Measurement of the Coherent-Scattering Amplitude of Tritium*

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The coherent scattering amplitude of tritium has been measured by a multiple-refraction method. A finely collimated beam of neutrons is passed through a powder alternately in vacuum and in a nonabsorbing fluid. The relative broadening of the neutron beam is determined by the coherent scattering amplitudes of the powder and the fluid. In this experiment, the powders were silicon and diamond, and the fluids were deuterium and tritium with densities up to 440 amagat. The coherent scattering amplitude of tritium was determined relative to deuterium using a value of $a_D = +6.21 \pm 0.04$ F from our mirror measurements. The value obtained for tritium was $a_T = +4.7 \pm 0.3$ F.

It should be noted that our value of $a_D = +6.21$ F is at variance with other measurements of a_D. Using the accepted value of $a_D = 6.7 \pm 0.1$ F, the coherent scattering amplitude of tritium is $a_T = 5.0 \pm 0.3$ F.

I. INTRODUCTION

The coherent neutron scattering amplitude of tritium has been measured by a multiple-refraction method previously described.[1,2] Briefly, a long-wavelength neutron passing through a powder in vacuum undergoes multiple small-angle refraction. The resultant scattering is a function of the index of refraction of the neutron in the powder material, as well as the size and number of particles traversed.[3]

The index of refraction μ is given by $\mu = 1 - \delta$, where $\delta = Na\lambda^2/2\pi$, N is the number of nuclei per cm^3 in the powder particles, a is the coherent scattering amplitude of the powder material, and λ is the neutron wavelength. A collimated neutron beam is thereby broadened in passing through the powder. The amount of broadening is proportional to the scattering amplitude through the relation $\omega^2 - \omega_0^2 \propto n\delta^2$ (Refs. 1 and 3). ω_0 is the full width at half-maximum (FWHM) intensity of the incident beam, ω is the FWHM intensity of the broadened beam, and n is the number of particles traversed. If the powder is immersed in a nonabsorbing fluid, the scattering is proportional to the difference between the index of refraction of the powder and the fluid.

Combining the measurements for the powder and powder plus the fluid, one obtains

$$\frac{(\omega_p^2 - \omega_0^2)}{(\omega_f^2 - \omega_0^2)} = \left[\frac{(Na)_p}{(Na)_p - (Na)_f}\right]^2, \quad (1)$$

where the subscripts p and f indicate powder and fluid, respectively. Knowing the scattering amplitude of the powder and the density of the fluid allows one to determine the amplitude of the fluid. By observing whether ω_f is larger or smaller than ω_p, one can determine the sign of the scattering amplitude. It has been demonstrated theoretically[4] and experimentally[5] that in measurements of the index of refraction of fluids with long-wavelength neutrons, the neutron interacts with the medium as a whole and the bound coherent scattering amplitude is measured.

II. EXPERIMENTAL ARRANGEMENT

Figure 1 shows the neutron optics arrangement. The neutron beam was a ribbon 2 in. wide and about 50 mils thick at the target. The geometrical collimation was 2 min of arc FWHM. The Pyrex optical flat was adjusted to remove the shorter wavelengths from the beryllium-filtered beam and to reflect the long wavelength beam out of the γ ray and fast-neutron background. The fluids used in these measurements were D_2 and T_2 gases, and the powders were silicon and diamond.

The powders were tightly packed in thin-walled aluminum cylinders $\frac{1}{2}$-in. i.d. by 3-in. length. The ends of the cylinders were fitted with fritted glass

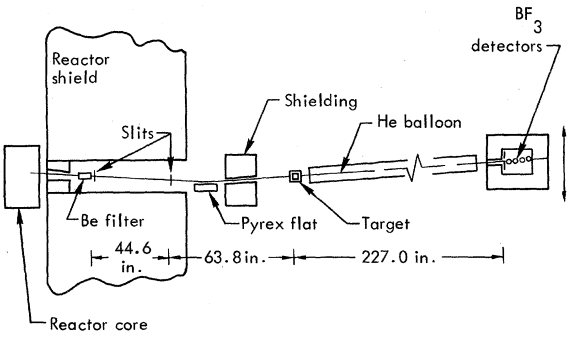

FIG. 1. Geometry of the experiment.

to allow free passage of the gases. These cylinders were placed in high-pressure, nonmagnetic, steel cylinders with windows 0.084 in. thick for the beam passage. Provision was made for vacuum and gas fill. Because of tritium gas hazards, the cylinders were placed in a large aluminum tank with $\frac{1}{16}$-in. beam windows. The beam width exiting the empty chamber was 2.45 min of arc FWHM, and the beam was attenuated by 40%.

The powders were better than 99.9% pure. The mean size of the silicon particles was about 5 mils and that of the diamond, about 2.5 mils. A typical T_2 analysis was 96.7% T_2, 3.2% D_2, and 0.1% H_2, with about 0.4% per month buildup of ^3He. The D_2 purity was 99.8%. Gas densities were measured by transducers calibrated against a Heise test gauge which in turn was calibrated against a primary standard. The absolute accuracy of the transducers was about 0.5%.

III. MEASUREMENT

Figure 2 shows ω_0, ω_p, and ω_f for typical neutron beam profiles. Figure 3 shows the beam width ω_f vs ρ, the gas density in amagat units, for silicon powder in D_2 and T_2 gases.

The neutron background varied from 2 to 5% of the peak intensity. Fast-neutron backgrounds were measured by inserting a cadmium absorber

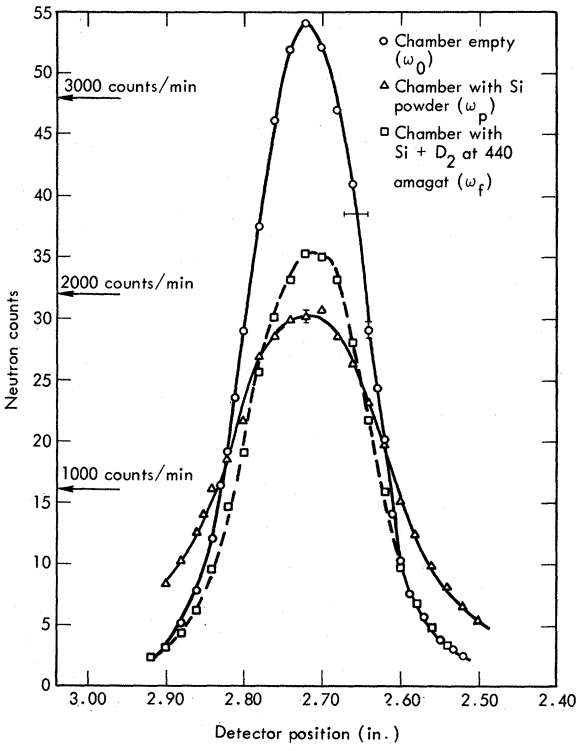

FIG. 2. Typical neutron beam profiles.

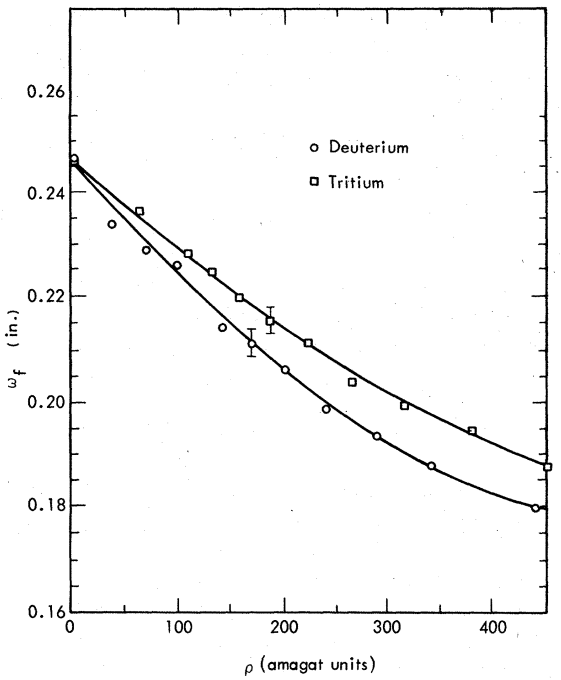

FIG. 3. Beam width ω_f vs ρ amagat for silcon powder in D_2 and T_2.

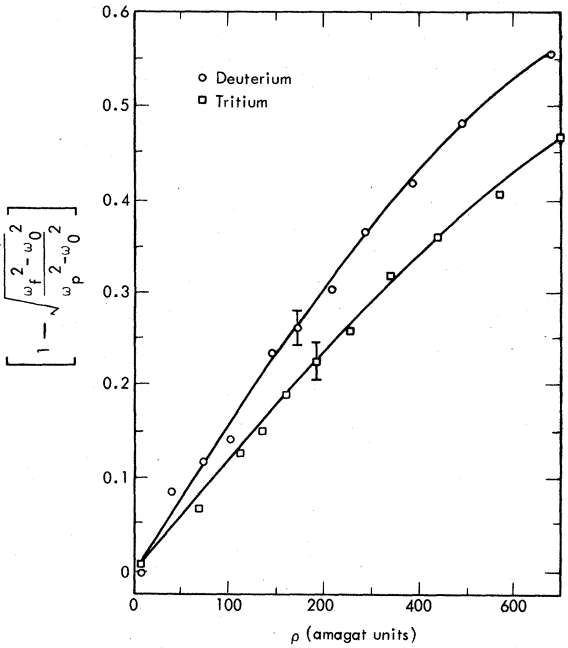

FIG. 4. $\left[1-\left(\frac{(\omega_f^2-\omega_0^2)}{(\omega_p^2-\omega_0^2)}\right)^{1/2}\right]$ vs ρ amagat for silicon powder in D_2 and T_2.

in front of the neutron beam. Thermal-neutron backgrounds were interpolated from measurements taken three FWHM's above and below the reflected beam.

Equation (1) can be written as

$$a_f = \frac{N_p a_p}{N_f}\left[1 - \left(\frac{\omega_f^2 - \omega_0^2}{\omega_p^2 - \omega_0^2}\right)^{1/2}\right]. \quad (2)$$

It is evident from Eq. (2) that high pressure and therefore large N_f gives more sensitivity to the experiment. Figures 4 and 5 show

$$1 - \left(\frac{\omega_f^2 - \omega_0^2}{\omega_p^2 - \omega_0^2}\right)^{1/2} \text{ vs } \rho \text{ (amagat units)}$$

for silicon and diamond powders, respectively.

Accurate determinations of N_f depend on knowledge of the virial coefficients for the fluids over the range of densities used in the experiment. They depend also on knowledge of the constancy of the internal volume of the gas cell. Of these two requirements, the second is probably the least attainable; that is, thin cell windows are required for low beam attenuation, while internal volume constancy demands very thick windows. In addition, the virial coefficients for tritium are not known at present, while those for deuterium are not usable beyond 50 amagat units.

The measurement of a_T relative to a_D, using the same powder, eliminates the need to know either N_p or a_p. The need for detailed knowledge of N_f for each gas is eliminated by taking the ratio of the derivatives of a_D and a_T with respect to ρ:

$$\frac{a_D}{a_T} = \frac{d}{d\rho}\left[1 - \left(\frac{\omega_D^2 - \omega_0^2}{\omega_p^2 - \omega_0^2}\right)^{1/2}\right] \Big/ \frac{d}{d\rho}\left[1 - \left(\frac{\omega_T^2 - \omega_0^2}{\omega_p^2 - \omega_0^2}\right)^{1/2}\right]. \quad (3)$$

The data in Figs. 4 and 5 were fitted with polynomials and the ratios a_D/a_T were calculated from the resulting parameters. The solid lines in Fig. 6 show these ratios, while the dashed lines show our limits of error.

Since the scattering amplitudes of silicon and carbon are positive, the narrowing of ω_f relative to ω_0 gives a positive scattering amplitude for tritium. Our data yielded the following values for tritium:

$$a_T = +4.7 \pm 0.3 \text{ F (bound)},$$

$$a_T = +3.5 \pm 0.2 \text{ F (free)},$$

$$\sigma_{coh} = 4\pi a_T^2 \text{ (free)} = 1.5 \pm 0.5 \text{ b (total coherent cross section)}.$$

A measurement of the total scattering cross section of tritium in the Angstrom region done by Vertebnyi *et al.*[6] has yielded a value of 1.3 ± 0.3 b. Comparison of the total cross section with the coherent cross section indicates that the incoherent scattering is quite small and that

$$a_{triplet} \simeq a_{singlet} = 3.5 \pm 0.2 \text{ F (free)}.$$

A theoretical calculation of the n-T scattering lengths has been made by Szydlik and Werntz,[7] giving the free values

$$a_{singlet} = 3.38 \text{ F}$$

and

$$a_{triplet} = 3.25 \text{ F},$$

in reasonable agreement with our value.

The scattering amplitudes used in determining

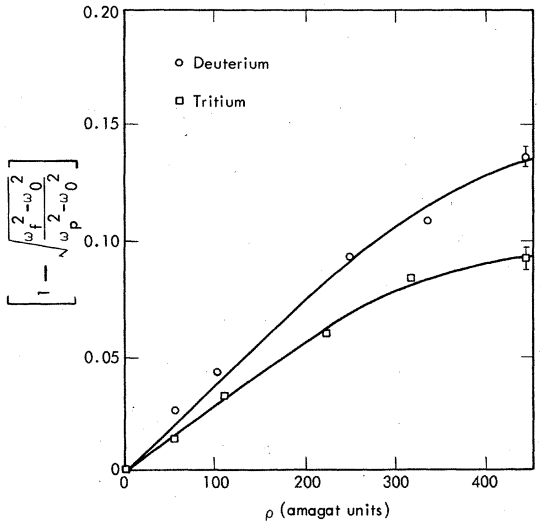

FIG. 5. $\left[1 - \left(\frac{(\omega_f^2 - \omega_0^2)}{(\omega_p^2 - \omega_0^2)}\right)^{1/2}\right]$ vs ρ amagat for diamond powder in D_2 and T_2.

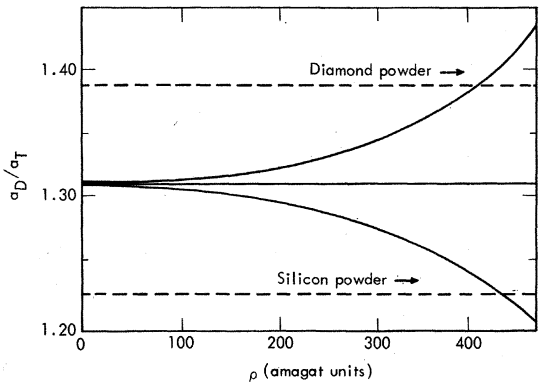

FIG. 6. a_D/a_T vs ρ amagat showing the extrapolation to zero pressure for silicon and diamond powders.

the tritium scattering[8,9] amplitude were, respectively,

$$a_D = 6.21 \pm 0.04 \text{ F (bound)},$$

$$a_H = -3.740 \pm 0.003 \text{ F (bound)}.$$

There is, at present, some controversy about the value of a_D measurements made by neutron mirror vs other methods. An example is the measurement of Koester.[10] Using Koester's value of 6.7 ± 0.1 F for a_D results in a tritium coherent scattering amplitude of 5.0 F (bound) and a total coherent scattering cross section of 1.74 b.

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Use of Eckart Interactions in Distorted-Wave Born-Approximation Calculations*

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The evaluation of the finite-range distorted-wave Born-approximation matrix elements is shown to be considerably simplified when one uses Eckart bound-state wave functions. The use of these wave functions to describe the stripping reaction $^{12}C(^6Li,d)^{16}O$ is found to give reasonable aggrement with the experimental results. Also, the use of the Eckart interaction as a test of the various methods for the evaluation of matrix elements is demonstrated.

I. INTRODUCTION

Transfer processes have been a valuable tool of nuclear spectroscopy. Single-particle-transfer reactions have provided much information about the nuclear shell structure of various nuclei. In the same manner, composite-particle transfer can provide considerable insight into nucleon clustering in nuclei. While the composite-particle-transfer reaction is capable of verifying various models for the cluster wave functions, the actual calculations are very difficult because of the complexity of the mathematics involved. Even in the cases where the distorted-wave Born approximation (DWBA) is thought to be valid, further approximations are usually required in order to evaluate the necessary matrix elements.

The matrix elements associated with the plane-wave Born approximation (PWBA) are much easier to perform, but this method neglects the important effects of both the nuclear and Coulomb distortions of the scattering wave functions. In fact, it is the Coulomb repulsion which probably insures the validity of the Born approximation at low energies. If the incident and target particles have a classical distance-of-closest-approach larger than the radius of their nuclear interactions, the matrix elements of the interaction should be small and the Born series convergent. This assumes that the long-range Coulomb effects have been included in the distorted waves. In this paper we deal primarily with the DWBA calculations.

The calculation of the six-dimensional DWBA matrix element can be reduced to the evaluation of a sum of two-dimensional radial integrals by first performing the angular integrations. However, one of the angular integrations is usually done numerically for every set of values of the ra-