$N < 93$, however, the values from Zeldes, Grill, and Simievic[15] are substantially greater. In fact, it appears that if lines were to be drawn through each set of predicted values, the slope for Ref. 15 would be steeper than that for Ref. 14. It is clear then that by measuring α-decay energies, e.g., for osmium and rhenium isotopes with $N < 93$, one could determine which of the two calculations does a better job of predicting nuclear masses in this region of the Periodic Table.

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Angular Distribution of Alpha Particles Emitted in the Fission of ^{252}Cf[†]

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The angular distribution of α particles emitted in the spontaneous fission of ^{252}Cf has been determined with respect to the light fragments and with respect to the heavy fragments. The full width at half maximum (FWHM) for the angular distribution of α particles is 23.5° – distinctly narrower than has been reported in the literature. The FWHM for the angular distribution of α particles emitted in coincidence with all the fragments is 37.5°. Trajectory and Monte Carlo calculations were done to find a set of initial parameters that give rise to an angular distribution and energy spectrum of α particles in agreement with experiment. The parameters so obtained were: D is the interfragment distance at scission (21.5×10^{-13} cm); t is the time of emission of α particle (0.4×10^{-21} sec); E_0 is the average initial energy of α particle at scission (2 MeV); σ is the standard deviation in the initial position of α particle (2.5×10^{-13} cm). These parameters are contradictory to predictions of the statistical model. Furthermore, we have found that parameters given by the statistical model at fission lead to a calculated energy spectrum that is not in agreement with experiment.

I. INTRODUCTION

The angular distribution of α particles emitted in fission is strongly peaked at about 90° with respect to the direction of the motion of the two major fission fragments. This sharp peaking results because the α particles are produced between the two larger fragments at approximately the moment of scission. The subsequent trajectory of the α particles (and other light charged particles) is strongly dependent on the configuration of the system at scission and is, therefore, a useful probe of this configuration. It has been shown by several investigators[1-4] that fission accompanied by emission of light particles is similar to the binary-fission process in many respects. Because of this similarity, it is believed that the scission configuration is approximately the same in both processes and that any conclusions drawn about scission from studies of the light particles, which are emitted only rarely, should be applicable to binary fission, the more probable process.

In particular, analyses of experimental data on fission accompanied by emission of α particles have yielded information on the separation and kinetic energies of the fragments at scission as well as on other parameters. Such information is useful as a check on the models used to explain the fission phenomenon. For instance, in his statistical-model calculations of fission, Fong[5] has assumed that the kinetic energy of the fragments at scission is less than 1 MeV. On the other hand, Nix and Swiatecki,[6] who calculated probability distributions in a fissioning system using the dynamics of the division of a liquid drop, have predicted a much higher value of 25–40 MeV for the translational kinetic energy of the fission fragments at scission.

There have been numerous attempts to determine this kinetic energy at scission. Boneh, Fraenkel, and Nebenzahl[7] found agreement between Fraenkel's experimental results and calculated quantities under the assumption that the fission fragments have about 40 MeV of kinetic energy when the α particle is released. Fong and his co-workers,[8,9] on the other hand, find agreement between experiment and calculation assuming that this energy is less than 1 MeV. Other investigators have drawn conclusions between these extremes: Raisbeck and Thomas,[10] in analyzing their own data, found that an assumed kinetic energy of 7.5 MeV gave good results. Krogulski and Blocki[11] conclude that a very low kinetic energy is inconsistent with the broad angular distribution found by Fraenkel. Musgrove[12] has fitted the energy spectrum of emitted α particles with a scission kinetic energy of 25 MeV, but has calculated an angular distribution significantly narrower than that reported by Fraenkel. Reliable data on the energy and angular distribution of the α particles and the correlation of these distributions with the other parameters of fission are needed to help settle these discrepancies.

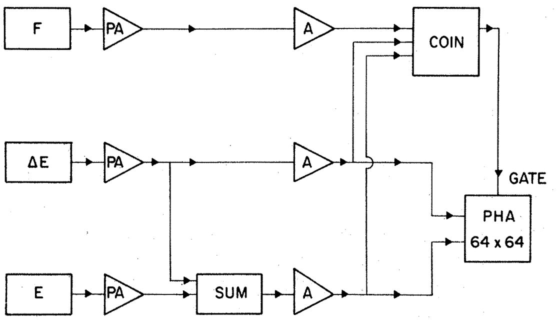

FIG. 1. Schematic diagram of the electronics used for the first series of experiments on ^{252}Cf fission. See text for a full description.

The energy and angular distributions of α particles emitted in fission have been investigated by many workers.[1,10,13-18] While there is general agreement in the energy-dsitribution data from all these experiments, there is a discrepancy in the angular-distribution data in the fission of ^{252}Cf obtained by Thomas and co-workers[10,17,18] and those obtained by Fraenkel.[1] Fraenkel has found the angular distribution to be rather broader than is consistent with the results obtained by Thomas and co-workers. This difference is significant because the different conclusions about the total kinetic energy of the fragments at scission depend on a knowledge of this distribution. It is difficult, however, to make a rigorous comparison between the two sets of data because, while Thomas and co-workers measured the angular distribution of α particles emitted in coincidence with all fission fragments, Fraenkel measured the α particles emitted in coincidence with light fragments only. Furthermore, Fraenkel's two fission detectors were 180° apart, although the fission fragments associated with long-range α emission are 185° from one another. This geometric arrangement should lead to a distortion of the angular distribution, but not to a significant broadening. In order to resolve the discrepancy between Fraenkel's results and those of Thomas *et al.*, we have remeasured the angular distributions of α particles accompanying the spontaneous fission of ^{252}Cf with respect to the direction of motion of the light fragment and with respect to the direction of motion of the heavy fragment.

II. EXPERIMENTAL PROCEDURE

The angular distribution of α particles was studied in two different experimental arrangements. The first series of experiments were done using the setup shown in Fig. 1. The source and the detectors were placed in a hemicylindrical vacuum chamber. The fission detector was fixed in a position at 37.5° to the plane of the source and the α-particle detector system was placed at 52.5° for 90° measurements and at other corresponding angles for other measurements. The angles subtended at the source were 15° for the fission detector and 7° for the α-particle detector. The source consisted of ^{252}Cf on aluminum backing and was covered with a nickel foil of thickness 450 μg/cm^2. The source strength was 4×10^7 fissions/min. The α particles were detected in a conventional counter telescope. The ΔE and E detectors were surface-barrier detectors of thickness 60 and 1500 μ, respectively. The ΔE detector was covered with aluminum foil of thickness 9.35 mg/cm^2 to cut off natural α particles and fission fragments emitted by ^{252}Cf. The fission fragments were detected by an ORTEC heavy-ion surface-barrier detector of thickness 85 μ.

The ΔE and E signals were sent through preamplifiers (PA), a summing amplifier, and then to an amplifier (A). The ΔE signal was also sent to another amplifier. The ΔE signal, $\Delta E+E$ signal, and the fission-fragment signal were sent to a triple-coincidence system and the output of the coincidence system was used to gate the spectrum in the analyzer. α particles whose energies were greater than 7 MeV when incident on the cover foil were accepted by this system. A resolution time of 340 nsec was used for the coincidence system. The ΔE and $\Delta E+E$ signals were stored in the 64×64 matrix of the analyzer. The display from such a system and the method of identification of the particles have been previously described.[17] An energy calibration of the detectors was obtained using α particles from a ^{241}Am standard and a pulser.

For each angle between the fission-fragment and the α-particle detectors, two types of measurements were made. In the first, α particles emitted in coincidence with all the fission fragments were counted. In the second, α particles emitted in coincidence with light fragments alone were counted. For this purpose, the single-channel analyzer (SCA) in the fission channel of the coincidence system was used, and was set to accept only fission fragments with kinetic energies corresponding to the light-fragment peak of the kinetic energy spectrum. The fission spectrum shifted during the course of the experiment, because of the exposure of the fission detector to radiation; this shift was monitored and the gate was suitably lowered to compensate for the shift. The chance coincidence rate was 15% of the true coincidence rate at 60° and was much less at 90°.

In the second series of experiments, the spectrum of fission fragments emitted in coincidence with α particles was taken at each angle using the setup shown in Fig. 2. The geometric arrangements were the same as for the first series. The α detector was a thin one, 60 μ thick, and the fission detector was again a heavy-ion surface-barrier detector 85 μ thick and was covered with an aluminum foil 9.35 mg/cm^2 thick. The angles subtended at the source were 15° for the fission detector and 7° for the α-particle detector. The α-particle and fission-fragment signals were sent through time-pickoff (TPO) units, which provided start and stop signals for the time-to-amplitude converter (TAC). The output from the converter was amplified and the signal was sent to a slow-coincidence system through a SCA. The SCA accepted only those events when the α-particle and fission-fragment signals came within 20 nsec

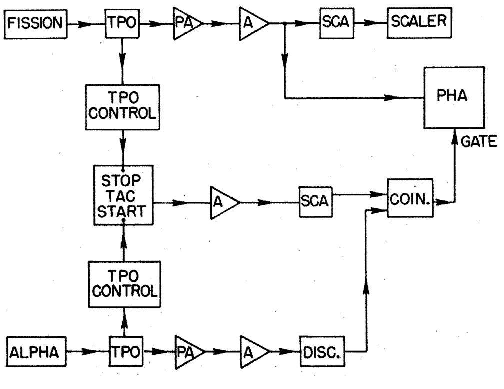

FIG. 2. Schematic diagram of the electronics used for the second series of experiments on ^{252}Cf fission. See text for a full description.

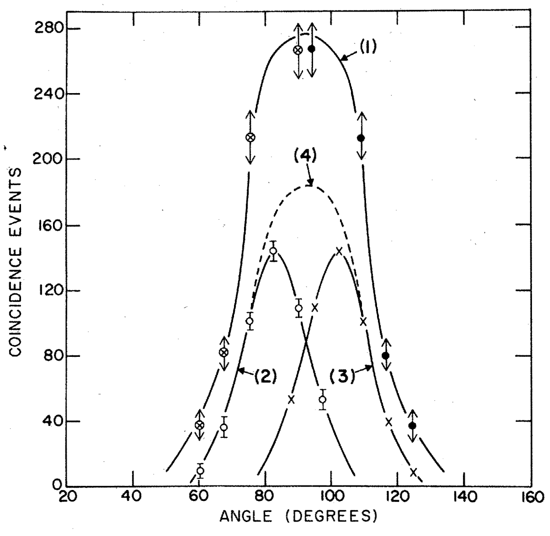

FIG. 3. Angular distribution of α particles emitted in coincidence with fission fragments. Curve (1), coincidence with all the fragments. The solid points are mirror reflections of the open points about 92°. Curve (2), coincidence with light fragments. Curve (3), coincidence with heavy fragments: obtained by mirror reflection of curve (2). Curve (4), coincidence with all the fragments obtained by combining curves (1) and (2).

of each other. The α-particle signal was sent to a discriminator and then to the slow-coincidence system. The discriminator level was set so that protons and tritons, which deposited less than 3 MeV in this detector, were cut off. α particles with energies greater than about 8 MeV when incident on the cover foil were sufficiently energetic to trigger the discriminator. The fission-fragment signal was amplified and analyzed by a pulse-height analyzer (PHA). A gating signal was generated when there was both a true fast coincidence and an α particle. This signal was used to gate the analyzer in which the fission spectrum was stored. The shift of the fission spectrum during the course of the experiment was taken care of in the following way: The fission-fragment signal was sent through a SCA and then to a scaler. The window was set so that only the light fragments were accepted. The count rate in the scaler was constantly monitored and as the count rate in the scaler decreased, the gain of the amplifier was increased to maintain a constant count rate and an undistorted spectrum.

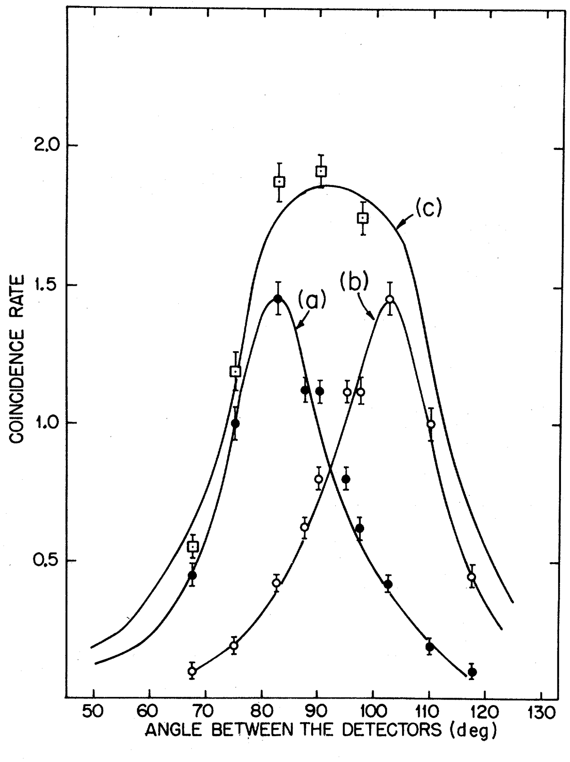

FIG. 4. Angular distribution of α particles emitted in coincidence with fission fragments. Curve (a), angular distribution of α particles measured in coincidence with light fission fragments. Curve (b), angular distribution of α particles measured in coincidence with heavy fission fragments. Curve (c), angular distribution of α particles emitted in coincidence with all the fission fragments.

III. RESULTS

The results or the angular-distribution measurements in the first setup are shown in the Fig. 3. Curve (1) shows the measured angular distribution of α particles emitted in coincidence with all the fission fragments. The full width at half maximum (FWHM) for this curve is 38° compared with the value of 35° obtained by Raisbeck and Thomas. Curve (2) shows the measured angular distribution of α particles emitted in coincidence with light fragments only. The FWHM for this curve is 23° in comparison to the value of 32° obtained by Fraenkel in a similar measurement. From curve (2) the angular distribution of α particles emitted in coincidence with heavy fragments alone can be deduced by reflecting this observed spectrum around an angle of about 92.5°. This reflected distribution is shown in Fig. 3 as curve (3). The combination of curves (2) and (3) gives the angular distribution of α particles emitted in coincidence with all the fission fragments and this is shown in curve (4). The FWHM for curve (4) is 36°, in agreement with the value of 35° obtained by Raisbeck and Thomas. There is a discrepancy in the value of the α count rate between curves (1) and (4), which means that some α particles were lost when gating on the fission spectrum (approximately 34% of the events). As noted above, the gating was done by setting the discriminator to accept only light fragments of the binary kinetic energy spectrum. The kinetic energy spectrum in long-range α fission is, however, shifted by about 6 MeV compared with the fission spectrum in binary fission (cf. Fig. 6 of Ref. 1). As a result some light fission fragments were lost in these coincidence experiments. To circumvent this difficulty, the binary fission spectrum was taken, the discriminator was put at the exact minimum between light and heavy fragments, and then lowered by 6 MeV. This discriminator was now assumed to be at the symmetry point in the kinetic energy spectrum of the long-range α fission. Under these conditions, the coincidence experiments were repeated at three angles. Even under this setup 28% of the events were lost. Since the discriminator level could not be easily set at the exact symmetry point of the fissioning spectrum, some coincident events were lost in this method of determining the angular distribution of α particles and so the second experimental arrangement given in Fig. 2 was used.

In the second experimental setup the spectrum of fission fragments emitted in coincidence with α particles is given as the output from the analyzer. By summing the portion of the spectrum corresponding to light fragments alone at each angle, we obtain the angular distribution of α particles with respect to light fragments. By summing the other portion of the spectrum we obtain the angular distribution of α particles with respect to heavy fragments. By combining these two sets of data we obtain the angular distribution with respect to all fragments. The results are shown in Fig. 4. Curve (a) shows the angular distribution of α particles emitted in coincidence with light fragments. Curve (b) shows the angular distribution with respect to heavy fragments. Curve (c) shows the angular distribution with respect to all fission fragments.

The angular distributions shown in Fig. 4 are broader than the true distributions because of the angular resolution of the two detectors. To correct for this we use the following procedure: We assume that the intrinsic angular distribution is described by a Gaussian curve of standard deviation σ. We fold such a distribution with the dispersion introduced by the detectors and compare the resulting distribution with the experimental results. We vary σ until there is a good match between the calculated and experimental data. In this way we have obtained approximate widths for the intrinsic angular distributions. For α particles emitted with respect to a specific fragment we find $\sigma = 10°$, corresponding to a FWHM of 23.5°. This value is significantly narrower than the value of 32° reported by Fraenkel for the FWHM. For α particles emitted with respect to all fragments we find $\sigma = 16°$, corresponding to a FWHM of 37.5°. This is to be compared with the values of 33°, reported by Atneosen, Thomas, and Garvey,[18] 28°, reported by Thomas and Whetstone,[17] and 35°, reported by Raisbeck and Thomas,[10] and a value of 49° inferred by Atneosen, Thomas, and Garvey from Fraenkel's data.

Although there is still a significant spread in the recent results reported for the angular distribution, it seems safe to conclude that the distribution is significantly narrower than that reported by Fraenkel. Inferences drawn on the basis of Fraenkel's angular data should, therefore, be viewed with caution. It should also be noted that measurements[19, 20] of the angular distribution of α particles with respect to light fragments in the thermal-neutron-induced fission of ^{235}U give a FWHM of 23 to 25°, in agreement with our measurement for ^{252}Cf.

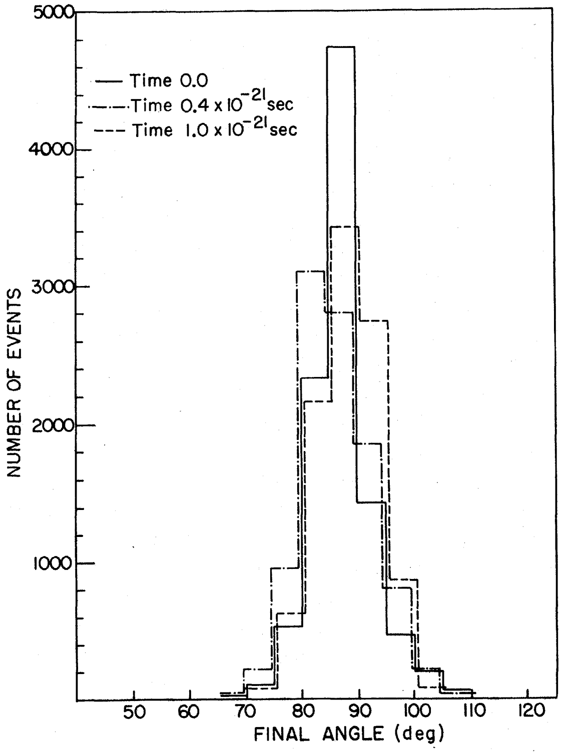

FIG. 5. Results of Monte Carlo calculations plotted as the distribution of angles (with respect to the light fragment) of α particles for various emission times. σ for the initial position of the α particle is 1.5×10^{-13} cm.

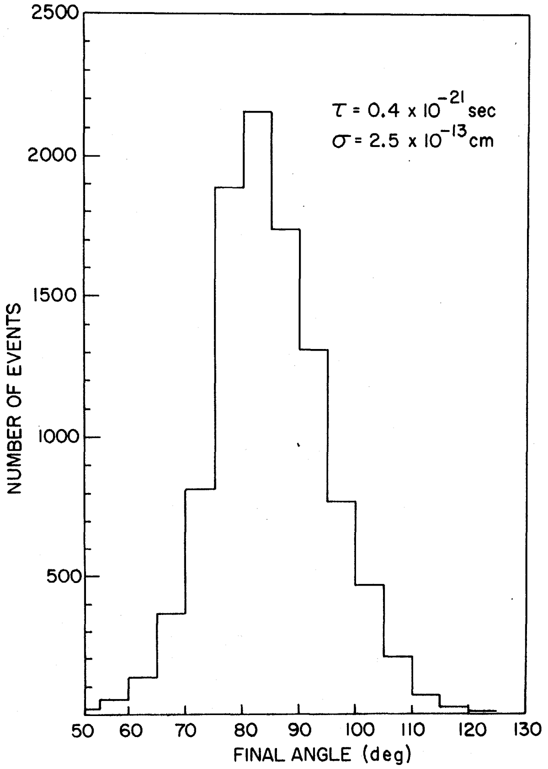

FIG. 6. Result of Monte Carlo calculation plotted as the distribution of angles (with respect to the light fragment) for emission time $t = 0.4 \times 10^{-21}$ sec and standard deviation in the position of α particle $\sigma = 2.5 \times 10^{-13}$ cm.

IV. TRAJECTORY CALCULATIONS

The problem of the emission of α particles in fission can be considered from two points of view, the first dealing with the mechanism leading to the release of the α particle and the second with the motion of the α particle (once it is released) in the Coulomb field of fission fragments.

The first problem has been discussed by many authors. Ramanna and co-workers[21, 22] explained their data on long-range α fission by assuming that the α particles in fission are evaporated from a highly deformed compound nucleus before scission. Halpern, rejecting the evaporation hypothesis, has suggested[23] that the α-particle ejection is caused by the sudden change in potential as the neck between the two fragments collapses at scission. An intermediate approach, considered by Feather,[24-26] is that there is a two-stage process: first, the division into two major fragments and, then, the subsequent emission of an α particle from one of them.

The second aspect has been to determine a set of initial dynamical variables from which the final energy and angular distributions of light particles could be calculated and compared with experiment. The purpose of such trajectory calculations is to determine what effects are due to the Coulomb interactions among the fragments in order to be able to focus more clearly on the release mechanism and the properties of the system at scission. It has been assumed, either explicitly or implicitly, that the set of initial dynamical variables so derived refer to the scission point. It should, however, be recognized that if long-range α-particle emission is a two-step process, these derived quantities refer to a time later than scission.

The first detailed trajectory calculations were done by Boneh, Fraenkel, and Nebenzahl[7] who attempted to fit the data on energy-angle correlations of α particles in ^{252}Cf fission. Raisbeck and Thomas[10] carried out similar trajectory calculations to fit their data on energy distributions of light charged particles emitted in the spontaneous fission of ^{252}Cf. They attempted to find, with partial success, one set of initial parameters that would fit the energy spectra for all of the different kinds of light charged particles emitted in fission. The calculated energy distributions were generally in agreement with the experimental data. However, the calculated angular distribution of α particles was narrower than that found experimentally. Since the present investigation gives unambiguous information on the angular distribution of α particles, it was considered worthwhile to extend their trajectory calculations in an attempt to see whether a better fit with the experimental data can be obtained.

The model assumes that the incipient fission fragments start moving apart under their mutual repulsion at some time zero. At some time later, t, a third particle appears at a specified point between the two major fragments and with a specified velocity. The final velocity of the three particles is calculated by numerically integrating the equations of motion. Sets of initial conditions are found that give rise to the energy and angle of emission of the third particle which are in agreement with the observed average values of these quantities. A Monte Carlo calculation of the final distributions of energies and angles of emission of α particles is then done by assuming certain distributions of the initial conditions (such as the position and energy of the α particle) about the averages. Sets of initial conditions that give agreement with the averages but not with the distributions are rejected.

The adjustable parameters used by Raisbeck and Thomas to fit the energy spectra are t, the time of emission of the α particle, and E_0, the average kinetic energy of the α particle at the time of emission. The initial momentum distribution for the α particle was taken to be Gaussian in each coordinate with the standard deviation chosen to give the correct value of the average initial energy E_0; the average initial position of the third particle was chosen to be on the axis between the two major fragments at the potential minimum; and the initial distribution of the third particle along this axis was taken to be Gaussian with an arbitrarily chosen standard deviation $\sigma = 1.5\times10^{-13}$ cm. The calculated energy spectra were not sensitive to the choice of σ. There is a one-to-one correspondence between the value of t and the distance D between the two major fragments that has been used by others to parametrize this problem.

Raisbeck and Thomas found a good fit to the α-particle spectrum with $t = 0.4\times10^{-21}$ sec ($D = 21.5 \times 10^{-13}$ cm) and an average initial α-particle energy of 2.0 MeV. They were not able to fit the energy spectrum satisfactorily with either significantly shorter times ($t = 0$, $D = 20.5\times10^{-13}$ cm) or significantly longer times ($t = 1.0\times10^{-21}$ sec, $D = 26 \times10^{-13}$ cm). As noted above, they calculated an angular distribution significantly narrower than was found experimentally.

Using the same procedure as did Raisbeck and Thomas, we have tried to find parameters that would give a better fit to the observed angular distribution without sacrificing the good fit to the energy spectra. We have begun by considering the same values of t, E_0, and σ that were investigated by Raisbeck and Thomas. The results of these calculations are shown in Fig. 5. The most probable angle of emission is about 87° for both $t=0$ and $t=1$ and 82° for $t=0.4$, compared to an experimental value of about 82°. The FWHM values are about 5° for $t=0$ and 15° for both $t=0.4$ and $t=1$ compared with an experimental value of 23°. The calculation shows that for $t=0$ and $t=1$, neither the most probable angle of emission nor the width of the distribution agree with the experimental values. For $t=0.4$, the most probable angle of emission agrees with experimental values, but the width is narrower than the experimental value. Further Monte Carlo calculations were done only for $t=0.4$.

According to Raisbeck and Thomas, change in the value of σ does not affect the energy distribution, but does affect the angular distribution. The result of the calculation with $\sigma=2.5\times10^{-13}$ cm is shown in Fig. 6. Here we see that the most probable angle of emission is 82° and the FWHM is 20°. The most probable angle of emission as well as the width of the angular distribution in this case is more consistent with the experimental value than in the other cases. Thus the following set of initial values is found to give rise to both an angular distribution and energy distribution in agreement with experiment: $t=0.4\times10^{-21}$ cm ($D=21.5\times10^{-13}$ cm), $\overline{E}_0=2.0$ MeV, $\sigma=2.5\times10^{-13}$ cm. Earlier and later times do not give energy spectra in agreement with experiment.

V. COMPARISON WITH OTHER CALCULATIONS

Many trajectory calculations have been published recently to fit the experimental data on the long-range α fission of Cf^{252}.[7,10-12,27,28] The initial conditions determined by these calculations are shown in Table I. We see that a wide range of conclusions has been drawn as a result of these calculations. Since many of these are based on fits to Fraenkel's experimental angular distribution or to his measurements of correlation between energy and angle, we feel that they cannot be taken as strong conclusions. Only the calculations we have done and those reported by Musgrove[12] and by Vitta[27] are in good agreement with both the experimental energy spectrum of the α particles and the experimental angular distribution of α particles. The initial conditions found by us and by Musgrove are in substantial agreement – the fission fragments have attained a significant portion of their final velocity when the α particle is released. Vitta, however, has concluded that both the energy spectrum and angular distribution can be fitted under the assumption of the statistical theory that the fragments have zero velocity when the α particle is released. In order to understand this discrepancy between his conclusion and ours, we must examine his calculation in detail.

Let us look at the calculation of the energy spectrum of α particles. In principle, the method is to establish the relationship between the final α-particle energy and the initial parameters, which are the initial kinetic energy, position, and direction of the α particle and the initial kinetic energies and positions of the fission fragments. The final energy spectrum is then determined from this relationship combined with appropriate distributions of the initial parameters. We can il-

TABLE I. Initial parameters derived from trajectory calculations. D is the international distance at scission; $\overline{E}_0$ is the average kinetic energy of α particles at scission; σ is the standard deviation in the initial position of the α particles; t is the time of emission of α particle; E_k is the total kinetic energy of both fission fragments at scission time.

Author	D (fm)	$\overline{E}_0$ (MeV)	σ (fm)	t (10^{-21} sec)	E_k (MeV)	Ref.
Rajagopalan and Thomas	21.5	2.0	2.5	0.4	7.5	Present work
Boneh, Fraenkel, and Nebenzahl	26.0	3.0	4.0	1.0	40	7
Raisbeck and Thomas	21.5	2.0	1.5	0.4	7.5	10
Musgrove	23.7	2.75	2.3	···	25	12
A. Katase	26.75	···	···	···	63	28
Krogulski and Blocki	26.0	2.0	···	···	40	11
Vitta	24.3	1.2	2.2		0	27

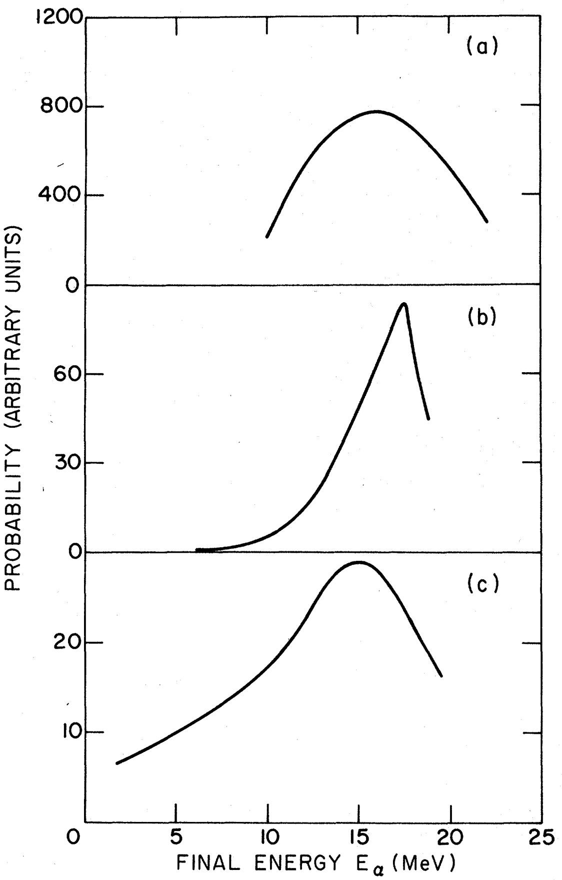

FIG. 7. Energy distribution of α particles. Curve (a), experimental energy distribution given by Raisbeck and Thomas. Curve (b), energy distributions calculated with the Jacobian. Curve (c), energy distribution calculated without the Jacobian. (See text.)

lustrate the salient feature of such a calculation by confining ourselves to one set of initial parameters for the fission fragments, and by noting that the final α-particle energy depends only weakly on its initial position and direction. The final spectrum of α-particle energies $P_\infty(\epsilon_\infty)$ is then determined by the initial spectrum of α-particle energies $P_0(\epsilon_0)$ and by the unique correspondence between initial energy ϵ_0 and final energy ϵ_∞. The probabilities are related by the equation:

$$P_\infty(\epsilon_\infty) = P_0(\epsilon_0) d\epsilon_0 / d\epsilon_\infty ,$$

where ϵ_0 is the inital energy that leads to the indicated final energy. In doing his calculations of the final spectrum, Vitta has ignored the Jacobian $d\epsilon_0/d\epsilon_\infty$. This oversight leads to a significant error in the final result as can be seen from Fig. 7. In this figure curve (a) represents the experimental energy spectrum of α particles. Curves (b) and (c) are based on Vitta's relationship between ϵ_∞ and ϵ_0 and his spectrum of initial α-particle energies (the prediction of the statistical model). Curve (b) is calculated with the expression given above, including the Jacobian. Curve (c), on the other hand, is calculated without the Jacobian (as has been done by Vitta). We note that the curve (c) is in reasonably good agreement with curve (a), leading to the erroneous conclusion that the statistical model can give the correct final-energy spectrum. We note that curve (b), which has been calculated correctly, is in poor agreement with the experimental spectrum.

We conclude therefore, that the initial parameters given by the statistical model of fission lead to an energy distribution that is not in accord with experiment. The apparent agreement that has been obtained by Vitta arises because of incorrect assumptions that were made in his calculations.

VI. ACKNOWLEDGMENTS

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Level Scheme of ^{235}U and the Distribution of Single-Particle Strength in Its Excited States[*]

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The spectra of low-lying states of ^{235}U have been studied using the reactions ^{233}U$(t,p)^{235}$U, ^{235}U$(d,d')^{235}$U, ^{234}U$(d,p)^{235}$U, ^{236}U$(d,t)^{235}$U, and ^{234}U$(n,\gamma)^{235}$U. Using intensity ratios and angular distributions from the charged-particle reactions, primary γ-ray excitations, patterns of γ-ray deexcitations, and rotational-band systematics, 80% of the levels observed up to 1500 keV have been assigned to 23 individual rotational bands. Some of the rotational bands can be associated with expected single-particle excitations, but in many cases it is shown that the single-particle strength is fragmented, giving rise to several rotational bands with similar properties. Mechanisms for this fragmentation are discussed and the distribution of single-particle strength over the states observed is examined and compared with theoretical predictions. The neutron binding energy for ^{235}U is determined to be 5297.6 ± 0.5 keV.

I. INTRODUCTION

In recent years level spectra of deformed nuclei in the rare-earth region have been extensively investigated,[1] and the Nilsson model[2] of single-particle excitations has been found to describe very well the low-energy structure of odd-A nuclei in this region. However, at excitation energies above about 1 MeV the Nilsson description must be modified to include collective three-quasiparticle excitations,[3,4] corresponding to the coupling of a Nilsson single-particle state to a one-phonon vibrational excitation of the core (a particle + phonon state). In the rare-earth region the lowest one-phonon states are usually of the quadrupole type, occurring typically around 1 MeV.

In the actinide region one-phonon core excitations of the octupole type also lie low in excitation energy, and are expected to make the structure of odd-A actinide nuclei complex at low excitation energies. There are, for example, six known vibrational states in ^{234}U below 1.5 MeV, the lowest being a $K^\pi = 0^-$ octupole state at 788 keV.[5,6] Braid, Chasman, Erskine, and Fried-