(65.2 \pm 3.2)\%$ M1 as determined from our $\gamma - \gamma(\theta)$ experiment. It is interesting to note that the Z_{β} has the opposite sign to that of the 1276.54-keV level as well as those in earlier work.³ One is tempted, then, to call this level a $2^+ \beta$ -vibrational band member. However, there is no close-lying 0⁺ level which would yield a moment of inertia anywhere close to that of the ground-state rotational band. In addition, Nielsen et al.⁹ noted that the electron-capture feeding to this level is a factor of 2 higher than that predicted by Alaga's intensity rules¹⁰ for a rotational band built on either of the 1433.99- or 1443.70-keV 0⁺ levels. A weak, unobserved 0^+ level with energy spacing consistent with a more reasonable moment of inertia would have even worse disagreement with the theoretical electron-capture feeding ratios. In addition, the relatively large intensity of the 269.41-keV transition from the 1443.86-keV level to the $2^+ \gamma$ -vibrational level at 1174.64 keV would indicate a questionably strong mixing of the β and γ bands if the 1443.86keV level is the $0^+ \beta$ -vibrational band head. It is also significant that the observed⁹ E0 strength of

the 1402.87-keV transition from the 1496.02-keV level is about a factor of 10 weaker than that of the 1183.40-keV transition.

Finally, this state is seen to be only very weakly populated in Coulomb excitation $[B(E2)_{exc} < 0.34$ single-particle units¹⁸]. Thus, it does not exhibit the collective character that one expects of a β vibration either.

Unfortunately, inadequate statistics prevented accurate admixture determinations for the 1468.13keV, $2^+ \rightarrow 2^+$ transition and the cascade transitions from the 1309.91-, 1513.64-, and 1566.45-keV levels to the 2^+ member of the ground-state band. Here, too, there are no band-head 0^+ states observed for this 2^+ level at 1561.27 keV and the possible 2⁺ levels at 1309.91, 1513.64, and 1566.45 keV. Any unobserved 0^+ levels would be subject to the same problem as above with the electron-capture feeding ratios to the 0^+ and 2^+ states. Also, the 1561.27-keV state and the three possible 2⁺ states are not excited in the Coulombexcitation studies¹⁸ so they have little, if any, collective character. The branching ratios from the 1561.27-keV level do not admit a one-parameter Z_{β} fit, however, for any assumed M1 component in the $2^+ \rightarrow 2^+$ transition. In order to obtain consistent branching ratios, the intensity of the 1468.13-keV, $2^+ \rightarrow 2^+$ transition would have to be increased by 40%, and a pure E2 transition assumed.

In conclusion we have shown that the E2 branching ratios out of the lowest $K^{\pi}I = 0^+2$ state at 1276.54 keV are not explained by the Bohr-Mottelson model with perturbational corrections,⁶ while one can obtain a consistent fit to these branching ratios for the $K^{\pi}I = 0^+2$, 1496.02-keV level. However, none of these $K^{\pi} = 0^+$ levels has any appreciable collective strength, so they are presumably not β vibrational in character. These states are most likely quasiparticle excitations. This characterization could also explain why large M1 admixtures are seen in the $2^+ - 2^+$ transitions from these levels in contrast to no M1 admixtures for similar transitions from the β -vibrational states in ¹⁵²Sm, ¹⁵⁴Gd, and ¹⁵⁶Gd, Refs. 8, 7, and 24, respectively. This point will be discussed in more detail in a forthcoming publication.²⁴

Very recently Ejiri and Hagemann²⁵ have observed a β band in ¹⁷⁴Hf with a 2⁺ state at 901 keV. They find there a very small (\leq few percent) *M*1 admixture in the 2⁺ \rightarrow 2⁺ transition, an enhanced *B*(*E*2) which indicates collective character, and branching ratios that can be fitted by a one- or at least a two-parameter band-mixing treatment of the data. Evidently, there is a very sharp rise in the β -band energy between ¹⁷⁴Hf and ¹⁷⁸Hf, since no such state is observed in the latter nucleus. *Present address: Eastern Kentucky University, Richmond, Kentucky.

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Identification of ⁹⁴Kr and ¹⁴³Xe, and Measurement of γ -Ray Spectra and Half-Lives

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of Nuclides in the Mass Chains 93, 94, and 143*

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Fission-produced nuclides 94 Kr and 143 Xe have been measured directly using on-line mass separation. The half-lives derived from the β -decay curve are 0.20 ± 0.01 and 0.3 ± 0.03 sec, respectively. γ -ray spectra in the mass chain 93 have been measured and various γ peaks were assigned to the individual nuclides.

Identification of short-lived nuclides produced in fission and knowledge of their nuclear properties are essential for determining independent fission yields and for a better understanding of the fission process. So far only those fission-produced krypton and xenon isotopes having half-lives longer than 1 sec have been identified.¹⁻⁴ The shorter-lived ones have escaped measurement be-

cause of the difficulty of achieving the extreme rapidity required in separating them from the fissioning source and transporting them to the counting assembly. The shortest-lived (and thus heaviest) ones measured directly so far were 93 Kr and 142 Xe.^{2, 5}

By using gas sweeping over a short distance between a fissioning ²³⁵U source and the ion source

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