This last is possible, as noted, for example in the case of ^{208}Pb. Thus a spectral assay of the nuclear γ-ray spectrum ought to be a sensitive test of single-nucleon K^- capture as described above. We ignore capture on two nucleons, which may contribute 20% to the K^- reaction cross section, as it will *not*, in general, lead to the same nuclear γ rays. It therefore will not seriously affect the argument. While theory does generally predict the orbit at which capture occurs, the nuclear γ spectrum would be a new kind of datum in that, for example, it is particularly sensitive to the relative rate of capture on neutrons and on protons. Such information is lacking presently except for the limited emulsion data reviewed recently by Burhop.[8] It is in fact precisely this kind of detailed knowledge which one must have before K^- capture can truly become a useful tool for the study of nuclear structure.

*Work was performed under the auspices of the U.S. Atomic Energy Commission.

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PHYSICAL REVIEW C VOLUME 5, NUMBER 1 JANUARY 1972

Relativistic Self-Consistent Meson Field Theory of Spherical Nuclei

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(Received 7 June 1971; revised manuscript received 12 October 1971)

A relativistic self-consistent theory is used in conjunction with meson field potentials having the form of the generalized one-boson-exchange potentials (GOBEP) to construct a relativistic self-consistent meson field theory of nuclear structure. A simple GOBEP model with qualitative features of successful *N-N* models, e.g., approximate cancellation of static terms arising from generalized (or regularized) scalar- and vector-meson fields, is used to calculate ground-state properties of the doubly-magic spherical nuclei ^{16}O, ^{40}Ca, ^{90}Zr, and ^{208}Pb, and one superheavy nucleus ^{298}114. Good agreement is obtained between theoretical and experimental total binding energies and radial charge distributions. The isotopic shift in charge distributions between the isotopes ^{40}Ca and ^{48}Ca and the single-particle eigenvalues agree quite well with the experimental numbers. The absence of explicit correlation corrections, the relationship of this model to earlier meson-theoretic descriptions, and physical interpretation in terms of nucleon form factors and relativistic interactions are discussed.

I. INTRODUCTION

The importance of velocity-dependent potentials in nuclear physics and the close relationship of velocity-dependent and nonlocal potentials have been discussed extensively in the literature[1] both in relation to the nucleon-nucleon (*N-N*) interaction and nucleon-nuclear (*N*-$\mathfrak{N}$) interactions. In certain forms of *N-N* interactions based upon meson theory, velocity-dependent terms arising as relativistic effects assume major physical importance because of the approximate cancellation of the attractive and repulsive static terms arising from couplings via scalar- and vector-meson fields.[2,3] Recent calculations [4-7] using such one-boson-exchange potentials (OBEP) in conjunction with reg-

ularized Yukawa functions[8] provide a quantitative description of N-N scattering data and the properties of the deuteron. The fact that S waves can be fitted precisely without introducing a hard core is a particularly important consequence of the velocity-dependent terms.

A model of the N-π interaction, proposed in 1955 by Johnson and Teller,[9] contains several similar physical features to such OBEP. In their original classical field theory of the nucleus, a large velocity-dependent N-π interaction arose through the inclusion of derivative couplings in the interaction Lagrangian density. Duerr,[10] however, showed that similar velocity-dependent effects could be obtained from a relativistic field theory based upon direct coupling to both scalar and vector meson fields. He used a Foldy-Wouthuysen (FW)[11] transformation to arrive explicitly at the equivalent Schrödinger interaction. Rozsnyai[12] used the FW-transformed Duerr formalism for numerical calculations of the spherical nuclei ^{16}O, ^{40}Ca, ^{90}Zr, ^{140}Ce, and ^{208}Pb, and found very reasonable single-particle energy levels.

Since velocity-dependent interactions arising as relativistic effects are very important in such N-N and N-π models, it is desirable to use relativistic wave equations. For the N-N problem this has been done via a Bethe-Salpeter equation[13] and the results substantiate to a certain degree the previous calculations which went to the nonrelativistic limit via a Breit-type reduction.[14]

The relativistic treatment of finite nuclei can be accomplished by use of the relativistic Hartree-Fock (RHF) theory. An N-particle Dirac Hamiltonian may be constructed in analogy to the two-particle Dirac Hamiltonian. The same position-space potentials represent the two-body interactions in the N-body Hamiltonian. The N-body equation is then replaced approximately by N single-particle Dirac equations via Hartree-Fock theory. In this way the velocity-dependent effects are included to all orders (within the Hartree-Fock approximation).

Some time ago Rozsnyai[15] used such a RHF theory and obtained good nuclear-saturation properties. The work was not published, because it was prior to the discovery of vector and scalar mesons and recent developments of OBEP and, hence, the physical significance of the formalism was still obscure.

In the present work we reexamine such an approach to the nuclear many-body problem in the light of recent discoveries of multipion resonances or mesons. Thus we consider in detail a relativistic self-consistent model of complex nuclei in which each nucleon is a Dirac particle which is a source of generalized (or regularized) vector and scalar fields and which is also acted upon by these fields. Because of the approximate cancellation of the static-interaction terms, relativistic terms of the nature of those investigated extensively by Breit[14] play major roles.

By analogy with atomic usage we will refer to a relativistic self-consistent meson field theory (RSC-MFT) of the nuclear many-body problem as RHF (for relativistic Hartree Fock) when exchange terms are included, or simply as RH (for relativistic Hartree) when the exchange terms are neglected.

II. THEORY

The relativistic forms of the position-space OBEP as given in the review article by Green and Sawada[4] are

$$V_s = -\gamma_1^0\gamma_2^0 J_s(|\vec{r}_1 - \vec{r}_2|)\,, \tag{1}$$

$$V_v = \gamma_1^0\gamma_2^0\gamma_1^\mu\gamma_2^\mu J_v(|\vec{r}_1 - \vec{r}_2|)\,, \tag{2}$$

$$V_p = \gamma_1^0\gamma_2^0\gamma_1^5\gamma_2^5 J_p(|\vec{r}_1 - \vec{r}_2|)\,. \tag{3}$$

The subscripts s, v, and p refer to scalar, vector, and pseudoscalar mesons, respectively. The γ's are Dirac matrices which will be written in the Dirac representation in terms of the Pauli spin matrices

$$\gamma^0 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}, \tag{4}$$

$$\vec{\gamma} = \begin{pmatrix} 0 & \vec{\sigma} \\ -\vec{\sigma} & 0 \end{pmatrix}, \tag{5}$$

$$\gamma^5 = -i\begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}. \tag{6}$$

The J's in Eqs. (1)–(3) are generalized Yukawa functions which may be obtained by assuming either higher-derivative field theories for the boson field,[4,8] or form factors for the meson-nucleon vertex.[5,16] The so-called dipole regularization form is used in this work and is appropriate to the form-factor interpretation. The associated generalized Yukawa function is

$$J(r) = g^2\hbar c\left[\frac{e^{-\mu r}}{r} - \frac{e^{-\Lambda r}}{r}\left(1 + \frac{\Lambda^2 - \mu^2}{2\Lambda}\,r\right)\right]. \tag{7}$$

The g^2 is the meson-nucleon coupling constant and μ is associated with the meson mass ($\mu = mc/\hbar$). The parameter Λ is related to the structure of the form factor as follows:

$$F(\vec{q}^2) = g\frac{\Lambda^2 - \mu^2}{\vec{q}^2 + \Lambda^2}\,. \tag{8}$$

The interactions of Eqs. (1)–(3) should be multiplied by the operator $\vec{\tau}_1 \cdot \vec{\tau}_2$ (isospin operators) for

the case of isovector mesons. The subscripts 1 and 2 in Eqs. (1)–(3) refer to nucleons 1 and 2 which are interacting via meson exchange. The metric of four space is chosen so that the four-vector product $\gamma_1^\mu \gamma_2^\mu$ obeys the following relation:

$$\gamma_1^\mu \gamma_2^\mu = \gamma_1^0 \gamma_2^0 - \vec{\gamma}_1 \cdot \vec{\gamma}_2 . \tag{9}$$

With these forms of the relativistic N-N interactions, one can proceed to the development of the appropriate RHF equations. Let V be the total relativistic two-body interaction

$$V = V_s + V_v + V_\rho . \tag{10}$$

Then the Hamiltonian operator for an A-body system with only two-body interactions is

$$H = \sum_{i=1}^{A} (c\vec{\alpha}_i \cdot \vec{p}_i + \beta_i M c^2) + \tfrac{1}{2} \sum_{i \neq j}^{A} V(|\vec{r}_i - \vec{r}_j|) . \tag{11}$$

The variational approach may be used to obtain the RHF equations. This involves taking the expectation value of H for a Slater determinant of appropriate single-particle wave functions and minimizing $\langle \psi | H | \psi \rangle$ with respect to the single-particle states. The resulting equations are the RHF equations

$$(c\vec{\alpha} \cdot \vec{p} + \beta M c^2)\phi_i(\vec{r}_1) + \sum_{j=1}^{A} \int \phi_j^\dagger(\vec{r}_2) V(|\vec{r}_1 - \vec{r}_2|) \phi_j(\vec{r}_2) d^3 r_2 \phi_i(\vec{r}_1)$$

$$- \sum_{j=1}^{A} \int \phi_j^\dagger(r_2) V(|\vec{r}_1 - \vec{r}_2|) \phi_i(\vec{r}_2) d^3 r_2 \phi_j(\vec{r}_1) = E_i' \phi_i(\vec{r}_1) . \tag{12}$$

Equation (12) is recognized as the single-particle Dirac equation with the relativistic single-particle direct potential

$$U(\vec{r}_1) = \sum_{j=1}^{A} \int \phi_j^\dagger(\vec{r}_2) V(|\vec{r}_1 - \vec{r}_2|) \phi_j(\vec{r}_2) d^3 r_2 , \tag{13}$$

and the exchange potential with the nonlocal kernel

$$K(\vec{r}_1, \vec{r}_2) = - \sum_{j=1}^{A} \phi_j^\dagger(\vec{r}_2) V(|\vec{r}_1 - \vec{r}_2|) \phi_j(\vec{r}_1) . \tag{14}$$

Remembering the various components of V, Eq. (13) may be expressed as

$$U(\vec{r}_1) = \gamma_1^0 U_s(\vec{r}_1) + \gamma_1^0 \gamma_1^\mu U_v^\mu(\vec{r}_1) + \gamma_1^0 \gamma_1^5 U_\rho(\vec{r}_1) , \tag{15}$$

where the single-particle relativistic potentials are defined as follows:

$$U_s(\vec{r}_1) = - \sum_{j=1}^{A} \int \phi_j^\dagger(\vec{r}_2) \gamma_2^0 J_s(|\vec{r}_1 - \vec{r}_2|) \phi_j(\vec{r}_2) d^3 r_2 , \tag{16}$$

$$U_v^\mu(\vec{r}_1) = \sum_{j=1}^{A} \int \phi_j^\dagger(\vec{r}_2) \gamma_2^0 \gamma_2^\mu J_v(|\vec{r}_1 - \vec{r}_2|) \phi_j(\vec{r}_2) d^3 r_2 , \tag{17}$$

and

$$U_\rho(\vec{r}_1) = \sum_{j=1}^{A} \int \phi_j^\dagger(\vec{r}_2) \gamma_2^0 \gamma_2^5 J_\rho(|\vec{r}_1 - \vec{r}_2|) \phi_j(\vec{r}_2) d^3 r_2 . \tag{18}$$

The exchange term in Eq. (12) may be cast in local (though state-dependent) form by multiplying and dividing by $\phi_i^\dagger(\vec{r}_1)\phi_i(\vec{r}_1)$ and deleting the factor $\phi_i(\vec{r}_1)$ to obtain the following state-dependent potential:

$$U_{ex}^i(\vec{r}_1) = - \sum_{j=1}^{A} \int \frac{\phi_j^\dagger(\vec{r}_2) V(|\vec{r}_1 - \vec{r}_2|) \phi_i(\vec{r}_2) d^3 r_2 \{\phi_j(\vec{r}_1) \phi_i^\dagger(\vec{r}_1)\}}{\phi_i^\dagger(\vec{r}_1) \phi_i(\vec{r}_1)} . \tag{19}$$

This form is inappropriate for numerical calculation because of a possibility of nodes in the denominator $\phi_i^\dagger(\vec{r}_1)\phi_i(\vec{r}_1)$. The nodes can be removed by a method similar to the Slater[17] approximation for the nonrelativistic system.

Perhaps the most interesting aspect of the relativistic exchange potential is the observation that the factor $\{\phi_j(\vec{r}_1)\phi_i^\dagger(\vec{r}_1)\}$ is a Dirac matrix. This matrix may be expanded to obtain:

$$\{\phi_j(\vec{r}_1)\phi_i^\dagger(\vec{r}_1)\} = \tfrac{1}{4}\sum_A \text{Trace}\,[\phi_j(\vec{r}_1)\phi_i^\dagger(\vec{r}_1)\gamma_A]\,\gamma^A$$

$$= \tfrac{1}{4}\sum_A [\phi_i^\dagger(\vec{r}_1)\gamma_A\phi_j(\vec{r}_1)]\gamma^A, \tag{20}$$

where the γ^A are the 16 linearly independent matrices formed from products of Dirac matrices.

One sees from Eq. (19) that even if the two-body interaction V is of restricted tensorial character (scalar for example), the single-particle exchange potential will have components transforming like all possible forms of Dirac tensors: scalar, vector, pseudoscalar, pseudovector, and tensor. Thus the general RHF equation will have all possible forms of single-particle potentials, i.e.,

$$\{c\vec{\alpha}\cdot\vec{p} + \beta[Mc^2 + U_s(\vec{r}) + \gamma^\mu U_v^\mu(\vec{r}) + \gamma^5\gamma^\mu U_A^\mu(\vec{r}) + \gamma^\mu\gamma^\nu U_T^{\mu\nu}(\vec{r})]\}\,\phi_i(\vec{r}) = E_i\phi_i(\vec{r}), \tag{21}$$

where only the six independent elements of the tensor $\gamma^\mu\gamma^\nu$ participate in the summation.

It should be remembered that those parts of the single-particle potentials which are due to the exchange potential are state-dependent.

Equation (21) is the most complicated Dirac equation possible. It needs to be simplified considerably in order to be useful for practical calculations. Such simplification occurs for nuclei with doubly closed shells, for then the single-particle interaction terms are required to commute with the total nucleon angular momentum operator $\vec{J}$:

$$\vec{J} = \vec{L} + \frac{\hbar}{2}\begin{pmatrix}\vec{\sigma} & 0\\ 0 & \vec{\sigma}\end{pmatrix}. \tag{22}$$

This restriction can be shown to induce the following simplifications in the single-particle potentials:

$$\begin{aligned}
&U_s(\vec{r}) = U_s(r),\\
&U_v^0(\vec{r}) = U_v^0(r),\\
&\vec{U}_v(\vec{r}) = \hat{u}_r U_v^r(r),\\
&U_p(\vec{r}) = 0,\\
&U_A^\mu(\vec{r}) = 0 \quad (\mu = 0, 1, 2, 3, 4),\\
&U_T^{ij}(\vec{r}) = 0 \quad (i, j = 1, 2, 3),\\
&U_T^{0j}(\vec{r}) = [\vec{U}_1(\vec{r})]_j,\\
&\vec{U}_1(\vec{r}) = \hat{u}_r U_1^r(r),
\end{aligned} \tag{23}$$

where $\vec{U}_r$ is a unit vector in the radial direction.

For a spherically symmetric field the single-particle wave functions in Eq. (21) are functions of good total angular momentum and can be written as follows[18]:

$$\phi(\vec{r}) = \frac{1}{r}\begin{pmatrix} F(r) & Y_{lm}^J(\theta,\phi)\\ iG(r) & Y_{l'm}^J(\theta,\phi)\end{pmatrix}. \tag{24}$$

The functions $F(r)$ and $G(r)$ in Eq. (24) are the large and small components, respectively, of the radical Dirac wave function for positive-energy states. The Y_{lm}^J are spin angular functions formed by coupling spin and orbital angular momentum functions in the usual way. Note that the orbital angular momentum quantum numbers differ between the large and small components. Their relation is expressed by the following:

$$J = l - \tfrac{1}{2}\tilde{\omega} = l' + \tfrac{1}{2}\tilde{\omega}, \tag{25}$$

where $\tilde{\omega}$ is defined by $\tilde{\omega} = \pm 1$ for states of parity $(-1)^{J\pm 1/2}$.

Equation (21), with the restrictions imposed by Eq. (23), may be reduced to radial form by standard methods[18]:

$$\begin{aligned}
\frac{dF}{dr} &= -\left[\frac{i(U_v^r - U_1^r)}{\hbar c} + \tilde{\omega}\frac{(J+\frac{1}{2})}{r}\right]F\\
&\quad + \left[\frac{2Mc^2 + U_s - U_v^0 + E}{\hbar c}\right]G,\\
\frac{dG}{dr} &= \frac{U_s + U_v^0 - E}{\hbar c}F - \left[\frac{i(U_v^r + U_1^r)}{\hbar c} - \tilde{\omega}\frac{(J+\frac{1}{2})}{r}\right]G.
\end{aligned} \tag{26}$$

In Eq. (26) the eigenvalue E is defined by

$$E' = E + Mc^2, \tag{27}$$

so that E is appropriate for comparison to the nonrelativistic single-particle eigenvalue. This form of the Dirac equation (Eq. 26) is used for generating numerical solutions. The i's occurring in Eq. (26) are no problem because the functions U_v^r and U_1^r are pure imaginary.

By using Eq. (24), the direct single-particle potentials, Eqs. (16)–(18), may be reduced to simple radial integrals. It is convenient to define the cur-

rent densities of various tensor character as follows:

$$\rho_s(\vec{r}) = \sum_{j=1}^{A} \phi_j^\dagger(\vec{r})\gamma^0\phi_j(\vec{r}), \tag{28}$$

$$\rho_v^\mu(\vec{r}) = \sum_{j=1}^{A} \phi_j^\dagger(\vec{r})\gamma^0\gamma^\mu\phi_j(\vec{r}), \tag{29}$$

$$\rho_p(\vec{r}) = \sum_{j=1}^{A} \phi_j^\dagger(\vec{r})\gamma^0\gamma^5\phi_j(\vec{r}). \tag{30}$$

For a closed-shell system, the only nonzero elements of Eqs. (28)–(30) are ρ_s and ρ_v^0, which are given by

$$\rho_s(r) = \sum_{J\ \text{shells}} \frac{2J+1}{4\pi r^2}(F^2 - G^2), \tag{31}$$

$$\rho_v^0(r) = \sum_{J\ \text{shells}} \frac{2J+1}{4\pi r^2}(F^2 + G^2). \tag{32}$$

With these angle-independent current densities, the integrals in Eqs. (16) and (17) may be simplified by the following relation:

$$\int \rho(r_2)J(|\vec{r}_1 - \vec{r}_2|)d^3r_2 = \frac{-2\pi}{r_1}\int_0^\infty r_2\rho(r_2)[e^{-\mu(r_1+r_2)} - e^{-\mu|r_1-r_2|}]\,dr_2, \tag{33}$$

where the function J is the Yukawa function

$$J(r) = \frac{e^{-\mu r}}{r}. \tag{34}$$

Equation (33) may be used to simplify Eqs. (16) and (17) even when the J's are generalized (or regularized) Yukawa functions, because all such functions [even Eq. (7)] are expressible as weighted sums of Yukawa functions.

One sees that for spherical nuclei the most important single-particle potentials are U_s and U_v^0. The other possible potentials (U_v^r and U_1^r) receive contributions only from the exchange potential.

The exchange potential [Eq. (19)] can also be greatly simplified for the case of closed-shell systems. The first problem is the removal of possible nodes in the denominator $\phi_i^\dagger(\vec{r}_1)\phi_i(\vec{r}_1)$. This is accomplished in nonrelativistic theory by applying the Slater approximation which yields an averaged exchange potential by summing both the numerator and denominator of Eq. (19) over the occupied states. In RHF theory the same can be accomplished by summing only over the magnetic substates of the J shell of which ϕ_i is a member. This is possible because the radial density of a J shell is proportional to $F^2 + G^2$. Nodes in the radial functions F and G are no problem as they never occur at the same point.

The approximate exchange potential then becomes

$$U_{\text{ex}}^i(\vec{r}_1) = -\sum_{j=1}^{A}\sum_{m_1}\int \frac{\phi_j^\dagger(\vec{r}_2)V(|\vec{r}_1 - \vec{r}_2|)\phi_i(\vec{r}_2)d^3r_2}{[(2J_i+1)/(4\pi r_1^2)](F_i^2 + G_i^2)} \times\{\phi_j(\vec{r}_1)\phi_i^\dagger(\vec{r}_1)\}. \tag{35}$$

This form [Eq. (35)] may be simplified greatly for the various elements of V by use of Eq. (20) and an angular expansion of the Yukawa function. In the present work the contribution of Eq. (35) is neglected; thus we pursue a RH theory. Preliminary calculations indicate that the exchange energy in our model is relatively small. Furthermore, exchange effects are approximately allowed for by our parameter adjustments.

The effect of isospin upon the relativistic single-particle potentials is the final topic of discussion in this section. It has already been pointed out that the two-body interactions, Eqs. (1)–(3), include the isospin operator $\vec{\tau}_1 \cdot \vec{\tau}_2$ for the case of isovector-meson exchange. The effect upon the single-particle potentials may be seen from Eq. (13). The single-particle states ϕ carry the isospinors,

$$p = \begin{pmatrix}1\\0\end{pmatrix} \quad \text{and} \quad n = \begin{pmatrix}0\\1\end{pmatrix}, \tag{36}$$

depending upon whether the state represents a proton or neutron, respectively. If one expands the isospin operator in terms of raising and lowering operators,

$$\vec{\tau}_1 \cdot \vec{\tau}_2 = \tau_{1z}\tau_{2z} + \tfrac{1}{2}(\tau_{1+}\tau_{2-} + \tau_{1-}\tau_{2+}), \tag{37}$$

one sees from Eq. (13) that only the $\tau_{1z}\tau_{2z}$ term contributes to the direct potential. The operator τ_{2z} operates upon the states which are summed to yield the self-consistent potential, while the τ_{1z} operates upon the wave function being calculated. Both states are eigenstates of the appropriate τ_z operator. The factor $\tau_{1z}\tau_{2z}$ is +1 if the states have the same isospin and −1 if they have opposite isospin. The resulting potential will be approximately proportional to $N - Z$ and will have the properties of a symmetry potential. The possible relation between the symmetry potential and isospin has long been recognized.[19]

The only important ingredient of the OBEP models which the present form of RH theory does not accommodate is the derivative coupling which seems to play an important role for the ρ meson. This neglect should not have much effect upon the properties of finite nuclei, for the ρ is an isovec-

tor meson and, hence, is important only for the symmetry potential which is itself a fairly small effect. Inclusion of the derivative-coupling terms would yield a RH equation with momentum-dependent single-particle potentials.

III. SUMMARY OF NUMERICAL TECHNIQUES

The calculations reported in this work were performed on the IBM 360 Model 65 computer at the University of Florida. All calculations were done in single precision. The Runge-Kutta (RK) integration method was used to solve the coupled Dirac equations (26) in position-space. The RK method requires the potentials halfway between the grid points and these were obtained by parabolic interpolation. The numerical solutions were matched in the tail regions to exact solutions to the Dirac equation.

The self-consistent potentials were calculated by use of the radial integrals in Eq. (33). These integrals were evaluated from a parabolic-fitting method which yields the following integration rule:

$$\int_a^b y(x)dx = \Delta[\tfrac{3}{8}y(1)+\tfrac{7}{6}y(2)+\tfrac{23}{24}y(3)+y(4)+\cdots y(n-3) + \tfrac{23}{24}y(N-2)+\tfrac{7}{6}y(N-1)+\tfrac{3}{8}y(N)]. \quad (38)$$

This rule embodies the simplicity of the trapezoidal rule and the accuracy of Simpson's rule. The calculations were done using 250 equally spaced mesh points.

The calculations were started using Woods-Saxon potentials and the equations were iterated until the total binding energy was stable to within 0.2%. The convergence was usually oscillatory so that numerical errors due to truncation of the calculations were minimized. Good convergence was usually achieved in 15 iterations or less.

IV. DISCUSSION OF THE EFFECTIVE INTERACTION

The calculations presented here were made with the model shown in Table I. Four parameters (denoted by superscript a) were adjusted in this investigation. The equality of the parameters for ω and ϵ serves here to accomplish the strong cancellation between vector and scalar static potentials which is present in all successful OBEP models. In addition, a residual attraction is incorporated by including a light scalar meson σ with twice the pion mass. The isovector ρ meson is included to provide a symmetry potential although the derivative-coupling terms along with the pion have been neglected on the basis that they would be small contributions for spherical nuclei. Dipole form factors are used as in the OBEP calculations of Ueda and Green.[5] The form-factor parameter (Λ) was chosen to be twice the nucleon mass, a value which yielded good saturation properties.

These model parameters are qualitatively similar to OBEP models which yield realistic N-N properties, but which gave underbinding in the RH formalism. The differences between the present effective interaction and realistic OBEP models are partially attributable to a difference in formalism, since the OBEP parameters were obtained by approximate calculations with the Schrödinger equation. The remaining differences are interpreted as phenomenological corrections for the neglect of correlation effects. The correlation effects are known to be quite strong for nonrelativistic local N-N models which are forced to include hard repulsive cores in order to fit phase shifts. It is suggested later that the absence of hard cores in the relativistic OBEP models simplifies the correlation effects to the point that they may be approximated by the simple numerical adjustments used here.

The philosophy of this work is then similar to that taken in the works of Köhler[20] and of Negele.[21] In the absence of an N-N model which yields a reaction matrix that properly saturates infinite nuclear matter, the process of deriving an effective interaction for finite nuclei is bypassed. Instead, a qualitatively realistic effective interaction is postulated which contains parameters that are adjusted to obtain fits to the properties of infinite nuclear matter or finite nuclei. The effective interaction of Köhler is obtained from very general properties of reaction matrices (velocity dependence, density dependence, state dependence, etc.), whereas that of Negele is obtained from minor variations of a reaction matrix derived from Reid's[22] potential in the local-density approximation. In both cases the effective interactions are very complicated in their dependence upon relative two-

TABLE I. Meson parameters of RHF model (MG).

Meson	$T\ J^p$	Mass	g^2	g'^2
ω	0 1^-	782.8	25.[a]	(17.)[b]
ϵ	0 0^+	782.8	25.	(17.)[b]
σ	0 0^+	277.4	0.675[a]	(0.649)[b]
ρ	1 1^-	763.	2.5[a]	(1.74)[b]

$\Lambda = 2M$[a]

[a] Adjusted parameter.

[b] The coupling constant in parentheses (g'^2) is related to the other constant (g^2) by the relation $g'^2=[(\Lambda^2-\mu^2)/\Lambda^2]^2 g^2$. The convention corresponding to g'^2 is more commonly used in particle physics.

body states and upon the local density.

The major advantage of the present work over the previous methods[20,21] is simplicity. By restricting these calculations to a relativistic formalism with relativistic OBEP N-N interactions, two major simplifications in the effective interactions are obtained. First, the complicated dependence upon relative two-body states is eliminated because the state dependence of the relativistic OBEP models is carried in the Dirac γ matrices which identify the type of meson producing the various interaction components. Second, the relativistic effects of OBEP models enable the hard cores to be eliminated. These hard cores were responsible for a sizable part of the correlation effects of nonrelativistic models. The correlation effects are thus simplified and can be approximated without the assumption of a complicated dependence upon the local density. Another advantage of the relativistic formalism is that spin-orbit forces are implicit and do not require further adjustable constants. This advantage is important in studies of superheavy nuclei.

The reduced importance of correlation effects for the OBEP models has been observed in infinite-nuclear-matter calculations of Haftel, Tabakin, and Richards[23] and of Wong and Sawada.[24] Using the OBEP model of Bryan and Scott,[7] Haftel, Tabakin, and Richards studied the variations due to neglect of the various components of the N-N force. Their result was that all components are necessary to yield a saturation curve in reasonable agreement with the full result. Thus neglect of the pion (as we have done) cannot yield realistic results (without further parameter variations) unless there is a fortuitous cancellation of the higher-order effects between the pion and other mesons. The more important aspect of their paper for the present work is the decrease in the value of the two-body wound integral (κ) over nonrelativistic models. This is especially well documented in the work of Wong and Sawada, where studies for the Hamada-Johnston, Reid soft-core, Bressel-Kerman-Rouben, Bryan-Scott, Ueda-Green I, and Ueda-Green III interactions are presented. For example, at nuclear-matter density the values of κ are 0.207, 0.137, 0.120, 0.087, 0.091, and 0.069 for the above interactions, respectively.

The wound integral κ is the expansion parameter in the compact cluster formalism[25] so that one expects significantly faster convergence of the correlation series for the OBEP models. Simpler approximation should thus be required to represent the higher-order effects.

It is quite possible that even these small values of κ are too large for the relativistic OBEP models because the nonrelativistic approximations used in usual nuclear matter calculations tend to break down and yield singularities at zero internucleon separation, a region to which κ is most sensitive. This is true, for example, for the spin-orbit and tensor interaction terms before the form-factor renormalizations are applied.

V. PRESENTATION AND DISCUSSION OF RESULTS

The present calculations (MG) of total binding energies and rms charge radii of doubly magic nuclei are presented in Table II and compared with experiment as well as theoretical calculations of Davies and McCarthy (DM),[26] of Köhler (K),[20] of Negele (N),[21] and of Nemeth and Vautherin (NV).[27] The experimental total binding energies are taken from the mass tables of Mattauch, Thiele, and Wapstra.[28] The experimental charge radii are taken from publications and preprints of the Stanford high-energy physics lab[29-33] except for the rms radius of ^{90}Zr which is taken from an unreferenced quotation in Köhler's work.[20]

The effective interactions of Negele and Köhler have been discussed earlier in this work. It is sufficient to mention that the work of Nemeth and Vautherin is only a minor modification of the work of Negele. The calculations of Davies and McCarthy are important for comparison with the other calculations because they represent the present status of fundamental nuclear-structure calculations. Their reaction matrix is derived from the Reid potential and is not altered to improve the agreement with experiment. There are, however, parameters in the Davies-McCarthy calculations which relate to many-body approximations that are not rigorously determined by the theory. Unfortunately, the results in Table II are strongly dependent upon variations in these parameters. These variations are not shown in Table II; however, the variations always improve either the binding energy or the rms radius at the expense of the other. The numbers in Table II thus give an adequate view of the present status of fundamental nuclear-structure theories, clearly unsatisfactory.

Of the phenomenological calculations in Table II, the best rms radii are given by the present calculations (MG), whereas the calculations of Köhler give the best values for total binding energy. Despite these minor variations, the phenomenological calculations all achieve quite realistic saturation properties which are far superior to those of the fundamental reaction-matrix calculation.

The binding energies of the present work are corrected for center-of-mass motion by the equation

$$E_{\text{c.m.}} = \tfrac{3}{4}\hbar\omega\,, \tag{39}$$

TABLE II. Calculations of total binding energy and rms charge radii of doubly major nuclei.

Element	−B.E./A						rms (charge) radii					
	DM	K	N	NV	MG	Exp.	N	DM	K	NV	MG	Exp.
^{16}O	6.76	8.10	6.75	7.73	7.35	7.98	2.71	2.59	2.84	2.76	2.70	2.73
^{40}Ca	4.99	8.53	7.49	8.32	8.25	8.55	3.41	3.19	3.67	3.45	3.49	3.49
^{48}Ca	4.00	8.70	7.48	7.87	8.55	8.67	3.45	3.25	3.75	3.52	3.49	3.48
^{90}Zr		8.59	7.85	8.07	8.87	8.71	4.18		4.58	4.23	4.30	4.30
^{208}Pb	2.49	7.90	7.53	7.31	8.05	7.87	5.37	4.87	5.94	5.44	5.54	5.50–5.54

where $\hbar\omega$ obeys the equation

$$\hbar\omega = 1.85 + 35.5A^{1/3}. \quad (40)$$

The same correction has been used by Negele.

The single-particle eigenvalues for these nuclei are listed in Tables III–VII. For the nuclei ^{16}O and ^{40}Ca (Tables III and IV) the proton eigenvalues are compared with experimental proton separation energies obtained from $(p, 2p)$ and $(e, e'p)$ experiments.[34] The numbers quoted here are actually those tabulated by Becker.[35] As may be seen from Tables III and IV, the eigenvalues from the RHF calculation are in moderate agreement with experimental separation energies even for the deep inner-particle states in ^{16}O and ^{40}Ca. This again is an advantage for the relativistic model, for large rearrangement energies are predicted on the basis of nonrelativistic calculations.[36]

The proton densities resulting from the RH calculations have been smeared by the Hofstadter-Wilson[37] proton form factor to obtain nuclear-charge distributions. These distributions are presented in Fig. 1, where they are compared with the analytic charge distributions obtained from elastic electron scattering experiments.[29–33] The agreement is seen to be quite good except for the interior where there are moderate shell oscillations in the theory. The analytic curves are improved by the additions of similar oscillations, but of smaller magnitude.[29] The experimental distributions are less well known in the interior in any case, and the effect of nonrelativistic center-of-mass corrections which have not been applied here might improve the agreement.

Even better determined by the scattering experiments than the total charge distribution is the isotopic shift in the charge densities between the calcium isotopes ^{40}Ca and ^{48}Ca.[30] This curve should provide a more stringent test for the RH theory of nuclear structure. The theoretical isotopic shift (difference) $4\pi r^2\Delta\rho(^{40}\text{Ca}-^{48}\text{Ca})$ is compared with the experimental curve in Fig. 2. While the theoretical curve is shifted to the right somewhat, it is apparent that the qualitative features of the two curves are the same. There are indications that a proper treatment of the Hartree-Fock exchange potential could improve the agreement between theory and experiment for the isotopic shift. The exchange potential of the pion has the right qualitative properties to remove such a discrepancy.

Recently, phenomenological saturating reaction

TABLE III. Eigenvalues and separation energies of states in ^{16}O.

State	Proton eigenvalue	Proton separation	Neutron eigenvalue	Neutron separation
$1s_{1/2}$	39.93	38 ± 4	44.12	
$1p_{3/2}$	20.03	$18.5^{+0.5}_{-0.2}$	23.96	
$1p_{1/2}$	10.83	$12.2^{+0.2}_{-0.1}$	14.67	15.69 (last particle)

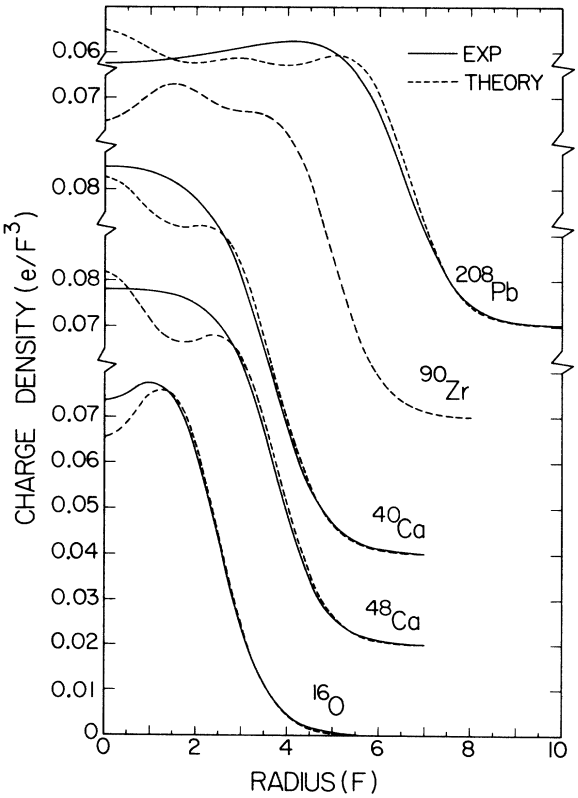

FIG. 1. Comparison of theoretical nuclear charge distributions with experimental (analytic) curves.

matrices have been applied to nuclei in the superheavy region.[19,20,38] So far, only exploratory calculations have been made in this region with the RH model. The parameters of Table I are again used, although it seems likely that the HF exchange potential should be used in this region because of the importance of single-particle levels. Such inclusions would necessitate a slightly different model. It also seems likely that improved numerical techniques will be required to do extensive studies in this region.

So far the calculations have centered upon the properties of the $^{298}114$ system. The RH model yields a binding energy (B.E./A) of 7.48 MeV, and an rms charge radius of 6.26 F. The single-particle eigenvalues of the top proton and neutron levels are −6.58 and −4.58 MeV, respectively. The splittings between occupied and unoccupied shells are 3.5 and 1.9 MeV for protons and neutrons, respectively. These results are comparable to those obtained in Refs. 20 and 38.

The RH theory is especially applicable to such superheavy nucleus calculations because the important spin-orbit interaction is inherent in the theory. The fact that the effective two-body interaction is independent of density also inspires confidence in extensions to untested regions.

VI. DISCUSSION AND CONCLUSIONS

A RH theory of nuclear structure has been presented which is based upon generalized OBEP-type N-N interactions. A simple model was investigated which produces the observed energy and density saturation properties of finite nuclei and also yields eigenvalues close to single-particle separation energies. The success of this simple effective interaction is probably related to the small magnitude of two-body correlation effects associated with OBEP interactions. This property is primarily due to the absence of unphysical hard cores which in this model are replaced by relativistic interactions implicit in the use of the Dirac equation.[14] These interaction terms are softened in their influence by our use of generalized (or regularized) fields, which is equivalent to using form factors. Earlier, Moszkowski[39] showed that short-range correlations are reduced when one replaces hard cores by phenomenological velocity-dependent N-N interactions.

TABLE IV. Eigenvalues and separation energies of states in ^{40}Ca.

State	Proton eigenvalue	Proton separation	Neutron eigenvalue	Neutron separation
$1s_{1/2}$	48.05	48.5 ± 5	56.09	
$1p_{3/2}$	33.48	36 ± 3	41.23	
$1p_{1/2}$	27.90	31.5 ± 3.5	35.68	
$1d_{5/2}$	18.24	16^{+3}_{-2}	25.68	
$1d_{3/2}$	8.99	$8.5^{+2}_{-0.2}$	16.37	
$2s_{1/2}$	7.71	12 ± 1	14.95	15.62 (last particle)

TABLE V. Eigenvalues of states in ^{48}Ca.

State	Proton eigenvalue	Neutron eigenvalue
$1s_{1/2}$	55.25	56.64
$1p_{3/2}$	42.28	42.13
$1p_{1/2}$	37.77	37.98
$1d_{5/2}$	27.48	26.73
$1d_{3/2}$	18.70	18.34
$2s_{1/2}$	14.24	16.28
$1f_{7/2}$		11.51
last-particle separation	15.26	9.94

The model is applied to the calculation of isotopic shifts in the calcium isotopes (^{40}Ca and ^{48}Ca) where qualitative agreement is obtained. The model is also applied to a nucleus in the superheavy region, where reasonable results are also obtained, although numerical approximations in the calculation are at present too crude to make definitive studies in this region. Improved approximations and numerical techniques may soon change this.

One might raise the question as to whether there are enough adjustable parameters in the present model to account for the good fit to binding energies and radii. As long ago as 15 years, Green and his collaborators[40-42] showed that one could get good radii and last-particle binding energies with a realistic nonrelativistic shell model with a diffuse independent-particle-model (IPM) potential. The models used in those early studies had about six parameters: (1) the radius parameter (r_0) in $R = r_0 A^{1/3}$; (2) a well depth V_0; (3) a spin-orbit

TABLE VI. Eigenvalues of states in ^{90}Zr.

State	Proton eigenvalue	Neutron eigenvalue
$1s_{1/2}$	54.00	62.75
$1p_{3/2}$	44.58	51.84
$1p_{1/2}$	41.93	49.45
$1d_{5/2}$	33.56	39.83
$1d_{3/2}$	28.05	34.66
$2s_{1/2}$	2[illegible].84	31.98
$1f_{7/2}$	21.40	27.15
$1f_{5/2}$	12.86	18.91
$2p_{3/2}$	8.52	16.58
$1g_{9/2}$		14.40
$2p_{1/2}$	6.05	13.99
last-particle separation	8.38	12.00

parameter; (4) a symmetry-potential parameter V_1[43]; (5) a surface thickness; and (6) a nonlocality distance.[44,45] This number of parameters is not greatly different from the number (four) used here. It must be emphasized, however, that these phenomenological IPM models which were adjusted to achieve reasonable last-particle binding energies suggested little as to the nature of the N-N interaction. Furthermore, they did not yield good total energies nor were they stable or self-consistent (although there were attempts to make them so[46]). It is rather interesting to note that the latest Brueckner-Hartree-Fock calculations by Becker and Patterson,[47] using occupation probability renormalization, indicate that single-particle eigenvalues should be close to last-particle separation energies (i.e., Koopman's theorem holds approximately). The fact that our RSC-MFT model has this property, and the fact that the shell structures (e.g., spin-orbit splittings and level sequences) are also in reasonable accord with observations, are gratifying.

In final conclusion, we must emphasize that we do not claim to have proceeded from a quantitative description of N-N interaction to a quantitative description of the nuclear many-body problem without introducing additional phenomenology. Nevertheless, we believe the qualitative resemblance of our N-N inputs to those in successful N-N models lends credence to the meaningfulness of the RSC-MFT model as a heuristic guide towards what may eventually be the final physical model of nuclear structure. The present interim model essentially is an updated synthesis of two old lines of thought, i.e., the vector-scalar generalized meson field theory of Green[2] as applied to the N-N interaction, and the vector-scalar classical meson field model of Johnson and Teller,[9] Duerr,[10] and Rozsnyai[12] as applied to the nuclear many-body problem. If one might be permitted to speculate as to the essential physical features of the future final model, one might suggest that the finite extension of nucleons (e.g., form factors) and relativistic effects surviving the cancellation of the static components of scalar and vector fields will play major roles just as they do in all current quantitative OBEP models of the N-N system.

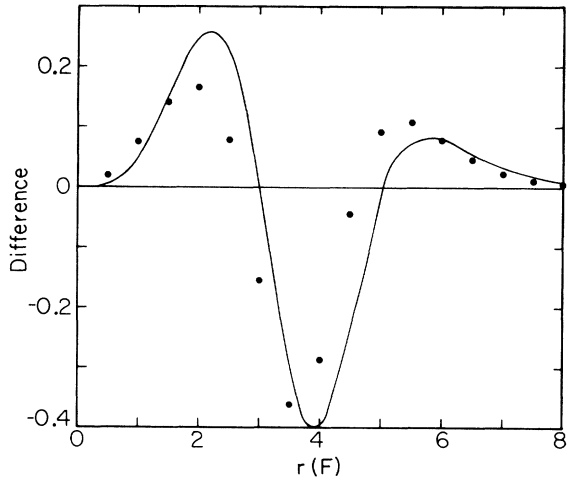

FIG. 2. Comparison of theoretical isotopic shift for calcium isotopes (^{40}Ca and ^{48}Ca) with experiment.

An unfortunate feature of such a physical model is that many complex relativistic or magnetic-like effects[48] may enter at the same order as the terms surviving the ω-ϵ cancellation. For example, the very broad width of the ϵ probably contributes in a way which cannot strictly be "mocked up" by the σ meson used here. Effects associated with uncorrelated multipion exchange, effects associated with tidal-like polarization of the soft nucleons, effects associated with differences between charge states of pions, and many other perturbation effects probably enter. Finally, the exchange effects which we have discarded here partly on the basis of our magnitude estimates and partly on the basis of our phenomenological adjustment of parameters undoubtedly contribute. More detailed studies of these latter effects are underway by one of us (LDM). In

TABLE VII. Eigenvalues of states in ^{208}Pb.

State	Proton eigenvalue	Neutron eigenvalue
$1s_{1/2}$	53.07	62.71
$1p_{3/2}$	47.76	56.57
$1p_{1/2}$	46.82	55.81
$1d_{5/2}$	40.75	49.00
$1d_{3/2}$	38.53	47.04
$2s_{1/2}$	33.38	43.34
$1f_{7/2}$	32.52	40.38
$1f_{5/2}$	28.51	36.66
$2p_{3/2}$	22.16	31.63
$2p_{1/2}$	20.77	30.24
$1g_{9/2}$	23.46	31.05
$1g_{7/2}$	17.27	25.14
$1h_{11/2}$	13.85	21.30
$2d_{5/2}$	10.81	19.98
$2d_{3/2}$	8.56	17.68
$3s_{1/2}$	6.84	16.59
$1h_{9/2}$		13.03
$1i_{13/2}$		11.39
$2f_{7/2}$		9.11
$2f_{5/2}$		6.51
$3p_{3/2}$		6.06
$3p_{1/2}$		5.31
last-particle separation	8.03	7.38

the meantime, the authors would like to express the view that because of the simplicity of the present model and economy of adjusted parameters in relationship to the many experimental properties that the model describes, that this RSC-MFT approach can contribute to an increased understanding of the structure of the nucleus and to a bridging of the gap between N-N studies and studies of the nuclear many-body problem. In this last connection we might note that by adding coupling to pions and by giving the ρ meson derivative coupling to nucleons, steps which would not influence spherical nucleus calculations, we can obtain an OBEP model which gives reasonable N-N phase shifts.[49]

VII. ACKNOWLEDGMENTS

The authors wish to acknowledge the support of the Air Force Office of Scientific Research Grant No. AFOSR-68-1397 in the early phases of this research, and the University of Florida Computing Center in the later phase. One of us (LDM) also wishes to acknowledge the support of fellowships granted by the National Science Foundation and the state of Florida which were held during the course of this work. Thanks are also due Dr. B. F. Rozsnyai and Dr. T. Sawada for helpful communications and conversations, Dr. F. Riewe, Dr. W. T. Miles, and Dr. E. F. Redish for reading the manuscript, and Professor Gregory Breit for his helpful comments.

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PHYSICAL REVIEW C VOLUME 5, NUMBER 1 JANUARY 1972

Level Structure of ^{178}Hf

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The level structure of ^{178}Hf has been studied via γ singles, γ-γ coincidences in the 2048 $\times$4096 channel mode, and γ-γ directional-correlation measurements with Ge(Li) detectors from the decay of ^{178}Ta. These data yielded 7 new transitions and the addition of 13 transitions to the decay scheme, which includes 5 new levels. The following levels were observed (energies in keV followed by the spin and parity): 93.13, 2^+; 306.52, 4^+; 1174.64, 2^+; 1199.24, 0^+; 1276.54, 2^+; 1309.91, $1^\pm$, 2^+; 1362.36, $2^{(-)}$; 1433.97, 0^+; 1443.86, 0^+; 1496.02, 2^+; 1513.64, $1^\pm$, 2^+; 1561.27, 2^+; 1566.45, $1^\pm$, $2^\pm$; and 1771.95, 0^+. γ-γ directional correlations were measured with the $2^+ \rightarrow 0^+$, 93.13-keV transition and eight transitions that feed the first excited state. From the γ-$\gamma(\theta)$ data, spins of 2 and 0 are established for the 1362.36- and 1771.95-keV levels and $M1$ admixtures of $85.6^{+1.2}_{-1.4}$ and 65.2 ± 3.2% for the 1183.40- and 1402.87-keV transitions. With these multipole admixtures, the branching ratios for the $K^\pi I = 0^+2$, 1276.54-keV level cannot be brought into agreement with theory for a single Z_β band-mixing parameter; while for the $K^\pi I = 0^+2$, 1496.02-keV level they can be.

I. INTRODUCTION

The Bohr-Mottelson model,[1] even with perturbational corrections for admixing of the β, γ, and ground-state bands,[2] appears to be unable at present to explain the branching ratios[3-5] from the β-vibrational states in ^{152}Sm, ^{154}Gd, and ^{156}Gd. The earlier proposal[4,6] that $M1$ admixtures in the $\Delta I = 0$ transitions between the β and ground bands may prove the way to bring theory and experiment into agreement has not been found true[7,8] for ^{152}Sm and ^{154}Gd. These two nuclei are often referred to as transitional nuclei, since they are just at the beginning of the region where permanent deformation is observed and are not highly deformed. When the effects of band mixing are included, the Bohr-Mottelson model has adequately described[6] the transitions from γ bands in highly deformed nuclei and is in near, though not exact, agreement with experiments[3] on the branching ratios from the γ bands in ^{152}Sm and ^{154}Gd. It seems most important that in strongly deformed nuclei we check very carefully the properties of β vibrations, as well as of all near-lying excited states which may present any perturbations to them. The nucleus ^{178}Hf is an excellent candidate for a detailed study of this problem.

The energy levels of ^{178}Hf populated by the decay of ^{178}Ta have been studied by Nielsen *et al.*[9] who observed nine excited states. Of particular interest in our present work is a study of the properties of the 0^+ and 2^+ levels of ^{178}Hf at 1199 and 1276 keV, respectively. The strong $E0$ component observed[9] in the $2^+ \rightarrow 2^+$, 1183-keV transition from the 1276-keV level to the 93-keV first excited state indicates a $K=0$ assignment for the 1276-keV level. Guided by this and the energy spacings, Nielsen *et al.*[9] proposed that the 1199- and 1276-keV levels are the 0^+ and 2^+ members of the β-vibrational band. The $K=0$ character of these levels also has been established by comparison of the experimental branching ratio of the β feed to these levels with the ratio predicted by the intensity rules of Alaga *et al.*[10] To add further support to their argument, Nielsen *et al.*[9] pointed out that the moment of inertia calculated from these two levels differed by only 20% from that calculated for the ground-state rotational band – in keeping with the observed trends.[11] A 4^+ member of this proposed β band has been reported in (n, e) work.[12]