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Neutron Radius of ^{208}Pb from 166-MeV Alpha-Particle Scattering

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The elastic scattering of α particles is analyzed with an optical potential obtained by a simple folding of the nuclear-matter distribution with an α-nucleon interaction. The ^{208}Pb neutron-matter radius is extracted using the charge-distribution parameters. The result $r_{nm}=5.75\pm0.09$ fm is compared with other determinations and some theoretical calculations.

The proton distribution in nuclei has been obtained from elastic electron scattering or from muonic x-ray spectra. The rms charge radius $r_c=\langle r_c^2\rangle^{1/2}$ and the surface thickness of the charge distributions are relatively well known.[1] The situation is quite different for neutron distributions. If r_p, r_n, and r_{nm} are, respectively, the rms radii of proton, neutron, and neutron-matter distributions, we have $\langle r_p^2\rangle=\langle r_c^2\rangle-0.64$ fm^2 and $\langle r_n^2\rangle=\langle r_{nm}^2\rangle-0.64$ fm^2, the difference corresponding to the nucleon dimension. Different methods have been used for the determination of r_{nm} by means of sometimes questionable approximations. As a consequence, the theoretical results are sometimes inconsistent. The most striking example is that of ^{208}Pb, where the experimental r_{nm} values vary between 5.44 and 6.35 fm. For theoretical calculations, one needs a more precise determination.

The 166-MeV α-particle beam of the Orsay synchrocyclotron was used to measure the elastic scattering differential cross section. It has been shown previously[2-4] that it is possible to calculate an optical potential for 166-MeV α particles which gives a good fit to differential cross sections. The real part of that potential is given by the equation

$$V_{\rm opt}^{\rm R}(r_\alpha)=U_{\rm R}\int V(\vec{r},\vec{r}_\alpha)\rho(\vec{r})d\vec{r}\,, \tag{1}$$

where $\rho(\vec{r})$ is the matter distribution of the nucleus, and $V(\vec{r},\vec{r}_\alpha)$ is an α-nucleon effective interaction obtained from a nucleon-nucleon interaction $V(\vec{r},\vec{r}_\alpha)=V_0\exp\{-[(\vec{r}-\vec{r}_\alpha)/\mu]^2\}$, with $V_0=-37$ MeV and $\mu=2$ fm. For simplicity, the imaginary part of the optical potential is supposed to be proportional to the real part, i.e., $W_{\rm opt}^{\rm I}(r_\alpha)=(U_{\rm I}/U_{\rm R})\times V_{\rm opt}^{\rm R}(r_\alpha)$. It has been shown previously that this

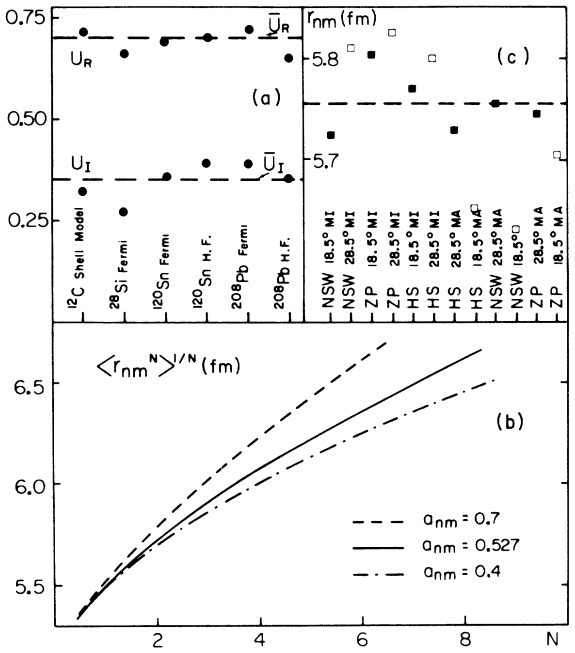

FIG. 1. (a) Determination of the mean values $\bar{U}_R$ and $\bar{U}_I$; (b) $\langle r_{nm}^N \rangle^{1/N}$ for ^{208}Pb, and (c) mean value of r_{nm} for ^{208}Pb. All radii, $\langle r_{nm}^N \rangle^{1/N}$ and r_{nm}, are in fm.

choice corresponds to a reasonable hypothesis.[2,4] A new determination of the two adjustable parameters U_R and U_I has been carried out, starting from more precise experimental results and reasonable matter distributions of different types (in order to avoid systematic errors). From Fig. 1(a) we find again[4] $\bar{U}_R = 0.70$ and $\bar{U}_I = 0.35$. The small dispersion of best values for each nucleus appearing in Fig. 1(a) proves the validity of the method used in applying Eq. (1). The mean values of these two parameters define an effective α-nucleon interaction for 166-MeV α particles. Now, using the preceding potential with $V(r, r_\alpha)$, $\bar{U}_R$, and $\bar{U}_I$ fixed, one seeks the best fit of experimental cross sections by varying $\rho_{nm}(r)$ $[\rho(r) = \rho_c(r) + \rho_{nm}(r)]$. The charge distribution used is that given by elastic electron scattering, while the neutron-matter distribution is similar in form:

$$\rho_{nm}(r) = \left(1 + w\frac{r^2}{c_{nm}^2}\right)\left[1 + \exp\frac{(r^\alpha - c_{nm}^\alpha)}{a^\alpha}\right]^{-1}. \quad (2)$$

We suppose that the parameters w, a, and α are equal for charge and neutron-matter distributions. The variation is done on the last parameter c_{nm} in order to minimize χ^2. Thus $\rho_{nm}(r)$ and r_{nm} are determined. Several critical points will be discussed now:

(A) An important assumption is made when the surface thicknesses of the charge and nuclear-matter distributions are set equal. Although there are strong arguments in favor of nearly equal surface thicknesses, we have studied the influence of a different choice on our conclusions. The different moments $\langle r_{nm}^N \rangle^{1/N}$ obtained starting from the Fermi distribution[5] (AN in Table I), but allowing a_{nm} to assume different values, are shown in Fig. 1(b). The parameter c_{nm} was always adjusted until the lowest χ^2 value was obtained. One sees that up to $N = 2$ the moments are well defined (better than 0.1 fm). Although the α scattering occurs on the surface of the nucleus, the potential between 6 and 10 fm is principally determined (owing to the fact that the effective interaction has a long range ~2 fm) by the matter density between 4 and 8 fm approximately. For the three preceding values of a_{nm}, the functions $\rho_{nm}(r) \times r^4$ are maximum at 6.5 fm and intersect at 7.5 fm.

(B) In order to investigate the precision of our result, we have determined r_{nm} starting with different (actually very similar) distributions proposed for $\rho_c(r)$. These distributions are referred to as NSW,[6] ZP,[7] and HS[8] (see Table I). In case ZP, a small correction (<0.02 fm) has been applied in order to correct the corresponding r_c value which is 5.48 fm instead of 5.50 fm, the more precise result.

(C) In the preceding method we assumed that the imaginary and real potentials were proportional. The calculations were repeated without this constraint. $W_{opt}^I(r)$ was assumed to exhibit a Woods-Saxon form $\bar{W}_I\{1 + \exp[(r - \bar{r}_I A^{1/3})/\bar{a}_I]\}^{-1}$, whose parameters are defined as mean values determined by a least-squares fit to several nuclei: $\bar{U}_R = 0.65$, $\bar{W}_I = 21.723$ MeV, $\bar{a}_I = 0.747$ fm, and $\bar{r}_I = 1.487$ fm. The values of $\bar{U}_R$ are nearly the same in the two methods. The last method is referred to as MA [macroscopic $W_{opt}^I(r)$] as opposed to MI [$W_{opt}^I(r)$ proportional to $V_{opt}^R(r)$, though $\bar{U}_I/\bar{U}_R$].

(D) The experimental measurements have been

TABLE I. Code and parameters of charge distributions used. All lengths (c_c and a_c) are in fm.

	Symbol	W_c	α_c	c_c	a_c
Anderson (Ref. 5)	AN	0	1	6.636	0.5273
Nolen, Jr., Schiffer, and Williams (Ref. 6)	NSW	0.4	1	6.30	0.57
Ziegler and Peterson (Ref. 7)	ZP	0	1	6.66	0.5
Heisenberg *et al.* (Ref. 8)	HS	0.338	2	6.3	2.888

done in 1° intervals with an angular resolution $\Delta\theta \approx 0.7°$. This resolution fills up the angular-distribution minima, so the corresponding experimental values are neglected in the fit. The method MA allows a better adjustment in a greater angular region. Beyond 30°, it may be that multiple scattering becomes important, in which case the experimental oscillations are reduced and then disappear. This angular region is neglected in the fit. The search for the best-fit χ^2 was done in two angular regions (up to 18.5° and up to 28.5°). The results are drawn in Fig. 1(c) for the three distributions defined in (B). The filled squares correspond to results of better confidence that represent MI up to 18.5° and MA up to 28.5°. The empty squares correspond to results of less confidence (MI up to 28.5° and MA up to 18.5°). It is worthwhile to remark that the mean value of r_{nm} corresponding to the six better fits is the same as the mean value corresponding to six other fits (in this case the dispersion is more important).

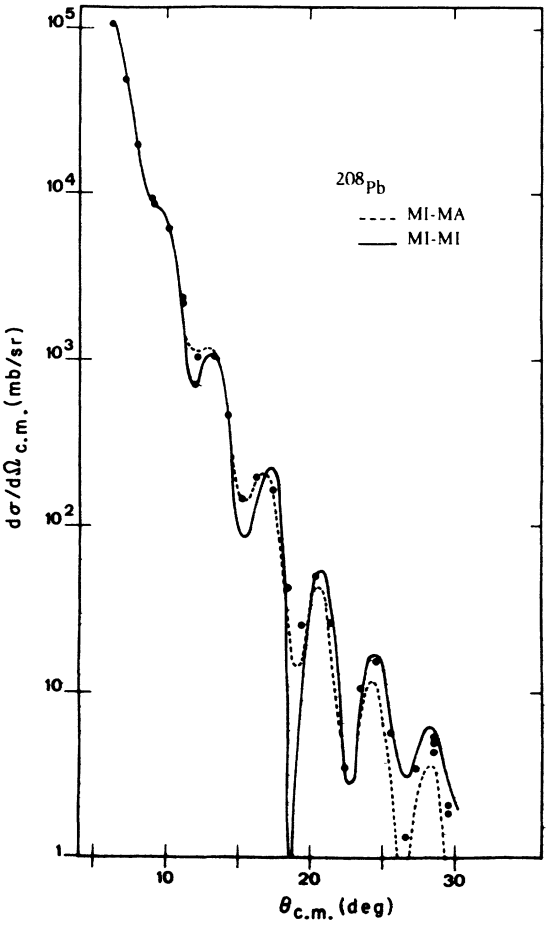

FIG. 2. Differential cross sections (diffractional angular range only).

(E) A variation of ±5% in $\overline{U}_R$ may be considered as the precision on that parameter (±0.035). It includes five of the six determinations drawn in Fig. 1(a). All parameters being unchanged, this corresponds to a variation of ±0.9% in r_{nm}. Indeed the steep slope of the distribution $\rho_{nm}(r)$, in the most useful region around 6.5 fm, explains why a noticeable variation of $\overline{U}_R$ may be compensated by a small shift of $\rho_{nm}(r)$ along r, r_{nm} remaining well defined.

(F) A variation of the range μ of the αN interaction would be compensated by different values of $\overline{U}_R$ and $\overline{U}_I$. The same result has been presented by Jackson[9,10] through detailed analysis.

With all these uncertainties taken into account, the mean value of the neutron-matter distribution of ^{208}Pb may be estimated as 5.75 ± 0.09 fm. Experimental results and best-fit curves are drawn in Fig. 2.

Several other determinations using different methods have been performed for ^{208}Pb, and the corresponding results are plotted in Fig. 3:

(a) From Coulomb displacement energies, the neutron excess density may be extracted, and therefore the neutron radius (it is supposed that neutron and proton cores are the same[11-14]). However, there are corrections missing which are difficult to compute with accuracy, but which have been estimated and discussed[15] for some other nuclei. The error bars presented in this last work are typically of ±0.05 fm.

(b) Starting from experimental results on elastic proton scattering, an analysis similar to ours

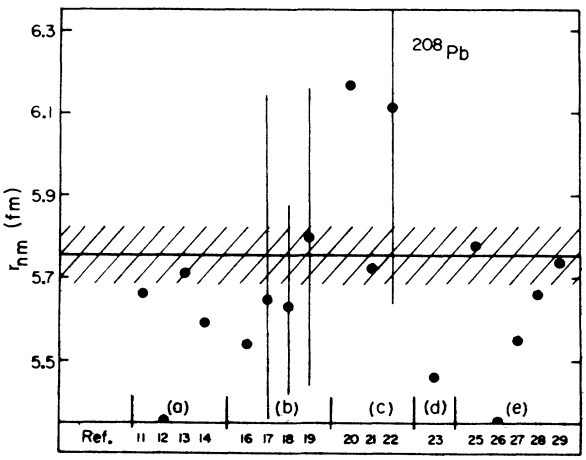

FIG. 3. Different values of the neutron-matter rms radius for ^{208}Pb. The dashed region corresponds to this work. Other results have been obtained by (a) Coulomb-energy differences, (b) elastic proton scattering analysis, (c) Woods-Saxon wells adjusted in order to give good single-particle energies, (d) pion scattering analysis, and (e) Hartree-Fock calculations.

has often been presented.[16-19] The complexity of the nucleon scattering description (exchange terms, uncertainty of the nucleon-nucleon interaction to be used) explains the great uncertainty in the final results.

(c) The adjustment of the local Woods-Saxon well parameters in order to reproduce the experimental separation energies in the vicinity of the Fermi limit allows the calculation of proton and neutron distributions, and hence r_{nm}.[20] In the work of Beiner,[21] these parameters are constrained by several relations between potentials and distributions deduced from consistency criteria. These parameters are valid for all spherical (and not light, $A \geq 32$) nuclei. In Elton's work,[22] the parameters determining the proton and neutron mean potential wells are independent, and are adjusted for each nucleus.

(d) An optical potential describing absorption and diffraction of $\pi^{\pm}$ has been calculated using proton and neutron densities and π-nucleon phase shifts.[23] The error bar quoted in Ref. 23 is rather small (±0.08 fm). However, there are great deviations among different determinations of the neutron radius from pion scattering results; for example, a previous work[24] claimed $r_n - r_p \approx 0.6$ fm instead of −0.1 fm for the result drawn in Fig. 3.

(e) Using two-body forces, theoretical calculations with the Hartree-Fock method have been done.[25-29] The radii were found to be dependent on the interaction chosen. Our analysis is in agreement with the theoretical results obtained with the interaction of Brink and Boeker,[25] and also with the new Skyrme interaction.[29] Of course it would be of great interest if theorists could assign some imprecision bars to their results. We understand that this is a very difficult task, since it implies the discussion of the choice of the two-body force and the higher-order corrections.

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