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# Effective Interactions Between Nucleons in the *s-d* Shell

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The effective interactions between nucleons in the *s-d* shells are investigated from an exact shell-model calculation via a least-squares fit of the experimental energy spectra of the normal-parity states in the $^{18,19,20}$O, $^{18,19,20}$F, and $^{20}$Ne nuclei. The effects of the $d_{3/2}$ single-particle orbit in shell-model calculations of light *s-d*-shell nuclei are discussed briefly. The two-body matrix elements are compared with those calculated by Kuo and by Arima *et al.*

A great deal of effort has been made during the last few years to compute the two-body matrix elements of the residual interaction of the nucleons in the *s-d* shells. Using the Hamada-Johnston potential, Kuo and Brown[1] have calculated the two-body matrix elements which closely resemble the values worked out by Kuo.[2] Elliott *et al.*[3] have determined the matrix elements of the relative motion from the experimental data of the nucleon-nucleon phase shifts in a basis of harmonic-oscillator wave functions. Without any assumption on the shape of the shell-model potential or on the residual interaction, however, the two-body matrix elements were calculated from a least-squares fit to energy-level data by Arima *et al.*[4] and Wildenthal *et al.*[5] In the last cases (in which only nuclei with $A = 18-20$ were studied by Arima *et al.*, and the nuclei with $20 \leqslant A \leqslant 28$ were investigated by Wildenthal *et al.*) an inert $^{16}$O core was assumed, and the $d_{5/2}$ and $s_{1/2}$ orbits were included in the active model space. All of the two-body matrix elements of the residual interaction computed with different methods were compared by Abulaffio.[6] The agreement is poor. Therefore, he suggested that Elliott's data[3] could be used to perform a mixed calculation with part of the two-body matrix element left as free parameters and part constrained to Elliott's values. In this way one could extend the basis without increasing the number of free parameters.

The calculations of this work are made within the framework of the conventional shell model. An inert $^{16}$O core is assumed. The $d_{5/2}$, $s_{1/2}$, and $d_{3/2}$ orbits are included in the active model space, but the number of particles in the $d_{3/2}$ orbit is always restricted to 0, 1, or 2. The two-body matrix elements which contain one or more than one nucleon in the $d_{3/2}$ orbit are taken to be those calculated by Kuo.[2] The 16 matrix elements which have the nucleons in the $d_{5/2}$ and $s_{1/2}$ orbits are left as the free parameters. The energies of the single-particle levels we adopted are from the observed values for $^{17}$O:

$$\epsilon(1d_{5/2}) = 0.0 \text{ MeV},$$

$$\epsilon(2s_{1/2}) = 0.87 \text{ MeV},$$

$$\epsilon(1d_{3/2}) = 5.08 \text{ MeV}.$$

These are held fixed throughout the calculation. With this restriction on the allowed configurations, the 16 matrix elements of the residual two-body interaction for the nucleons in the $(d_{5/2}, s_{1/2})$ orbits are varied until a best fit to the 37 states of well-known spin and parity in $^{18,19,20}$O, $^{18,19,20}$F, and $^{20}$Ne is obtained.

The results of our least-squares fit to the energy-level data are shown in columns 7 and 8 of Table I, where column 7 gives the theoretical energies and column 8 displays the intensities of the configuration including $1d_{5/2}$ and $2s_{1/2}$ only. The experimental energy $E_k^{\text{exp}}$ in column 4 is an excitation energy if the state $k$ is excited and is the total interaction energy of the extracore nucleons if the state $k$ is the ground state. The values shown in columns 5 and 6 are those calculated by Arima *et al.*[4] The root-mean-square deviation between our calculated energies and the experimental energies is only 0.24 MeV, which is smaller than for the calculated values shown in columns 5 and 6 of Table I. The present model is at its worst for the nucleus $^{20}$Ne. Nevertheless, the interesting rotational structure in this nucleus is well reproduced by our truncated shell-model calculation. In addition, the third $0^+$ state at 7.21 MeV and third $2^+$ state at 7.85 MeV in $^{20}$Ne,[7] which are not included in the least-squares fit,

TABLE I. Comparison of experimental and theoretical energies.

|  |  |  |  | Arima *et al.* |  | Ours |  |
|---|---|---|---|---|---|---|---|
| Nucleus | $J$ | $T$ | Exp. energy (MeV) | Except 1.7-MeV state in $^{18}F$ | Including 1.7-MeV state in $^{18}F$ | Energy (MeV) | Intensity of $(d_{5/2}, s_{1/2})$ configuration |
| $^{18}F$ | 1 | 0 | −5.00 | −5.34 | −4.74 | −4.85 | 0.62 |
|  | 3 | 0 | 0.94 | 0.98 | −0.06 | 0.77 | 0.93 |
|  | 5 | 0 | 1.13 | 1.08 | 1.09 | 1.38 | 1.00 |
|  | 1 | 0 | 1.70 | 8.75 | 2.34 | 1.75 | 0.77 |
|  | 2 | 0 | 2.53 | 2.51 | 2.74 | 2.47 | 0.54 |
| $^{18}O$ | 0 | 1 | −3.91 | −3.74 | −3.84 | −3.97 | 0.93 |
|  | 2 | 1 | 1.98 | 2.00 | 1.96 | 2.00 | 0.94 |
|  | 4 | 1 | 3.55 | 3.66 | 3.49 | 3.62 | 0.88 |
|  | 0 | 1 | 3.63 | 3.65 | 4.31 | 3.84 | 0.99 |
|  | 2 | 1 | 3.92 | 3.98 | 4.25 | 4.04 | 0.97 |
|  | 3 | 1 | 5.37 | 5.77 | 5.92 | 5.71 | 1.00 |
| $^{19}F$ | $\frac{1}{2}$ | $\frac{1}{2}$ | −11.30 | −11.49 | −11.57 | −11.13 | 0.68 |
|  | $\frac{5}{2}$ | $\frac{1}{2}$ | 0.20 | 0.46 | 0.27 | −0.02 | 0.73 |
|  | $\frac{3}{2}$ | $\frac{1}{2}$ | 1.56 | 2.17 | 2.48 | 1.58 | 0.37 |
|  | $\frac{9}{2}$ | $\frac{1}{2}$ | 2.79 | 2.76 | 2.86 | 2.56 | 0.78 |
| $^{19}O$ | $\frac{5}{2}$ | $\frac{3}{2}$ | −3.72 | −3.64 | −3.59 | −3.75 | 0.90 |
|  | $\frac{3}{2}$ | $\frac{3}{2}$ | 0.10 | 0.07 | 0.35 | 0.16 | 0.93 |
|  | $\frac{1}{2}$ | $\frac{3}{2}$ | 1.47 | 1.48 | 1.25 | 1.36 | 0.91 |
|  | $\frac{9}{2}$ | $\frac{3}{2}$ | 2.78 | 2.65 | 2.39 | 2.56 | 0.85 |
|  | $\frac{5}{2}$ | $\frac{3}{2}$ | 3.15 | 3.17 | 2.99 | 3.05 | 0.94 |
| $^{20}Ne$ | 0 | 0 | −24.00 | −22.98 | −23.23 | −23.62 | 0.54 |
|  | 2 | 0 | 1.63 | 1.61 | 1.53 | 2.20 | 0.53 |
|  | 4 | 0 | 4.25 | 4.41 | 4.45 | 4.54 | 0.45 |
|  | 0 | 0 | 6.75 | 6.74 | 6.17 | 6.47 | 0.62 |
|  | 2 | 0 | 7.46 | 7.43 | 7.02 | 7.38 | 0.71 |
|  | 0 | 0 | 8.60 | 8.63 | 8.94 | 8.66 | 0.96 |
|  | 6 | 0 | 8.79 | 8.49 | 8.24 | 8.14 | 0.39 |
|  | 4 | 0 | 9.11 | 9.22 | 8.88 | 9.63 | 0.51 |
|  | 8 | 0 | 11.99 | 12.03 | 12.38 | 11.83 | 0.29 |
| $^{20}F$ | 2 | 1 | −13.76 | −14.21 | −14.26 | −14.06 | 0.69 |
|  | 3 | 1 | 0.65 | 0.44 | 0.76 | 0.79 | 0.77 |
|  | 1 | 1 | 1.06 | 0.96 | 0.40 | 0.74 | 0.60 |
| $^{20}O$ | 0 | 2 | −7.19 | −7.16 | −7.06 | −7.29 | 0.81 |
|  | 2 | 2 | 1.67 | 1.93 | 2.01 | 1.87 | 0.82 |
|  | 4 | 2 | 3.57 | 3.56 | 3.32 | 3.94 | 0.87 |
|  | 2 | 2 | 4.07 | 3.82 | 3.55 | 4.01 | 0.85 |
|  | 0 | 2 | 4.45 | 4.45 | 4.49 | 4.39 | 0.88 |

cannot be accounted for on the basis of our assigned configurations. However, the third $0^+$ state can be categorized approximately as a state which results from the weak coupling of the first excited $0^+$ state of $^{16}$O (the four-particle–four-hole state) to the ground state of $^{20}$Ne obtained in the conventional $s$-$d$-shell model.[8] The present calculation also does not include in the least-squares fit the second $2^+$ state in $^{20}$F, since the spin assignment of this level may be doubtful in view of the corresponding analog state in $^{20}$Ne.[9] These three levels in $^{20}$Ne and $^{20}$F were not taken into account by Arima *et al.*[4] either.

The two-body matrix elements of the residual interaction in the present work are shown in column 7 of Table II. In the same table, the results calculated by Kuo[2] and Arima *et al.*[4] are also shown in columns 4, 5, and 6. Our two-body matrix elements are close to all of them for $T = 1$, but the agreement is rather poor for $T = 0$. It is worthwhile to mention that both the root-mean-square deviation and the two-body matrix elements do not change even with the second $1^+$ state of $^{18}$F at 1.7 MeV excluded in the least-squares fit. This is the most significant difference between our results and those of Arima *et al.*, where both the root-mean-square deviation and two-body matrix element change considerably depending on whether or not the 1.7-MeV $1^+$ level in $^{18}$F is included in the least-squares fit.

The importance of the effect of the $d_{3/2}$ single-particle orbit in the shell-model calculations of light $s$-$d$-shell nuclei has been pointed out by McGrory.[10] In our calculations, the $d_{3/2}$ single-particle orbit plays an important role for the low-isospin states. As shown in column 8 of Table I, it is apparent that the $d_{3/2}$ single-particle orbit should not be neglected in the shell-model calculations except for the oxygen isotopes, for which the isospin $T$ is the highest and the intensities from the configuration without the $1d_{3/2}$ orbit are always larger than 80% for the low-lying states under consideration. For the lowest $\frac{3}{2}^+$ state in $^{19}$F and the lowest $6^+$ and $8^+$ states in $^{20}$Ne, the intensities of the configuration including $1d_{5/2}$ and $2s_{1/2}$ only are smaller than 40%. The fact that the agreement of our two-body matrix elements with those of Arima *et al.* are quite good for $T = 1$, but rather poor for $T = 0$, can be understood in view of the effect of the $d_{3/2}$ single-particle orbit.

In conclusion, the experimental spectra of the low-lying states of $^{18,19,20}$O, $^{18,19,20}$F, and $^{20}$Ne are well reproduced by the truncated shell-model calculation. It is interesting that the 1.7-MeV $1^+$ state in $^{18}$F can be adequately reproduced without taking into account the core excitation. However, it has been suggested that this state probably arises because of the 4p-2h configuration relative to the $^{16}$O core.[1,4] Our calculated electric-quadrupole-transition probability for decay from

TABLE II. Two-body matrix elements of the residual interaction.

| Configuration | $J$ | $T$ | Kuo | Arima *et al.*<br>Except 1.7-MeV state in $^{18}$F | Arima *et al.*<br>Including 1.7-MeV state in $^{18}$F | Ours |
|---|---|---|---|---|---|---|
| $\frac{5}{2}\frac{5}{2}\frac{5}{2}\frac{5}{2}$ | 0 | 1 | −2.44 | −3.41 | −3.12 | −2.32 |
| | 2 | 1 | −1.04 | −1.21 | −0.52 | −1.16 |
| | 4 | 1 | −0.05 | −0.08 | −0.35 | 0.22 |
| $\frac{5}{2}\frac{5}{2}\frac{5}{2}\frac{1}{2}$ | 2 | 1 | −0.85 | −0.88 | −1.13 | −0.65 |
| $\frac{5}{2}\frac{5}{2}\frac{1}{2}\frac{1}{2}$ | 0 | 1 | −0.97 | −1.04 | −1.61 | −1.18 |
| $\frac{5}{2}\frac{1}{2}\frac{5}{2}\frac{1}{2}$ | 2 | 1 | −1.29 | −1.17 | −1.80 | −0.85 |
| | 3 | 1 | 0.17 | 1.16 | 1.21 | 0.93 |
| $\frac{1}{2}\frac{1}{2}\frac{1}{2}\frac{1}{2}$ | 0 | 1 | −1.95 | −2.17 | −1.98 | −2.33 |
| $\frac{5}{2}\frac{5}{2}\frac{5}{2}\frac{5}{2}$ | 1 | 0 | −1.03 | 0.01 | −4.33 | −1.99 |
| | 3 | 0 | −0.86 | 0.38 | −1.06 | −1.09 |
| | 5 | 0 | −3.66 | −4.26 | −3.65 | −3.62 |
| $\frac{5}{2}\frac{5}{2}\frac{5}{2}\frac{1}{2}$ | 3 | 0 | −1.57 | −3.53 | −3.64 | −3.31 |
| $\frac{5}{2}\frac{5}{2}\frac{1}{2}\frac{1}{2}$ | 1 | 0 | −0.60 | −4.27 | −0.89 | 1.82 |
| $\frac{5}{2}\frac{1}{2}\frac{5}{2}\frac{1}{2}$ | 2 | 0 | −0.62 | −3.70 | −2.87 | −0.93 |
| | 3 | 0 | −3.69 | −2.60 | −2.13 | −0.20 |
| $\frac{1}{2}\frac{1}{2}\frac{1}{2}\frac{1}{2}$ | 1 | 0 | −3.18 | −3.67 | −4.56 | −3.51 |

the 2.5-MeV $2^+$ state to the 1.7-MeV $1^+$ state is about 400 times smaller than the experimental result.[11] This indicates that the 4p-2h configuration is not negligible in the consideration of the low-lying states of $^{18}$F. In addition, the effect of the $d_{3/2}$ single-particle orbit in shell-model calculations of light $s$-$d$-shell nuclei is quite important, especially for the low-isospin states. The use of the two-body matrix element in this work in the calculations of $A > 20$ nuclei will be given elsewhere.

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PHYSICAL REVIEW C VOLUME 5, NUMBER 6 JUNE 1972

# Effective Interactions of Nucleons in the Upper *s*-*d* Shell*

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The effective interactions of the nucleons in the upper $s$-$d$ shell are studied in a truncated shell-model calculation by a least-squares fit to the experimental energy levels of the normal-parity states in the nuclei $^{36}$S, $^{36,\,37}$Cl, $^{37,38}$K, and $^{36,\,38}$Ar. The resulting effective interactions and two-body matrix elements are discussed briefly.

The two-body matrix elements of the residual interaction of the nucleons in the $s$-$d$ shell were calculated from the Hamada-Johnston potential or nucleon-nucleon phase shifts.[1-3] The matrix elements have been used in practical calculations with reasonable success. However, the agreement between the two-body matrix elements derived from different methods is poor.[4] Without any assumption on the residual interaction, the two-body matrix elements were calculated from a least-squares fit to the energy-level data of $A = 18$–$20$ nuclei by Arima *et al.*[5] and considerable improvement in experimental data fits was obtained. In that calculation, only $d_{5/2}$ and $s_{1/2}$ orbits were included in the active model space. The active model space was extended by Lee, Hsieh, and Yang to include the $d_{3/2}$ orbit.[6] It was shown that the effects of the $d_{3/2}$ orbit could not be overlooked. In this work we apply a similar technique to study the residual interaction of the nucleons in the upper $s$-$d$ shell.

In the present calculations, an inert $^{40}$Ca core is assumed. The $d_{3/2}$ and $s_{1/2}$ orbits are taken to be the active model space at first, then the effects of the $d_{5/2}$ orbit are included later, but the number of holes in this orbit is always restricted to 0, 1, or 2. The calculation is therefore similar to that of Refs. 5 and 6, except that holes are considered instead of particles, and the roles of the $d_{5/2}$ and $d_{3/2}$ orbits are interchanged. The 15 two-body matrix elements which have the nucleons in the $s_{1/2}$ and $d_{3/2}$ orbits are left as free parameters. The matrix elements for the nucleons in the $d_{5/2}$ and $s_{1/2}$ orbits are fixed to be those determined in Ref. 6 except for $\langle s_{1/2}{}^2|V|s_{1/2}{}^2\rangle_{T=0,J=1}$ and $\langle s_{1/2}{}^2|V|s_{1/2}{}^2\rangle_{T=1,J=0}$, which are considered to be free parameters. The remaining matrix elements are adopted from Kuo.[2] In the previous calculation, the single-particle levels were taken from the observed values for $^{17}$O. Unfortunately, it is rather difficult to choose the single-particle levels from the energy levels of $^{39}$Ca. We therefore