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Nuclear Structure of the $N=82$ Isotones ^{140}Ce and ^{139}La from the (α,α') Reaction*

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Differential cross sections for elastic and inelastic scattering of 45-MeV α particles by the $N=82$ isotones ^{140}Ce and ^{139}La have been measured. Both collective- and microscopic-model distorted-wave Born-approximation analyses have been performed to ascertain the collective nature of the levels excited; the wave functions used in the microscopic-model calculations were the eigenfunctions of the pseudo L-S coupling scheme recently developed by Hecht and Adler. The results of the collective-model analysis of the ^{140}Ce(α,α') data indicate that none of the levels excited are well described as mass vibrations. The microscopic-model calculations indicate that significant contributions from the core are needed in order to fit the magnitudes of the cross sections for transitions to the first 2^+ (1.597 MeV) and 4^+ (2.084 MeV) states of ^{140}Ce. The collective-model analysis of the ^{139}La(α,α') data permits parity assignments to be made for all levels excited; unfortunately, it is not possible to understand the features of the (α,α') spectrum in terms of a weak-coupling model. The microscopic-model calculations for the ^{139}La(α,α') experiment indicate that transitions which require a "pseudospin flip" are strongly inhibited. The features of the experimental ^{139}La(α,α') spectrum are consistent with this prediction. The $\Delta B=0$ selection rule suggested by Hecht and Adler (B is the total pseudospin) is approximately obeyed for both reactions.

I. INTRODUCTION

$N=82$ isotones have, in recent years, been extensively studied both experimentally and theoretically. The experiments which have been performed have elucidated the level structure and the single-particle (shell-model) nature of the levels of these nuclei; the purpose of the present experiments is to complement these results by examining the collective nature of the levels of the nuclei ^{140}Ce and ^{139}La.

In order to understand the specific nature of a collectively enhanced level it is necessary to distinguish between core excitations and collective enhancement due to coherence among the "active" nucleons. The techniques used here to arrive at this distinction have been developed by Bernstein,[1] Satchler and Park,[2] and Morgan and Jackson.[3] Bernstein[1] has shown that isoscalar transition rates, as measured by collective analysis of an (α,α') experiment, should be equal to measured electromagnetic transition rates for transitions to states well described as mass vibrations. Studies by Satchler and Park[2] and by Morgan and Jackson[3] indicate that microscopic analyses of inelastic scattering experiments may provide a test of the assumed nuclear wave functions; if one chooses a reasonable effective interaction between the α particle and the "active" nucleons, a microscopic analysis should correctly predict the transition strength if the assumed nuclear wave functions are correct. In the present work the view will be adopted that enhancement of this "reasonable" interaction most probably indicates contributions from the core.

Wave functions for these nuclei are expected to involve mixed proton configurations in the usual j-j coupling representation, since the two most important orbitals, $1g_{7/2}$ and $2d_{5/2}$, lie quite close to each other. This expectation has been verified by Wildenthal[4] in his highly successful calculations employing the Oak Ridge shell-model code; the wave functions of most of the levels have many important components corresponding to several n', n'' and J', J'' values of the configurations $[(g_{7/2})^{n'}_{J'}(d_{5/2})^{n''}_{J''}]_J$.

A pseudo L-S coupling scheme applicable to $N=82$ nuclei has recently been devised by Hecht and Adler.[5] This scheme provides a set of basis states which serve as a much improved set over the j-j coupling representation. In the approximation in which the $g_{7/2}$ and $d_{5/2}$ orbitals are considered degenerate, and no other orbitals contribute significantly to the structure of the low-lying levels, the spectra are those of a pseudo f shell. If, in addition, the two-body interaction can be approximated as a pseudospin-preserving interaction [for example the surface δ interaction (SDI)] then each wave function can be described by one component corresponding to a specific value of the pseudo-f-shell seniority, total pseudospin, and total pseudo orbital angular momentum. In terms of such wave functions a microscopic analysis of the (α, α') experiment becomes relatively simple.

An additional advantage of the (α, α') experiment is that seniority $v=3$ levels of odd-A nuclei, inaccessible to proton-transfer experiments, can be studied if there is sufficient collective enhancement. Many of the low-lying levels of ^{139}La observed[6] in the (β-γ) decay of ^{139}Ba have not been

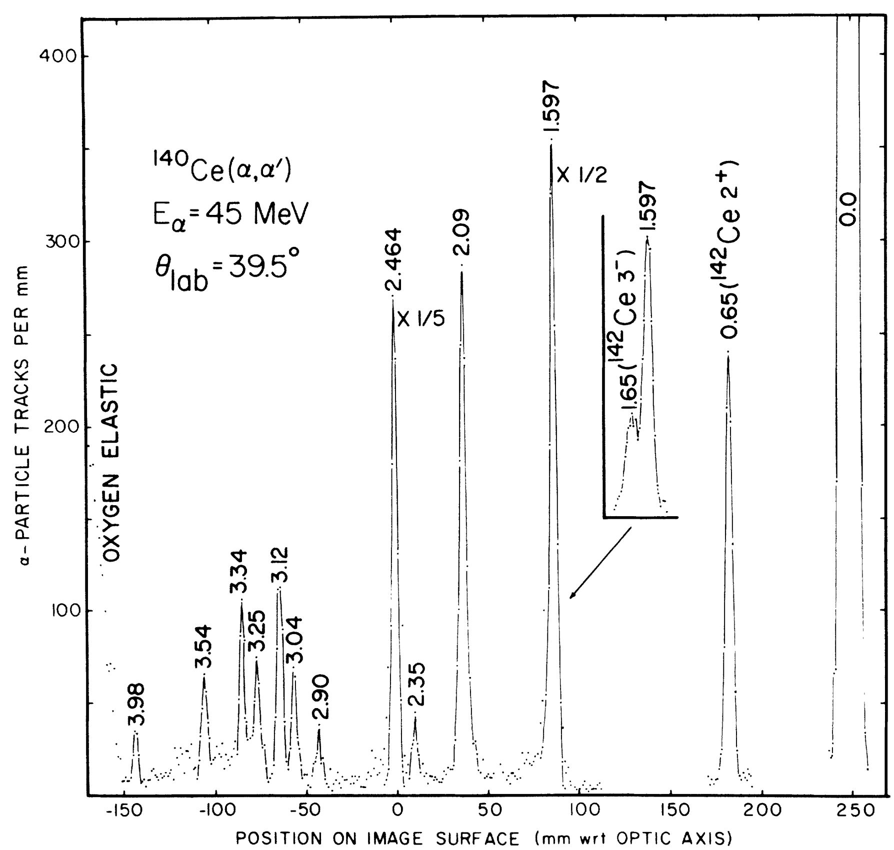

FIG. 1. Spectrum of α particles scattered from ^{140}Ce.

observed in proton-transfer reactions and are therefore expected to be dominated by seniority $v=3$ configurations. Because the (α, α') experiment provides results complementary to the results of proton-transfer experiments the ^{140}Ce and ^{139}La(α, α') experiments can add significantly to the knowledge of these nuclei.

II. EXPERIMENTAL TECHNIQUES

The data presented here were obtained using the 45-MeV α-particle beam from the University of Michigan 83-in. sector-focused cyclotron.[7]

The targets were evaporated metal foils sandwiched between thin (<10-μg/cm^2) layers of carbon to inhibit corrosion; thicknesses of the foils ranged from 200–1000 μg/cm^2. Both cerium and lanthanum targets were fabricated from the natural metals; the isotopic purities were thus 99.9% for ^{139}La and 88.5% for ^{140}Ce. Self-supporting targets were prepared for acquisition of small-angle data (<30°), where the elastic carbon peak obscured the spectra. Target thicknesses were measured by performing small-angle elastic scattering. These results were checked by measuring the energy loss of 5.48-MeV α particles and by weighing carefully measured areas of the foils. Uncertainty in target thickness is estimated to be less than 5%.

Day-to-day normalization of the data was done by measurement of the 2^+ and 3^- transition yields at 40° for ^{140}Ce and by measurement of the elastic yield at 40° for ^{139}La. Spectroscopic information was obtained by visual normalization of distorted-wave Born-approximation (DWBA) calculations to the experimental angular distributions. The overall normalization uncertainty is estimated to be less than 10%.

Spectra were obtained at the image surface of the first of three 180° analyzing magnets. Most spectra were recorded in Ilford K0 nuclear emulsions, but some small-angle data were obtained using a position-sensitive detector placed on the image surface. Energy resolution [full width at half maximum (FWHM)] was typically 35 keV. Typical spectra for the ^{140}Ce(α, α') and ^{139}La(α, α') experiments are shown in Figs. 1 and 2.

For most ^{140}Ce spectra the resolution was inadequate to separate the 1.597-MeV level from the 1.66-MeV 3^- level in the impurity ^{142}Ce. From a high-resolution spectrum it was determined that the intensity of the ^{142}Ce 3^- level was 14% of that of the ^{140}Ce 3^- level at 2.464 MeV. Making the reasonable assumption that the two 3^- angular distributions are identical in shape, it was possible to separate the intensities of the two close-lying states near 1.6 MeV.

Inadequate energy resolution presented more serious problems to the analysis of the ^{139}La spectra. Several of the levels had to be analyzed as multiplets. Energy assignments for the ^{139}La levels were made on the basis of a ^{139}La(d, d') spectrum at 22.7-MeV incident deuteron energy.

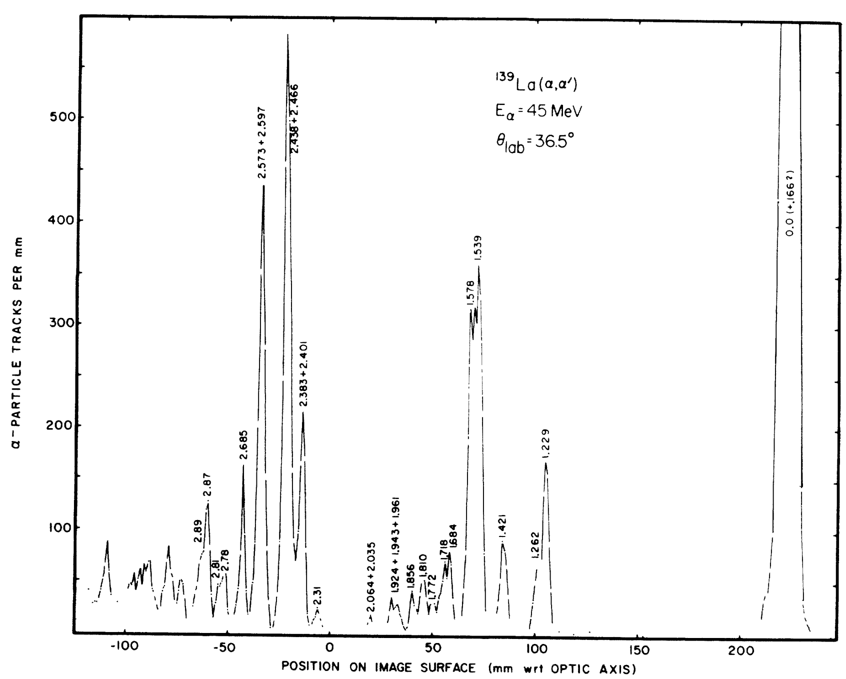

FIG. 2. Spectrum of α particles scattered from ^{139}La.

Energy resolution of approximately 12 keV (FWHM) was achieved. This spectrum is shown in Fig. 3.

Low-background data could be obtained at small angles because of the nature of the beam-preparation system; the two 110° focusing magnets have an intermediate focus for energy selection and therefore provide a well-focused, well-resolved beam spot on the target without the use of slits in the scattering chamber. Removal of the scattering-chamber slits resulted in nearly an order of magnitude reduction of background and permitted data to be obtained at angles as small as 12° for all levels and at angles as small as 8° for the stronger transitions in ^{140}Ce.

III. ELASTIC SCATTERING

The elastic scattering data were analyzed using the standard Woods-Saxon volume-absorption optical potential:

$$V(r) = -V(e^x+1)^{-1} - iW(e^{x'}+1)^{-1} + V_C(r),$$
$$x = (r - r_0A^{1/3})a, \qquad (1)$$
$$x' = (r - r_0'A^{1/3})/a'.$$

$V_C(r)$ is the Coulomb potential assuming a uniform charge distribution of radius $1.4A^{1/3}$ F. The results of an optical-model analysis using the search code HUNTER[8] are shown in Figs. 4 and 5. Only the cerium (natural Ce metal target) data were searched on; a χ^2 of 34, assuming a 10% error for all data, was obtained.

Jackson and Morgan[3] have found that the choice of optical-model parameters may affect the quality of the fit obtained from a microscopic-model inelastic scattering calculation. This effect was investigated in the present work; the results indicate that the most satisfactory fit to the inelastic scattering data is obtained using the optical potential which most nearly belongs to the $V \simeq 200$-MeV family. This was the principal criterion used to select the optical-potential parameters shown in Figs. 4 and 5 from the several sets which provided approximately equivalent fits to the elastic scattering data.

IV. INELASTIC SCATTERING THEORY

A. Distorted-Wave Theory

The method used to analyze the experimental data was the DWBA. The transition amplitude for inelastic scattering may be written as

$$T_{if} = \int \chi_f^{(-)*}(\vec{k}_f, \vec{r}) \langle \Phi' | v | \Phi \rangle \chi_i^{(+)}(\vec{k}_i, \vec{r})\, d\vec{r}, \qquad (2)$$

where $\chi(\vec{k}, \vec{r})$ are the distorted waves determined by the optical parameters.

The matrix element $\langle \Phi' | v | \Phi \rangle$ depends both on the model used for the nuclear wave functions Φ and the interaction v assumed to cause the transi-

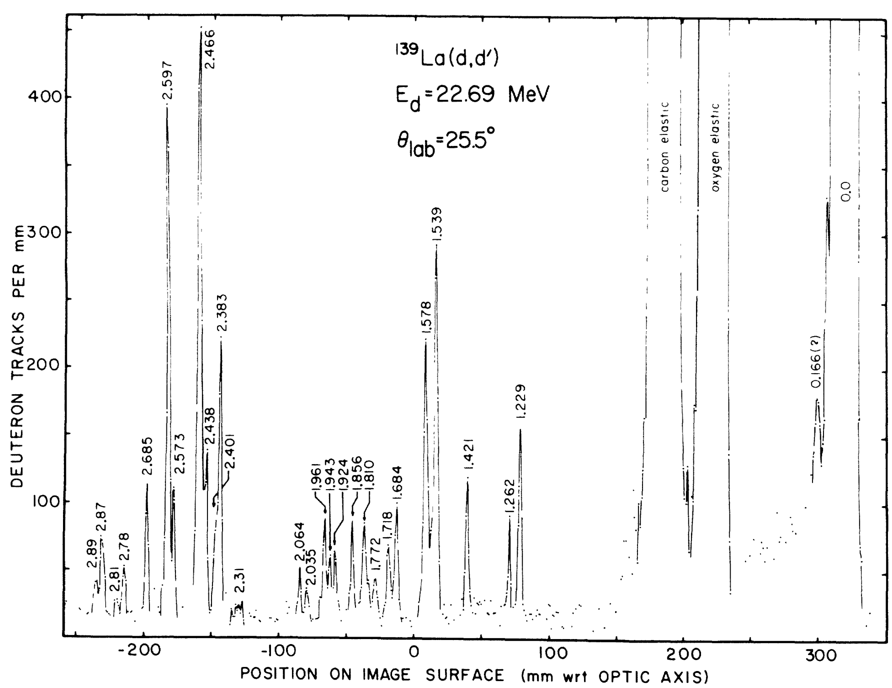

FIG. 3. Spectrum of deuterons scattered from ^{139}La.

tion. The data presented here have been analyzed using both a collective nuclear model with a collective interaction v and the nuclear shell model with a Gaussian microscopic interaction.

To facilitate the evaluation of $\langle \Phi' | v | \Phi \rangle$ for each model it is customary to perform a multipole expansion of v and use the Wigner-Eckhart theorem[9]:

$$\langle \Phi' | v | \Phi \rangle = \sum_{L\mathfrak{M}} i^{-L} Y_L^{\mathfrak{M}*}(\theta, \phi) \langle JLM\mathfrak{M} | J'M' \rangle \frac{\langle J' \| v_L \| J \rangle}{(2J'+1)^{1/2}} . \tag{3}$$

The reduced matrix element $\langle J' \| v_L \| J \rangle$ will be referred to here as the form factor $F_L(r)$. Note that the Clebsch-Gordan coefficient in Eq. (3) restricts L:

$$J + J' \leq L \leq |J - J'| . \tag{4}$$

The sum over L in Eq. (3) is therefore unnecessary for 0^+ targets; for odd-A nuclei, however, several L values may be allowed and L admixtures should therefore be expected.

B. Collective Model

If the nuclear levels are described as vibrational states, then the interaction v is identified as the first term of a Taylor-series expansion of the optical potential.[10] The form factor therefore becomes

$$F_L(r) = -i^L \left(\frac{2J'+1}{2L+1} \right)^{1/2} \left(\beta_L^r R_r \frac{\partial V}{\partial r} + i \beta_L^i R_i \frac{\partial W}{\partial r} \right). \tag{5}$$

V and W are the real and imaginary parts of the optical potential. The collective analysis presented here has been performed assuming equal real and imaginary deformations, i.e.,

$$\beta_L^r R_r = \beta_L^i R_i \equiv \delta_L . \tag{6}$$

The importance of the inclusion of Coulomb excitation has been noted elsewhere.[11] The form factor for this effect has been calculated as in Ref. 10. It may be written, in the notation adopted here, as

$$F_L^C(r) = i^L \frac{3ZZ'e^2}{(2L+1)^{3/2}} (2J'+1)^{1/2} \frac{R_C{}^L}{r^{L+1}} \beta_L^C, \quad r > R_C$$
$$= 0, \quad r < R_C . \tag{7}$$

It has been assumed that

$$\beta_L^C = \frac{\delta_L}{R_r} . \tag{8}$$

C. Shell Model

If one assumes that the interaction between the α particle and the ith nucleon is Gaussian,

$$v_{i\alpha} = -V_0 g(r_{i\alpha}) = -V_0 e^{-\gamma r_{i\alpha}{}^2} , \tag{9}$$

then the net interaction may be written

$$v = \sum_i V_0 g(r_{i\alpha})$$
$$= -4\pi V_0 \sum_{L\mathfrak{M}} Y_L^{\mathfrak{M}*}(\theta, \phi) \sum_i g_L(r_i, r) Y_L^{\mathfrak{M}}(\theta_i, \phi_i); \tag{10}$$

g_L may be expressed in terms of spherical Bessel functions. (See Appendix.) Thus the form factor is

$$F_L(r) = -\frac{(2J'+1)^{1/2} 4\pi V_0}{\langle JLM\mathfrak{M} | J'M' \rangle} \times \langle \Phi' | \sum_i i^L g_L(r_i, r) Y_L^{\mathfrak{M}}(\theta_i, \phi_i) | \Phi \rangle . \tag{11}$$

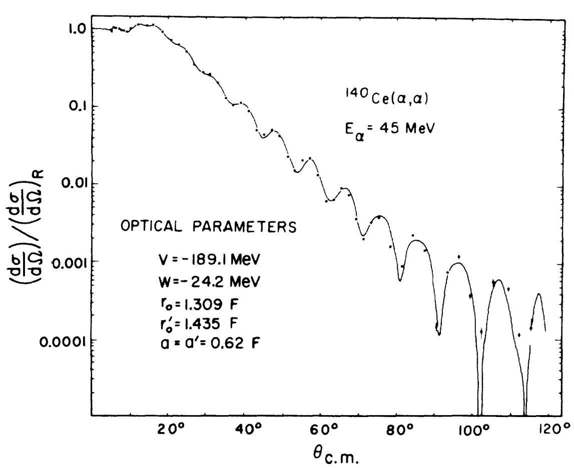

FIG. 4. Angular distribution for elastic scattering of 45-MeV α particles from ^{140}Ce.

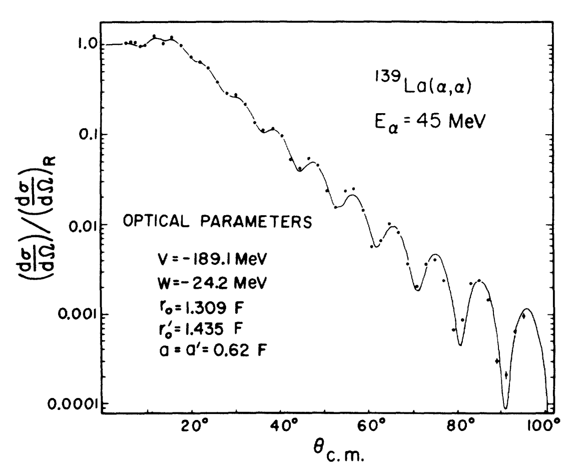

FIG. 5. Angular distribution for elastic scattering of 45-MeV α particles from ^{139}La.

The nuclear wave functions Φ are determined by the Hecht coupling scheme, as will be described in the following section. Evaluation of the matrix element in Eq. (11) yields

$$F_L(r) = -\sqrt{4\pi}\; V_0 \sum_{jj'} A_{jj'} I_L^{jj'}(r). \tag{12}$$

Here j and j' refer to shell-model orbitals. In the calculations described in this work, only the $1g_{7/2}$ and $2d_{5/2}$ orbitals are considered. Thus the series in Eq. (12) contains four terms. The $A_{jj'}$ are, of course, dependent on all quantum numbers necessary for the description of Φ and Φ'. An explicit expression for $A_{jj'}$ is given in the Appendix. The $I_L^{jj'}(r)$ are the usual radial form factors,

$$I_L^{jj'}(r) = i^L \int u_{j'}(\rho) u_j(\rho) g_L(\rho, r) \rho^2 d\rho. \tag{13}$$

The radial wave functions u_j were calculated assuming that the protons move in a Woods-Saxon well with a Thomas spin-orbit term,

$$V(r) = U\left(1 + \frac{\lambda}{42.5} \frac{\vec{L}\cdot\vec{S}}{r} \frac{d}{dr}\right)\left[1 + \exp\left(\frac{r - r_0 A^{1/3}}{a}\right)\right]^{-1}, \tag{14}$$

where

$\lambda = 25$,

$r_0 = 1.24$ F,

$a = 0.65$ F.

U was adjusted to reproduce the assumed binding energy of a proton in a state of excitation energy E:

$$E_B \equiv (S_p - E), \tag{15}$$

where S_p is the separation energy of a proton in the ground state. A uniform charge distribution of radius $1.25A^{1/3}$ F was assumed.

Previous studies[2,3] indicate that, provided Φ and Φ' are the "correct" nuclear wave functions, the interaction given by Eq. (9) with

$\gamma = 0.25$ F^{-2},

$V_0 \simeq 30$–50 MeV

should correctly predict the magnitude of the (α, α') cross section. Therefore the magnitude of V_0 necessary to normalize the DWBA calculations to the experimental data is probably indicative of the importance of core excitations, which are not included in Φ and Φ'.

V. SHELL-MODEL WAVE FUNCTIONS

One of the interesting aspects of the $N=82$ nuclei is that for each group of levels with seniority v some are depressed in energy. This effect is correctly predicted by Wildenthal's shell-model calculations.[4] Such a property, however, suggests the possibility of the existence of a generalized seniority quantum number which counts the number of protons not members of favored $J \neq 0$ pairs, in the same way that the usual seniority quantum number counts the number of protons not members of favored $J=0$ pairs.

Hecht and Adler have recently devised a pseudo-angular-momentum coupling scheme[5] in which one of the pseudo-angular-momentum quantum numbers may be associated with this generalized seniority quantum number. Application of the coupling scheme to the $N=82$ nuclei is achieved by assuming that the close-lying $1g_{7/2}$-$2d_{5/2}$ single-proton orbitals may be represented as a degenerate pseudospin-orbit doublet with pseudo orbital angular momentum $c=3$ and pseudospin $b=\frac{1}{2}$. The favored pairs of protons are those coupled to total pseudospin $B=0$; $2B$ therefore plays the role of a generalized seniority quantum number. A state with seniority v has possible values of the total pseudospin B ranging from $v/2$ to 0 for an even number of protons, or to $\frac{1}{2}$ for an odd number of protons. The possible values of the total pseudo orbital angular momentum C have been tabulated by Racah in his studies of f-shell atomic spectroscopy.[12]

The eigenfunctions of this coupling scheme will be used for the microscopic analysis of the ^{140}Ce

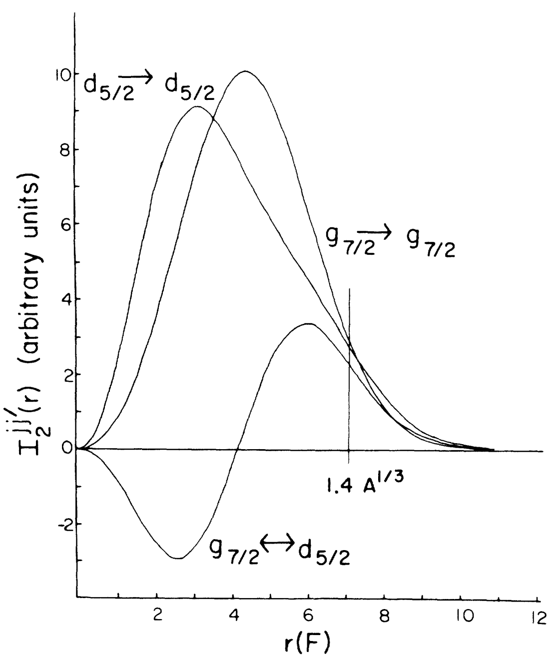

FIG. 6. ^{140}Ce(α, α') radial form factors for an $L=2$ transition.

and ^{139}La(α, α') experiments. The notation used to specify these wave functions will be $|J;v(BC)\alpha\rangle$, where α is an additional quantum number occasionally needed if more than one state specified by J, v, B, and C exists. (If α is unnecessary to describe a given state it will be deleted.)

The functions $|J;v(BC)\alpha\rangle$ are eigenfunctions of any two-body interaction which preserves pseudospin. One such interaction is the SDI. The SDI has proven to be a very good effective interaction for shell-model calculations for $N=82$ isotones[4]; thus, in the approximation that the $g_{7/2}$ and $d_{5/2}$ orbitals are considered degenerate and that the higher-lying orbitals do not contribute significantly to the structure of the low-lying energy levels, the eigenfunctions $|J;v(BC)\alpha\rangle$ are expected to be quite good wave functions for the protons outside the ^{132}Sn core. The eigenvalues of the SDI are degenerate for $B=v/2$ and cluster together quite closely for $B<v/2$. This suggested the existence of a simpler effective interaction whose eigenvalues are all degenerate for a given v and B. This interaction has been called the generalized pairing interaction by Hecht and Adler and has eigenvalues given by

$$E_{\rm GP}=-G\frac{2c+1}{2c+3}\left[\tfrac{1}{4}(n-v)(4c+4-n-v)\right.$$
$$\left.-B(B+1)+\tfrac{3}{4}n+\tfrac{1}{4}n(n-1)\right]. \quad (16)$$

One interesting property of the eigenfunctions $|J;v(BC)\alpha\rangle$ is that a selection rule for the (α, α') reaction is obtained,

$$\Delta B=0, \quad (17)$$

provided the radial form factors $I_L^{jj'}(r)$ are independent of j,j'. Figure 6 displays the four possible $I_L^{jj'}$ for an $L=2$ transition. Since there is approximate j,j' independence in the nuclear-surface region and since α particles, being strongly absorbed, are known to interact primarily in this surface region, this selection rule is expected to be approximately obeyed.

VI. COLLECTIVE-MODEL ANALYSIS

A. ^{140}Ce

Angular distributions were measured and analyzed for transitions to 11 levels in ^{140}Ce. These are shown in Figs. 7–10.

The three 2^+ levels excited are probably the same states observed[13] at 1.5966, 2.8997, and 3.1183 MeV in the β-γ decay of ^{140}La. The level at 1.597 MeV has previously been assigned to be a 2^+ state. The levels at 2.90 and 3.12 MeV had previously been assigned to be $(1, 2)^+$; on the basis of the present experiment this uncertainty is removed. The correct J^π is 2^+.

The level observed at 2.35 MeV was very weakly excited. Since the excitation of this level may be a violation of the $\Delta B=0$ selection rule, it was of interest to determine whether it was the 2.348-MeV 2^+ state or the 2.350-MeV $(5)^-$ state, both of which were observed in the β-γ decay of ^{140}La. Figure 8 indicates that an $L=2$ assignment is clearly favored.

Angular distributions for $L=4$ transitions are shown in Fig. 9. The level at 3.54 MeV is weak and data at small angles were difficult to measure; therefore, only a tentative $J=4$ assignment can be made, since good small-angle data are a prerequisite to making spin assignments. The level at 3.34 MeV is fitted quite well by the $L=4$ calculation and is therefore probably not the 3.32-MeV $(1, 2)^+$ level observed in the decay work. The 4^+ level at 2.09 MeV is quite strongly excited. A 6^+ level has been observed[13] in the β-γ decay of ^{140}La to be about 25 keV above this 4^+ level. This 6^+ level, which would not be resolved from the 4^+ level in the present experiment, is excited very weakly if at all. Any appreciable 6^+ strength would be easily detected, since the addition of a 6^+ angular distribution, characterized by a strong

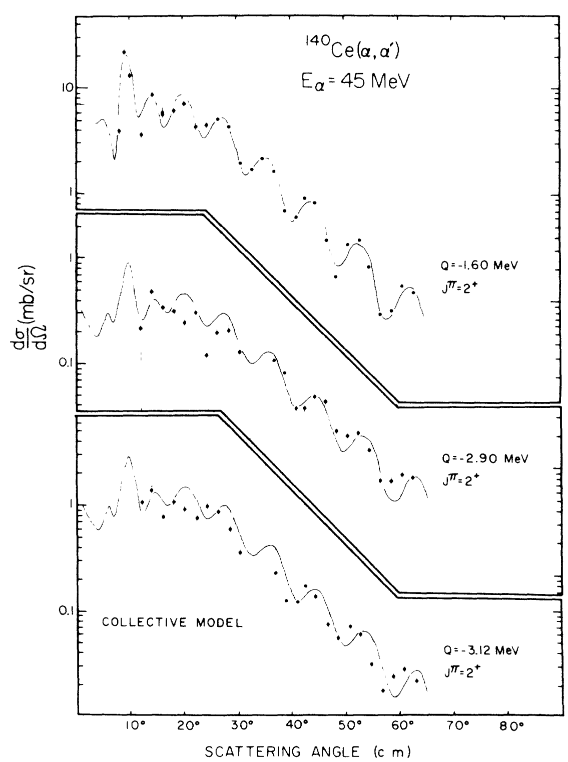

FIG. 7. Angular distributions for (α, α') transitions to 2^+ levels of ^{140}Ce.

maximum at 20°, would wash out the structure of the 4^+ angular distribution in this region.

Figure 10 shows the angular distributions of the negative-parity levels excited. The 3^- level at 2.464 MeV, generally believed to be an octupole vibration, is strongly excited. The 3^- level at 3.04 MeV is a new level, since only positive-parity levels had previously been observed in this region of the spectrum. The 5^- level at 3.25 MeV may be the negative-parity level observed at this energy in the reaction[14,15] $^{139}La(^3He, d)^{140}Ce$. The 3^- level at 3.98 MeV had not been previously observed.

The spectroscopic results for ^{140}Ce are presented in Table I. The reduced isoscalar transition rates $B_u(ISL)$ were calculated for an assumed uniform mass distribution using the form suggested by Bernstein[1]:

$$B_u(ISL)=\frac{Z^2}{4\pi}\frac{(L+3)^2}{(2L+1)}\left(\frac{\delta_L}{1.2A^{1/3}}\right)^2. \tag{18}$$

In this form $B_u(ISL)$ is in Weisskopf single-particle units. Owen and Satchler[16] have shown that the uniform-mass-distribution approximation results in underestimation of transition rates, the error increasing with increasing multipolarity; therefore, Table I also lists the reduced isoscalar transition rates $B_F(ISL)$ for an assumed Fermi mass distribution

$$\rho(r)=\rho_0[1+e^{(r-c)/a}]^{-1}, \tag{19}$$

with

$$c=(1.15A^{1/3}-0.53A^{-1/3})\ \text{F},$$

$$a=0.568\ \text{F}.$$

The $B_F(ISL)$ were calculated using the tables of Bernstein.[1]

Also listed in Table I are the reduced electromagnetic transition rates measured by Pitthan[17] using the (e, e') reaction. The listed value of $B(E2)$ is in agreement with measurements using other techniques.

It is interesting to compare the isoscalar and electromagnetic transition rates for transitions to the first 2^+, 4^+, and 3^- levels; as Bernstein has pointed out, these should be equal for levels well described as mass vibrations. It is not surprising to find that the $B(EL)$ and $B(ISL)$ transition rates are quite different for the first 2^+ and 4^+ levels: both of these levels have been excited in proton-transfer experiments and have been predicted by considering only the active protons; therefore, one does not expect these levels to be vibrational states. The result that the 3^- level is apparently not describable simply as an octupole vibration is somewhat surprising; this level has not been observed in proton-transfer experiments and has been assumed to be a simple mass vibration.

B. ^{139}La

For odd-A nuclei extraction of δ_L is usually not possible. The reason is evident from Eq. (5); the

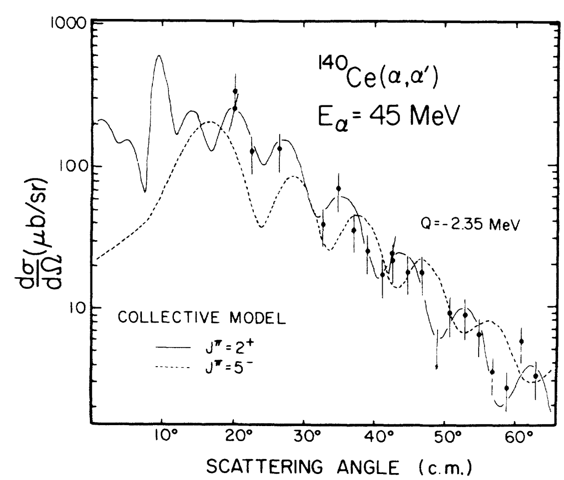

FIG. 8. Angular distribution for the $\Delta B \neq 0$ transition to the 2^+ state at 2.35 MeV in ^{140}Ce.

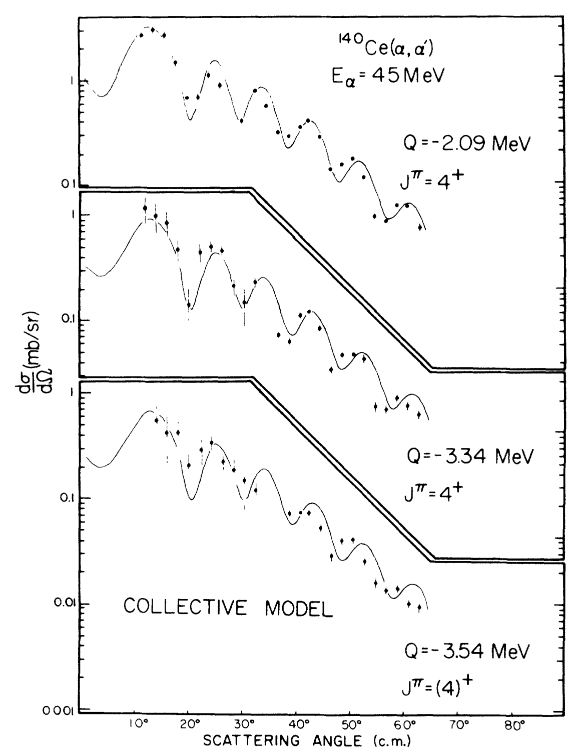

FIG. 9. Angular distributions for (α,α') transitions to 4^+ levels of ^{140}Ce.

form factor, and therefore the cross section, is dependent on the spin of the final state J', and this spin is not usually known. Therefore the cross-section parameters will be the partial deformation lengths $\delta_L^{J'}$ defined by

$$\delta_L^{J'} \equiv \left(\frac{2J'+1}{(2J+1)(2L+1)}\right)^{1/2} \delta_L \ . \qquad (20)$$

According to the weak-coupling model, there may exist multiplets of states in odd-A nuclei that result from a coupling of the odd particle or hole to collective core states in the adjacent even-even nucleus. One expects the cross section for excitation of the core to be approximately independent of the addition or subtraction of one nucleon. As a consequence of this expectation and because in the odd-A nucleus the core strength is expected to be spread among the members of the multiplet, the result that

$$(\delta_L)^2 \simeq \sum (\delta_L^{J'})^2 \qquad (21)$$

is obtained. The sum is over all states excited by angular momentum transfer L, and δ_L is the deformation length for the core state in the neighboring even-A nucleus.

One does not anticipate that the members of a weak-coupling multiplet will be easily identified in the spectrum of ^{139}La because of the many possibilities for mixing. The $\frac{5}{2}^+$ first excited state is at 166 keV, and configurations with this state coupled to core states are therefore expected to mix with "pure" core-ground-state coupled states of the same spin and parity. Also complicating the situation is the closeness of the collective 4^+ and 2^+ levels in the ^{140}Ce core; members of multiplets arising from coupling between these two levels and the ground and first excited states are also expected to mix.

TABLE I. Spectroscopic results for ^{140}Ce.

E (MeV)	J^π	δ_L [a] (F)	$B_u(ISL)$ [b]	$B_F(ISL)$ [b]	$B(EL)$ [c]
			(Weisskopf single-particle units)		
1.597	2^+	0.46	7.4	7.4	18 ± 2
2.09	4^+	0.42	6.8	7.8	21 ± 4
2.35	(2^+)	0.08	0.22	0.22	
2.464	3^-	0.67	15.8	16.5	26 ± 3
2.90	2^+	0.13	0.55	0.55	1.3 ± 0.5
3.04	3^-	0.15	0.80	0.83	
3.12	2^+	0.22	1.7	1.7	2.6 ± 0.5
3.25	5^-	0.30	3.6	4.9	
3.34	4^+	0.24	2.2	2.5	
3.54	$(4)^+$	0.21	1.7	1.9	
3.98	3^-	0.21	1.56	1.63	

[a] Estimated uncertainty of ±5%.
[b] Estimated uncertainty of ±10%.
[c] Reference 17.

All levels below 2.31 MeV which have been appreciably excited by the (α, α') reaction are positive-parity levels. Angular distributions for the first four groups analyzed are shown in Fig. 11. The doublet at 1.23-1.26 MeV is very well fitted by the $L=2$ DWBA calculation, and it therefore appears very unlikely that either level could have been excited by a pure $L=4$ transition; thus, neither level is the $\frac{1}{2}^+$ state observed near 1.2 MeV in the proton-transfer experiments,[14, 15, 18] since a transition from the $\frac{7}{2}^+$ ground state to a $\frac{1}{2}^+$ level cannot proceed via an $L=2$ transition.

The level at 1.42 MeV is also well described by the $L=2$ DWBA calculation. The $1h_{11/2}$ one-quasiparticle level observed near 1.42 MeV in proton-transfer experiments is therefore not appreciably excited, since a negative-parity mixture in the angular distribution would be easily detected.

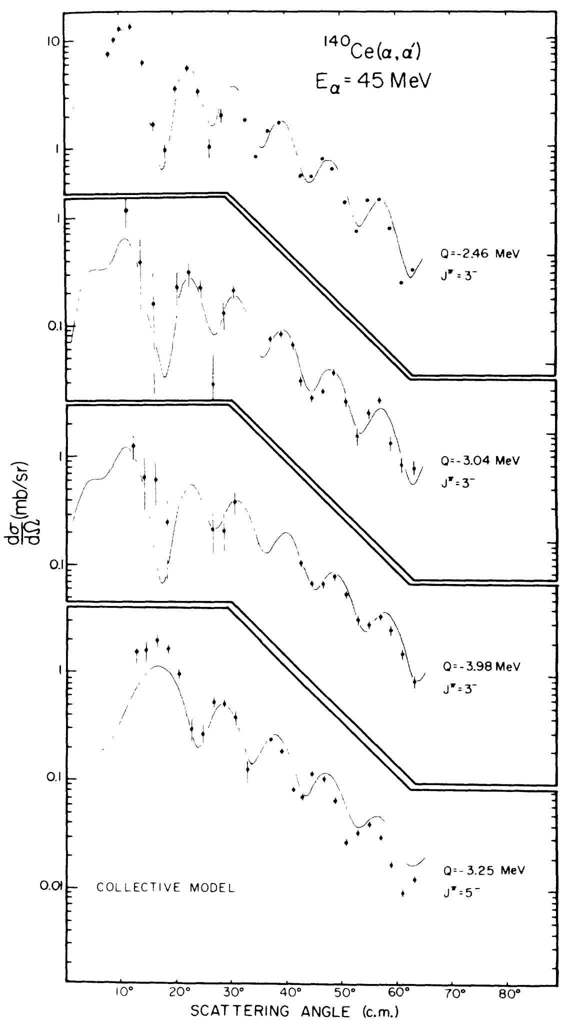

FIG. 10. Angular distributions for (α, α') transitions to negative-parity states of ^{140}Ce.

This result is in contrast to the results of the $^{141}Pr(\alpha,\alpha')$ experiment[19] in which the $1h_{11/2}$ one-quasiparticle state at 1.12 MeV was found to have a sizable fraction of the 3^- collective strength.

The doublet at 1.54-1.58 MeV is also predominantly populated by an $L=2$ transition. Although some excitation of the $v=1$, $2d_{3/2}$ level at 1.56 MeV cannot be ruled out, it is felt, on the basis of high-resolution spectra, that most of the strength comes from the levels at 1.54 and 1.58 MeV. These two levels are the most strongly excited positive-parity states in the spectrum.

The transition to the doublet at 1.68-1.72 MeV is also dominated by an $L=2$ transition, but some $L=4$ admixture is possible.

The angular distributions for the more weakly excited positive-parity groups are shown in Fig. 12. The triplets at 1.77, 1.81, 1.86 MeV and at 1.92, 1.94, 1.96 MeV both appear to be relatively pure $L=4$ transitions.

The doublet at 2.04-2.06 MeV is very weak and it is only possible to assign a positive parity to each of these levels.

Since the group observed at 2.31 MeV was extremely weak in the (d, d') spectrum, it is not possible to determine the number of levels excited.

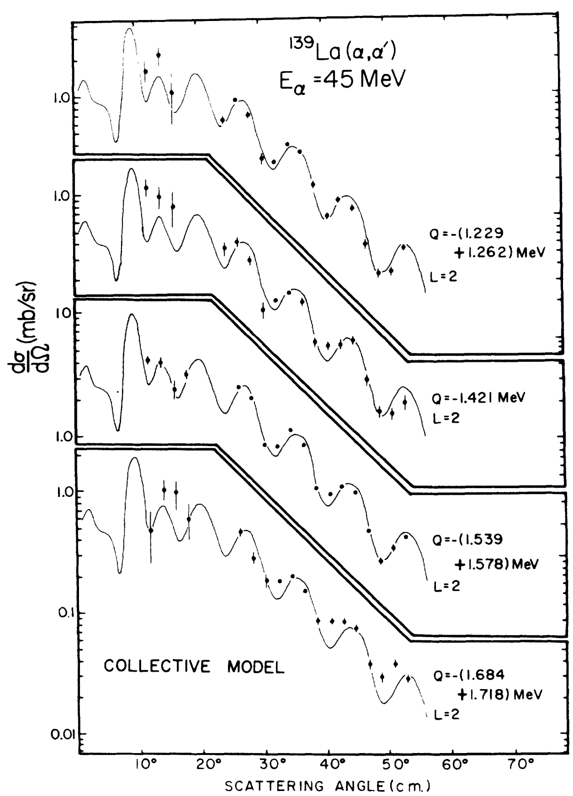

FIG. 11. Angular distributions for (α,α') transitions to positive-parity levels below 1.72 MeV in ^{139}La.

This group is dominated by positive-parity transitions.

The angular distributions for the groups of levels belonging to a negative-parity multiplet are shown in Fig. 13. Seven levels are definitely resolved in the (d, d') spectrum, and the level at 2.466 MeV is slightly broadened and could be an unresolved doublet. Although the resolution is inadequate to determine the individual intensities of this septet, it appears that the large percentage of the total strength which is possessed by the levels at 2.466 and 2.597 MeV precludes the possibility of a $2J+1$ intensity rule being obeyed.

Any positive-parity levels above the negative-parity multiplet are expected to be either levels with wave functions dominated by $v=3$, $B=\frac{3}{2}$ configurations or levels with $v>3$, which could not be populated by the (α,α') reaction. Therefore any transitions to positive-parity levels observed in this region of the spectrum are probably violations of the $\Delta B=0$ selection rule. The angular

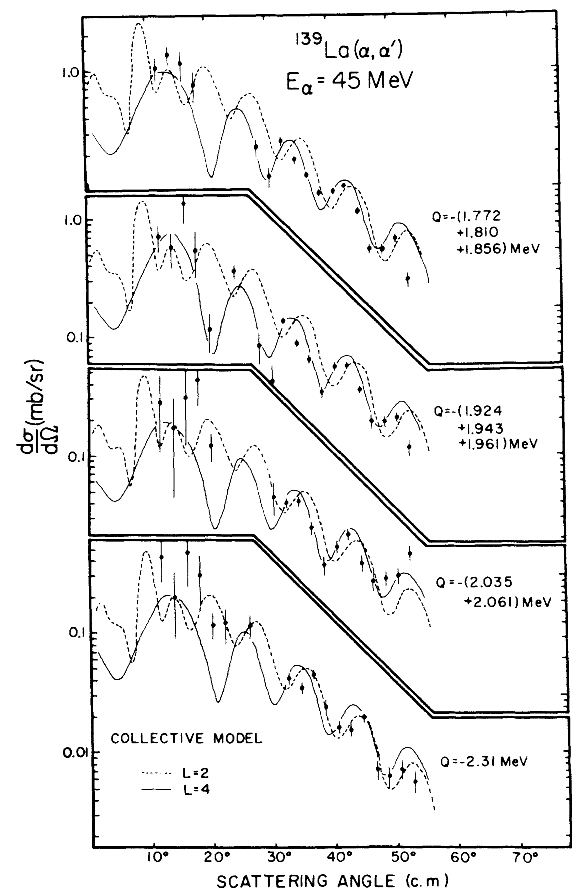

FIG. 12. Angular distributions for (α,α') transitions to positive-parity states between 1.75 and 2.31 MeV in the spectrum of ^{139}La.

distribution for the doublet at 2.78-2.81 MeV is essentially structureless indicating that these two levels have differing parities. Examination of line shapes for all spectra reveals that the level at 2.81 MeV has positive parity. The estimated strength of this transition is less than 5% of the total $L=2$ strength to states below 2.3 MeV. The angular distribution for the doublet at 2.87-2.89 MeV shows a slight loss of structure but is otherwise rather well fitted by an $L=2$ calculation; this indicates that the stronger level at 2.87 MeV has positive parity and that the level at 2.89 MeV probably has negative parity. The excitation strength for the 2.87-MeV level is approximately 10% of the $L=2$ strength to levels below 2.3 MeV. Above 2.9 MeV the density of states becomes very high and no analysis was possible. No evidence for positive-parity states comparable in strength to the levels at 2.81 and 2.87 MeV was found.

A summary of the experimental results for ^{139}La is presented in Table II. Where possible the relative intensities of individual members of multiplets analyzed together have been extracted. No attempt to determine $L=4+2$ admixtures has been made; groups for which it is uncertain whether the predominant L transfer is 2 or 4 have been analyzed for each case.

The strengths listed in Table II may be compared with the strengths for ^{140}Ce. For the $L=2$ transition to the 1.597-MeV level in ^{140}Ce the quantity $(\delta_2)^2$ was 0.215 F^2. If one sums the squares of the corresponding partial deformation lengths for excitation of levels below 2.3 MeV, excluding the levels between 1.77 and 1.96 MeV which appear to be dominated by $L=4$ transition strength, one finds that

$$\sum(\delta_2^{J'})^2 = 0.186\ \mathrm{F}^2 .$$

TABLE II. Spectroscopic results for ^{139}La.

E [a] (MeV)	Assumed L transfer	$(\delta_L^{J'})^2$ (F^2)	B_u(ISL) [b] (Weisskopf single-particle units)	B_F(ISL) [b] (Weisskopf single-particle units)
1.229 [c]	2	0.028	1.05	1.05
1.252 [c]	2	0.011	0.51	0.51
1.421	2	0.017	0.81	0.81
1.539 [d] 1.578 [d]	2	0.105	5.1	5.1
1.684 [c]	2	0.012	0.56	0.56
1.718 [c]	2	0.008	0.40	0.40
1.772 [d] 1.810 [d] 1.856 [d]	2 4	0.028 0.056	1.33 2.92	1.33 3.39
1.924 [d] 1.943 [d] 1.961 [d]	2 4	0.016 0.032	0.78 1.66	0.78 1.93
2.035 [d] 2.061 [d]	2 4	0.005 0.011	0.24 0.59	0.24 0.68
2.31 [e]	2 4	0.005 0.012	0.24 0.62	0.24 0.72
2.383 [d] 2.401 [d] 2.438 [d] 2.466 [d]	3	0.225	11.2	11.7
2.573 [d] 2.597 [d]	3	0.129	6.39	6.69
2.685	3	0.032	1.59	1.66
2.78 [d]	(3)			
2.81 [d]	(2)			
2.87 [c]	2	0.022	0.74	0.74
2.89 [d]	(3)			

[a] Energies deduced from a (d, d') spectrum.
[b] Estimated uncertainty of ±10%.
[c] Member of an unresolved multiplet whose intensity has been deduced from a high-resolution spectrum.
[d] Member of an unresolved multiplet.
[e] There is an unresolved multiplet at approximately this energy.

This represents about 87% of the $L=2$ strength for ^{140}Ce. The large number of states excited, however, precludes the possibility that the ^{139}La(α, α') spectrum can be understood easily in terms of a simple weak-coupling model; in the absence of mixing one expects to excite a quintet of levels which result from coupling the $\frac{7}{2}^+$ ground state to the 2^+ core.

The $(\delta_4)^2$ for the 2.084-MeV level in ^{140}Ce was found to be 0.18 F^2. The sum of the corresponding quantities for those transitions dominated by $L=4$ transitions (1.77–1.96 MeV) is

$$\sum (\delta_4^{J'})^2 = 0.087 \text{ F}^2 ,$$

only about half of the 4^+ core strength. The remainder of this strength could be present as $L=4$ admixtures in the strong $L=2$ transitions and would be very difficult to observe.

The total strength of the negative-parity levels between 2.3 and 2.7 MeV is

$$\sum (\delta_3^{J'})^2 = 0.39 \text{ F}^2 .$$

This represents about 90% of the $L=3$ strength for the transition to the 2.464-MeV 3^- level in ^{140}Ce; for this state $(\delta_3)^2$ was 0.436 F^2.

VII. MICROSCOPIC-MODEL ANALYSIS

A. Nuclear Wave Functions

Because of the success of shell-model calculations[4,14,15] in predicting spectroscopic properties of the $N=82$ nuclei, the zero-order eigenfunctions of the pseudo L-S coupling scheme are expected to be reasonably good representations of the wave functions of the "active" protons. Therefore, large enhancements of the α-particle–proton interaction are very probably indicative of contributions from the core.

B. ^{140}Ce

Of the levels excited in the present experiment, pseudo L-S wave functions may be confidently assigned to only two: The 2^+ level at 1.597 MeV and the 4^+ level at 2.09 MeV, by virtue of their strong excitations, must be excited by $\Delta B=0$ transitions; since the ground state has total $B=0$, these two levels must have dominant components of their wave functions of $|2;2(02)\rangle$ and $|4;2(04)\rangle$, respectively. The $B=0, 6^+$ level, $|6;2(06)\rangle$, although not appreciably excited in the present experiment, may also be analyzed by putting an upper limit on its collective enhancement. The weakly excited (2^+) level at 2.35 MeV is probably a $B=0$, $v=2$ state and therefore could have a dominant component of its wave function of either $|2;2(11)\rangle$ or $|2;2(13)\rangle$. (Calculations by Jones and Borgman[14,15] indicate that appreciable mixing between these two $B=1$, zero-order wave functions should be expected.)

The results of the calculations for the $\Delta B=0$ transitions are shown in Fig. 14. Collective Coulomb excitation was assumed for the $L=2$ transition using

$$\beta_2^C R_r = 0.46 \text{ F}$$

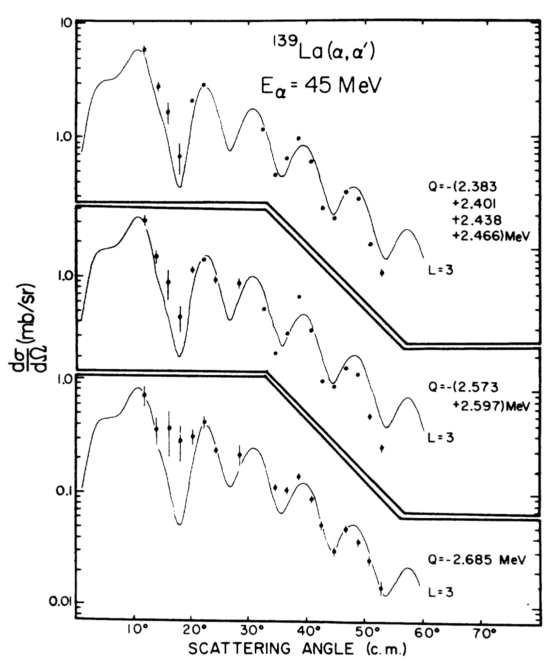

FIG. 13. Angular distributions for (α,α') transitions to negative-parity states in ^{139}La.

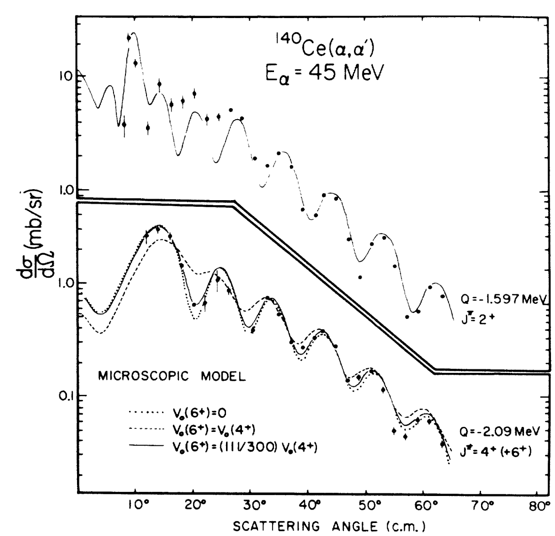

FIG. 14. Microscopic-model DWBA predictions for transitions to the 1.597- and 2.09-MeV levels of ^{140}Ce.

and was neglected for transitions to the 4^+ and 6^+ levels.

The interaction strength V_0 required to normalize the $L=2$ calculation to the 2^+ data was approximately 320 MeV, considerably larger than the 30–50 MeV expected if the assumed wave function were perfectly descriptive of the state. Thus, although the collective analysis clearly rules out the possibility of this level being described as a mass vibration, the large enhancement of V_0 indicates that the core probably plays a role in the structure of the 1.597-MeV 2^+ state which is considerably larger than had previously been assumed.

The cross section for excitation of the observed group at about 2.09 MeV is shown fitted with three microscopic DWBA calculations. A pure $L=4$ calculation provides an adequate fit to the data, but the good statistics of the 18° datum allows one to seek a better fit by assuming that the 6^+ level is weakly excited. If one assumes equal interaction strengths V_0 for the 4^+ and 6^+ excitations, the fit is appreciably worsened. Finally, assuming $V_0 = 300$ MeV for the $L=4$ calculation and $V_0 = 111$ MeV for the $L=6$ calculation an optimum fit is obtained; it is felt that the upper limit of $V_0(6^+)$ is 150 MeV.

The calculations for the $\Delta B \neq 0$ transitions to pure $|2;2(1,1)\rangle$ and $|2;2(1,3)\rangle$ levels indicate that the cross sections should be approximately 10 and 5%, respectively, as large as the cross section for excitation of the $|2;2(0,2)\rangle$ level; these calculations were performed using the same enhanced strength factor V_0 as was used for the 1.597-MeV 2^+ level. These are to be compared with the experimental cross section for the 2.35-MeV level, which was about 3% of that for the 1.597-MeV level.

It had been hoped that the apparent violations of the $\Delta B=0$ selection rule would be explainable in terms of $\Delta B=0$ transitions between smaller components of the wave functions. These calculations, however, indicate that breaking of the selection rule may be due instead to small differences in the tails of the radial form factors.

Jones and Borgman[14,15,20] have calculated more realistic wave functions by removing the degeneracy of the $1g_{7/2}$ and $2d_{5/2}$ orbitals and allowing excitations into the $2d_{3/2}$, $3s_{1/2}$, and $1h_{11/2}$ orbitals. If transitions among the smaller pieces of the wave functions add coherently, the conclusions drawn from the zero-order calculations concerning the importance of the core would be in error. The transition to the 2^+ level at 1.6 MeV was examined using these mixed (B,C) configuration wave functions; all $\Delta B=0$ transitions connecting the $v=0$ and $v=2$ pieces of the ground state to the $v=2$ and $v=4$ pieces of the 2^+ level were included. It was found that no strength was gained; in fact, approximately 10% more enhancement was required than for the zero-order case. The result that the core probably plays an important role in the structure of this level is therefore unchanged.

TABLE III. Relative transition strengths for $|\frac{7}{2};1(\frac{1}{2}3)\rangle \to |J';3(B'C')\alpha'\rangle$.

$2J'$	$2B'$	C'	α'	$\lvert\sum A_{jj'}\rvert^2_{L=2}$	$\lvert\sum A_{jj'}\rvert^2_{L=4}$	$J'=C'\pm B'$
3	1	1		0.69	0.21	+
5	1	2	(20)	0.71	0.61	+
5	1	2	(21)	1.46	0.29	+
7	1	3		1.31	0.15	+
9	1	4	(20)	1.48	0.15	+
9	1	4	(21)	0.058	1.51	+
11	1	5	(20)	0.53	0.81	+
11	1	5	(21)	2.43	1.39	+
13	1	6		0.0	1.00	+
15	1	7		0.0	2.22	+
17	1	8		0.0	0.0	+
1	1	1		0.0	0.30	−
3	1	2	(20)	0.078	0.30	−
3	1	2	(21)	0.16	0.24	−
5	1	3		0.16	0.09	−
7	1	4	(20)	0.14	0.19	−
7	1	4	(21)	0.01	0.58	−
9	1	5	(20)	0.03	0.21	−
9	1	5	(21)	0.12	0.36	−
11	1	6		0.0	0.16	−
13	1	7		0.0	0.16	−
15	1	8		0.0	0.0	−

C. ^{139}La

The $\Delta B=0$ transitions which can occur for ^{139}La are considerably more numerous than for ^{140}Ce. The ^{139}La ground state, in zero order, has the wave function $|\frac{7}{2};1(\frac{1}{2}3)\rangle$. Transitions can therefore occur to levels with seniority $v=3$; $B=\frac{1}{2}$; $C=1, 2\ [\alpha=(21)],\ 2\ [\alpha=(20)],\ 3, 4\ [\alpha=(20)],\ 4\ [\alpha=(21)],\ 5\ [\alpha=(20)],\ 5\ [\alpha=(21)], 6, 7,$ and 8. (Racah's U notation,[12] in terms of the two quantum numbers associated with the symmetry group G_2, is used for α.) Coupling B to C to obtain J, there are therefore 22 $v=3, B=\frac{1}{2}$ levels. Since the ground state is a $\frac{7}{2}^+$ level, only 16 states can be reached by an $L=2$ transition ($\frac{3}{2} \leq J' \leq \frac{11}{2}$) and 21 states can be reached by an $L=4$ transition ($\frac{1}{2} \leq J' \leq \frac{15}{2}$). To perform the DWBA calculations for each of these 37 transitions of interest would not be particularly informative, since many multiplets are not resolved. Furthermore, any mixing between states of the same spin and parity would alter the relative intensities and the L admixtures. Therefore, detailed agreement between microscopic analyses and experimental results is neither expected nor sought; rather it is hoped that a qualitative comparison of the experimental results with microscopic calculations using the eigenfunctions $|J; v(BC)\alpha\rangle$ will elucidate the general features of the excitation spectrum and perhaps the structure of some of the energy levels of ^{139}La.

In order to compute the relative intensities, Eq. (12) will be used. Furthermore, it will be assumed that the radial form factors $I_L^{jj'}(r)$ are independent of j,j'; this assumption is justified by the approximate j,j' independence near the nuclear surface as shown in Fig. 6. In this approximation, Eq. (12) becomes

$$F_L(r) = -\sqrt{4\pi}\, V_0 I_L(r) \sum_{jj'} A_{jj'} . \quad (22)$$

The cross section for a given transition may therefore be approximated as

$$\frac{d\sigma}{d\Omega} \propto |\sum_{jj'} A_{jj'}|^2 . \quad (23)$$

In Table III are listed the calculated values of $|\sum A_{jj'}|^2$ for all $\Delta B = 0$, $L = 2, 4$ transitions. These have been grouped into levels with $J' = C' \pm \frac{1}{2}$ to emphasize an interesting feature of these calculations; nearly all of the predicted transition strength is to levels with $J' = C' + \frac{1}{2}$. Noting that the ground state is also a level of this type, i.e.,

$$J = \tfrac{7}{2} = 3 + \tfrac{1}{2} ,$$

this result may be summarized by stating that transitions involving a "pseudospin flip" are predicted to be strongly inhibited.

Examination of the experimental results (see Table II or Fig. 2) reveals that most of the $L = 2$ strength is concentrated in the first six levels excited. It is also interesting to note that there is no appreciable transition strength to the previously observed levels at 1.38 and 1.48 MeV; these levels are presumed[21-23] to be high-spin states ($J' \geq \frac{7}{2}$) and would ordinarily be expected to be strongly excited. These experimental results are explainable in terms of the theoretical predictions shown in Table III; strong excitations can be identified as having $J' = C' + \frac{1}{2}$, while the missing levels probably have $J' = C' - \frac{1}{2}$.

Another interesting result of the calculated intensities is the relative purity of the strong transitions. Of the six strongest $L = 2$ transitions only transitions to the $|\frac{11}{2}; 3(\frac{1}{2}5)(21)\rangle$ and $|\frac{5}{2}; 3(\frac{1}{2}2)(20)\rangle$ levels have comparably large $L = 4$ components. This prediction also agrees with the experimental results; $L = 2$ DWBA calculations shown in Fig. 11 fitted the experimental data for the strong groups and little evidence for $L = 4$ admixtures was seen. The strongest $L = 2$ transition is also predicted to be one of the strongest $L = 4$ transitions. Unfortunately the strongest two states in the spectrum are unresolved, thereby making this prediction difficult to test, since the $L = 4$ admixture would become smaller relative to the strong $L = 2$ transitions.

The success of the qualitative microscopic calculations is somewhat surprising in view of the fact that one might have expected considerable mixing to occur between states of the form $|J = C + \frac{1}{2}; 3(C\frac{1}{2})\alpha\rangle$ and $|J = C' - \frac{1}{2}; 3(C'\frac{1}{2})\alpha\rangle$. The most probable reason why the levels apparently remain relatively pure is that the direct matrix element connecting them is proportional to $(n - 7)$, n being the number of active protons. Thus, since $n = 7$ for ^{139}La, mixing can occur only through higher-lying (e.g., $v = 5$) levels.

Finally, in regard to Table III, note that if all the $L = 2$ strengths are summed the predicted total strength is

$$\sum |\sum_{jj'} A_{jj'}|^2_{L=2} = 9.37 .$$

If one compares this with the $L = 2$ strength predicted for ^{140}Ce,

$$|\sum_{jj'} A_{jj'}|^2 = 10.67 ,$$

it is seen that the total $L = 2$ strength for ^{139}La is predicted to be 87.5% of that for ^{140}Ce (this is just 7/8, the ratio of the numbers of "active" protons in the two nuclei). Since this is in good agreement with the experimental results, one may conclude that the role of the core for $L = 2$ transitions is probably quite similar for ^{139}La and ^{140}Ce.

VIII. DISCUSSION

A. ^{140}Ce

Table IV presents the energy levels of ^{140}Ce as determined by several experiments. These include the present work, the β-γ decay study of Baer, Reidy, and Wiedenbeck,[13] the (^3He, d) results of Borgman *et al.*,[14,15] and the (d, ^3He) results of Jones *et al.*[14,15] A collation of these spectra is compared in Fig. 15 with the shell-model calculations of Wildenthal[4] and with the eigenvalues of the generalized pairing interaction (GPI).

The most striking feature of Fig. 15 is that of the many low-lying (<2.7 MeV) positive-parity levels predicted by the theoretical calculations, only the 5^+ level has not been observed experimentally. It can be concluded that the shell-model structure of these low-lying positive-parity levels is reasonably well understood. The results of the present experiment do not refute the validity of the shell-model calculations but indicate that such calculations do not represent complete wave functions for these levels if the core is ignored.

The structure of the low-lying negative-parity levels is somewhat less well understood. The present experiment indicates that the 3^- level at 2.464 MeV is not simply an octupole vibration. The negative-parity level at 2.35 MeV, assigned to be $(5)^-$ by Baer, Reidy, and Wiedenbeck,[13] was not excited by the (α, α') reaction and is therefore probably not a predominantly core-excited state. This level has not been observed in any other experiment and its structure is therefore quite uncertain.

Many levels above 2.9 MeV have been observed. This region of the spectrum is of interest primarily because the $2d_{3/2}$, $1h_{11/2}$, and $3s_{1/2}$ single-particle strengths are observed here.

B. ^{139}La

Table V presents the results of several experimental investigations of ^{139}La. These include the present work, a collation of β-γ decay studies of Hill and Wiedenbeck and of Berzins, Bunker, and Starner,[6] the (n, n') and $(n, n'\gamma)$ results of Van Der

TABLE IV. Summary of spectroscopic information for ^{140}Ce as determined by the present experiment and other recent experiments.

Present (α, α')		Baer, Reidy, and Wiedenbeck (Ref. 13) β-γ decay		Borgman *et al.*[a] (Refs. 14, 15) (^{3}He, d)		Jones *et al.*[a] (Refs. 14, 15) (d, ^{3}He)		Collation of results[b]	
E	J^π	E	J^π	E	l	E	l	E	J^π
0.0		0.0	0^+	0.0	4	0.0	2	0.0	0^+
1.597	2^+	1.597	2^+	1.60	2	1.60	2, 4	1.597	2^+
		1.904	0^+	1.90	2	1.90	2	1.904	0^+
2.09	4^+	2.084	4^+	(2.10)	2	2.08	2, 4	2.084	4^+
(2.09)	(6^+)	2.108	$(6)^+$	(2.10)	2	2.11	2, 4	2.108	6^+
2.35	(2^+)	2.348	2^+	2.35	2	2.35	2, 4	2.348	2^+
		2.350	$(5)^-$					2.350	$(5)^-$
		2.412	3^+	2.41	2	2.41	2, 4	2.412	3^+
2.464	3^-	2.464	3^-					2.464	3^-
		2.481	$(4)^+$	2.47	2			2.481	$(4)^+$
		2.516	$(4^+, 3^\pm)$	(2.52)	2	(2.52)	2, 4	2.516	$(4)^+$
		2.522	2^+	(2.52)	2	(2.52)	2, 4	2.522	2^+
		2.548	$(1, 2)^+$			2.54	2, 4	2.548	$(1)^+$
				2.63	4			2.63	$(6)^+$
2.90	2^+	2.900	$(1, 2)^+$	2.90	2	2.90		2.900	2^+
				3.00		3.00	2	3.00	+
3.04	3^-			(3.06)				3.04	3^-
3.118	2^+	3.118	$(1, 2)^+$	3.12	2	3.13		3.118	2^+
				3.19				3.19	
3.25	5^-			3.25	5			3.25	5^-
		3.320	$(1, 2)^+$	(3.33)	2, 0			3.320	$(1, 2)^+$
3.34	4^+			(3.33)	2, 0			3.34	4^+
				3.42	5			3.42	−
				3.47	5			3.47	−
				3.53	5			3.53	−
3.54	$(4)^+$							3.54	$(4)^+$
				3.64				3.64	
				3.69	5			3.69	−
				3.78	2, 0			3.78	+
				3.89	5			3.89	−
3.98	3^-							3.98	3^-
				4.00	2, 0			4.00	+
				4.13	0			4.13	+
				4.17	0			4.17	+
				4.26	0			4.26	+
				4.36	2			4.36	+

[a] Parentheses on energies indicate only uncertainty in identification of these levels with levels observed in the other experiments.
[b] Theoretical spectra have been used as a guide for selecting J^π values for some levels.

Merwe *et al.*,[21] the (γ,γ') results of Moreh and Nof[22] and of Szichman *et al.*,[23] the $(^3\mathrm{He},d)$ results of Wildenthal, Newman, and Auble,[18] and the $(d,{}^2\mathrm{He})$ results of Jones *et al.*[14,15] Identification of levels excited in each experiment with those excited in the other experiments was done by careful review of all experimental results; this identification should be considered probable, not definite. These results are compared with the shell-model calculations of Wildenthal[4] and with the eigenvalues of the GPI in Fig. 16. The low-lying levels (<2.5 MeV) are seniority $v=3$ and seniority $v=1$ levels involving the $1g_{7/2}$, $2d_{5/2}$, $3s_{1/2}$, $2d_{3/2}$, and $1h_{11/2}$ orbitals. In the proton-transfer experiments the $2d_{3/2}$ and $3s_{1/2}$ strengths are observed to be fragmented among the predominantly $v=3$ levels with J^π of $\frac{3}{2}^+$ and $\frac{1}{2}^+$, respectively; comparison with the theoretical predictions indicates that these strengths are found in all available $\frac{3}{2}^+$ and $\frac{1}{2}^+$ levels.

TABLE V. Summary of spectroscopic information for ^{139}La as determined by the present work and several other recent experiments. In each column are listed the measured E and $2J^\pi$ values when determined.

(α,α') [a]	β-γ decay (Ref. 6)	(n,n') and $(n,n'\gamma)$ (Ref. 21)	(γ,γ') (Ref. 23)	(γ,γ') (Ref. 22)	$(^3\mathrm{He},d)$ (Ref. 18)	$(d,{}^3\mathrm{He})$ (Refs. 14 and 15)	Collation of results
0.0 +	0.0 7^+	0.0 7^+	0.0 7^+	0.0 7^+	0.0 7^+	0.0 7^+	0.0 7^+
	0.166 5^+	0.166 5^+	0.166 5^+	0.166 5^+	0.166 5^+	0.166 5^+	0.166 5^+
		1.206			1.21 1^+	1.22 1^+	1.206 1^+
1.229 +	1.219	1.217 (7^+)	1.219 $9^{(+)}$	1.220 $(5, 9^+)$			1.219 $(9)^+$
1.262 +	1.257	1.255 (5^+)	(1.257)	1.257			1.257 $(5)^+$
	1.382	1.383 (9^+)	1.381 (7–9)	1.384 7			1.382 7^+
1.421 +	1.421	1.420 (5^+)	1.421 (7, 11)	1.419 7, (11)			1.421 $(7)^+$
		1.439			1.42 11^-	1.42 11^-	1.43 11^-
	1.476	1.475 (7^+)	1.477 (7–11)	1.480			1.476 $(7)^+$
1.539 +	1.536	1.538	1.540 (7–11)	1.536 7			1.536 7^+
	1.559	(1.559)			1.56 3^+	1.56 3^+	1.559 3^+
1.578(11) +	1.578	1.577	1.581 (7–11)	1.580 9			1.578 $(9)^+$
1.684 +	1.683	1.682	1.687 $7^{(+)}$	1.684			1.683 7^+
1.718 +		1.714		1.714			1.714 +
		1.756		1.756			1.756
(1.772) +	1.762				1.78 3^+	1.75 3^+	1.762 3^+
(1.772) +	1.768		1.770 $9^{(+)}$				1.768 $9^{(+)}$
(1.772) +					1.78 1^+	1.75 1^+	1.77 1^+
1.810 +		1.820		1.820			1.820 +
		1.835		1.838			1.838
1.856 +	1.857		1.856 (7–11)		1.85 3^+	1.85 3^+	1.857 3^+
			1.894 (7–11)				1.894 (7–11)
1.924 +	1.922 $(5, 7^+)$			1.919			1.922 $(5, 7)^+$
1.943 $(13, 15)^+$							1.943 $(13, 15)^+$
1.961 +	1.963			1.956	1.96 3^+	1.95 3^+	1.963 3^+
2.035							2.035 +
2.064 +	2.061		2.060 (5–9)	2.061			2.061 $(5–9)^+$
				2.123			2.123
				2.232 (7, 11)			2.232 (7, 11)
					2.24 3^+		2.24 3^+
(2.31) +					2.31 1^+		2.31 1^+
2.383 −							2.383 −
2.401 −							2.401 −
2.438 −							2.438 −
2.466 −							2.466 −
2.573 −							2.573 −
2.597 −							2.597
2.685 −							2.685 −
2.78 (−)							2.78 (−)
2.81 (+)							2.81 (+)
2.87 +							2.87 +
2.89 (−)							2.89 (−)

[a] Energies determined by a ^{139}La(d,d') spectrum.

In the following discussion, reference to Table III will be helpful. The ground state and first excited state at 0.166 MeV have J^π of $\frac{7}{2}^+$ and $\frac{5}{2}^+$, respectively; these levels are the one-quasiparticle $1g_{7/2}$ and $2d_{5/2}$ levels. In terms of the *B-C* model these two levels constitute the "ground-state doublet" with wave functions $|\frac{7}{2};1(\frac{1}{2}3)\rangle$ and $|\frac{5}{2};1(\frac{1}{2}3)\rangle$.

The triplet of levels near 1.2 MeV can be identified as follows: The level at 1.206 MeV is a $\frac{1}{2}^+$ level by virtue of its excitation by the (^{3}He, *d*) reaction through the seniority $v=1$, $3s_{1/2}$ component of its wave function. Since the spectroscopic factor for this level is small, it is likely that the dominant component of the wave function of this level is $|\frac{1}{2};3(\frac{1}{2}1)\rangle$. The level at 1.219 MeV is probably a $\frac{9}{2}^+$ state as determined by the (γ,γ') experiment.[22] Since this level is excited by the (α,α') reaction, it is possible that the dominant configuration of this level is $|\frac{9}{2};3(\frac{1}{2}4)\alpha\rangle$. The level at 1.257 MeV was assigned $\frac{5}{2}^+$ by Van Der Merwe *et al.* on the basis of a Hauser-Feshbach analysis of the (n,n') data. Since this level was also populated by the (α,α') reaction, a possible dominant configuration consistent with this result would be $|\frac{5}{2};3(\frac{1}{2}2)\alpha\rangle$.

The level at 1.382 MeV was assigned to have spin of $\frac{7}{2}$ by Moreh and Nof. Since this level was not populated in the present experiment, the probable dominant configuration of the wave function is one with $J'=C'-\frac{1}{2}$, $|\frac{7}{2};3(\frac{1}{2}4)\alpha\rangle$.

The $\frac{11}{2}^-$ level observed near 1.42 MeV in the proton-transfer experiments is probably the same level observed at 1.439 MeV in the (n,n') experiment for which the appropriate neutron group was observed but no depopulating γ rays were seen. Since this level was not observed in the (α,α') experiment, it may be concluded that a $1h_{11/2}$ one-quasiparticle configuration is dominant with no appreciable collective strength of the form $(3^-\times 1g_{7/2})_{11/2}$.

The spin assignments for the positive-parity levels at 1.421 and 1.476 MeV are uncertain. The former level is populated by the (α,α') reaction, whereas the latter is not.

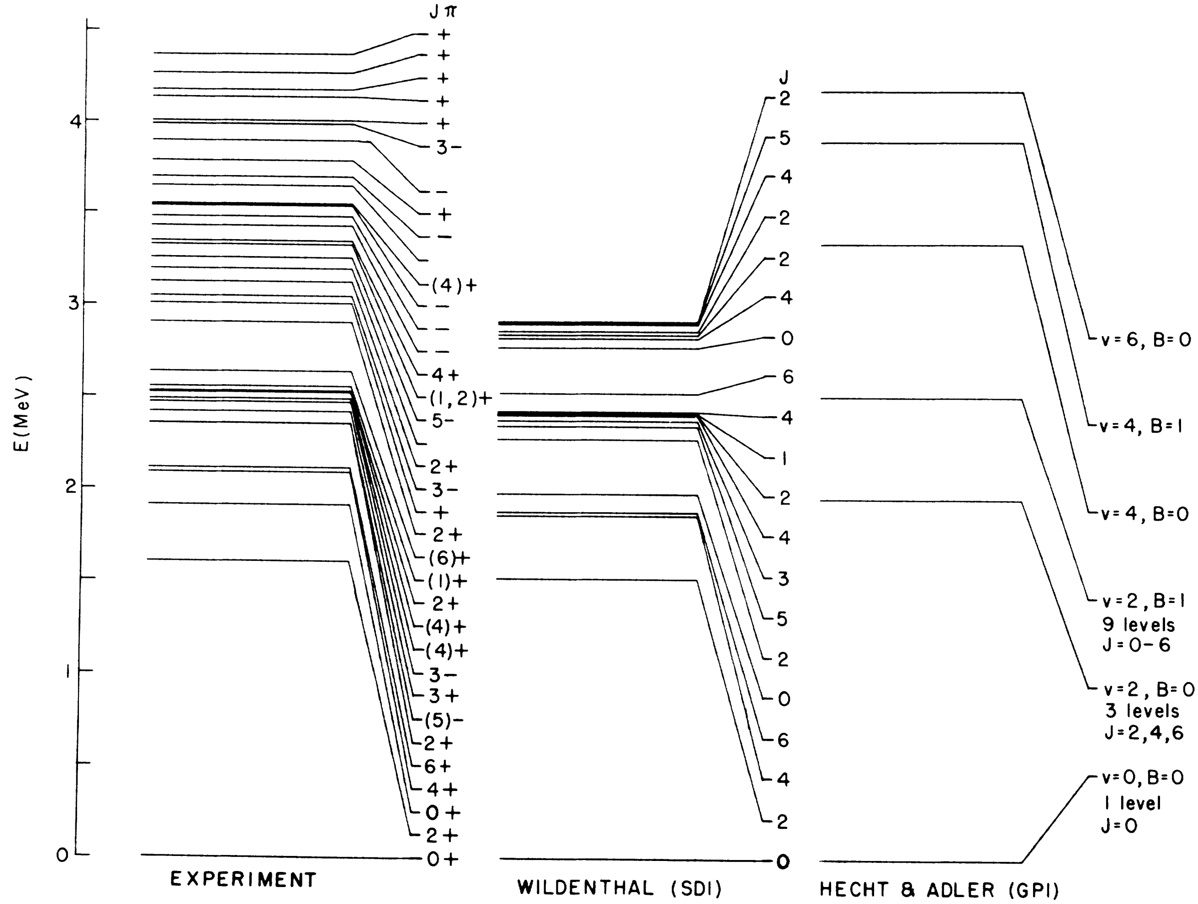

FIG. 15. Comparison of the experimentally observed spectrum of ^{140}Ce with the predictions of shell-model calculations.

The levels at 1.536 and 1.578 MeV were the two most strongly excited levels in the (α, α') spectrum. Theoretically, it is expected (see Table III) that one of these levels should have a spin of $\frac{11}{2}$. Moreh and Nof, however, assign to these levels a J^π of $\frac{7}{2}^+$ and $\frac{9}{2}^+$, respectively. This certainly is the major discrepancy between the present experiment and the (γ, γ') experiments: At least one of the strong (α, α') transitions should be to an $\frac{11}{2}^+$ level, yet the (γ, γ') results indicate that none of the levels which are strongly excited have this spin. The level at 1.536 MeV, moreover, has a γ-decay branch to the $\frac{5}{2}^+$ first excited state and could therefore not be an $\frac{11}{2}^+$ state. The level at 1.578 MeV decays only to the ground state and is therefore a possible candidate for the $\frac{11}{2}^+$ level; the $\frac{9}{2}^+$ assignment of Moreh and Nof, however, is quite convincing.

The $\frac{3}{2}^+$ level at 1.559 MeV is only weakly excited by the (α, α') reaction, and a quantitative measurement of its strength is not possible because of the nearby strong transitions. The spectroscopic factor measured by the $(^3He, d)$ reaction is small, indicating that the $v = 1, 2d_{3/2}$ component of the wave function is not the dominant component.

The phase of the (α, α') angular distributions indicates that the levels at 1.683 and 1.714 MeV have positive parity.

The $\frac{3}{2}^+$ component of the $\frac{3}{2}^+$-$\frac{1}{2}^+$ unresolved doublet reported near 1.76 MeV in the proton-transfer experiments is probably identifiable as one of the members of the 1.762-1.768-MeV doublet seen in the γ-decay work. The rather large $(^3He, d)$ spectroscopic factor indicates that the dominant component of the wave function of this level is seniority $v = 1, 2d_{3/2}$. Hill and Wiedenbeck have tentatively assigned $\frac{3}{2}^+$ to the 1.762-MeV level because of its nearly exclusive γ decay to the 166-keV first excited state.

The $\frac{1}{2}^+$ member of this doublet, however, cannot be identified as the other member of the γ-decay doublet, since the log*ft* values rule out β-decay feeding to $\frac{1}{2}^+$ levels. There are, therefore, at least three levels very close to 1.76 MeV in the spectrum; the spin-parity assignments for two of them being $\frac{3}{2}^+$ and $\frac{1}{2}^+$. The α-particle group ob-

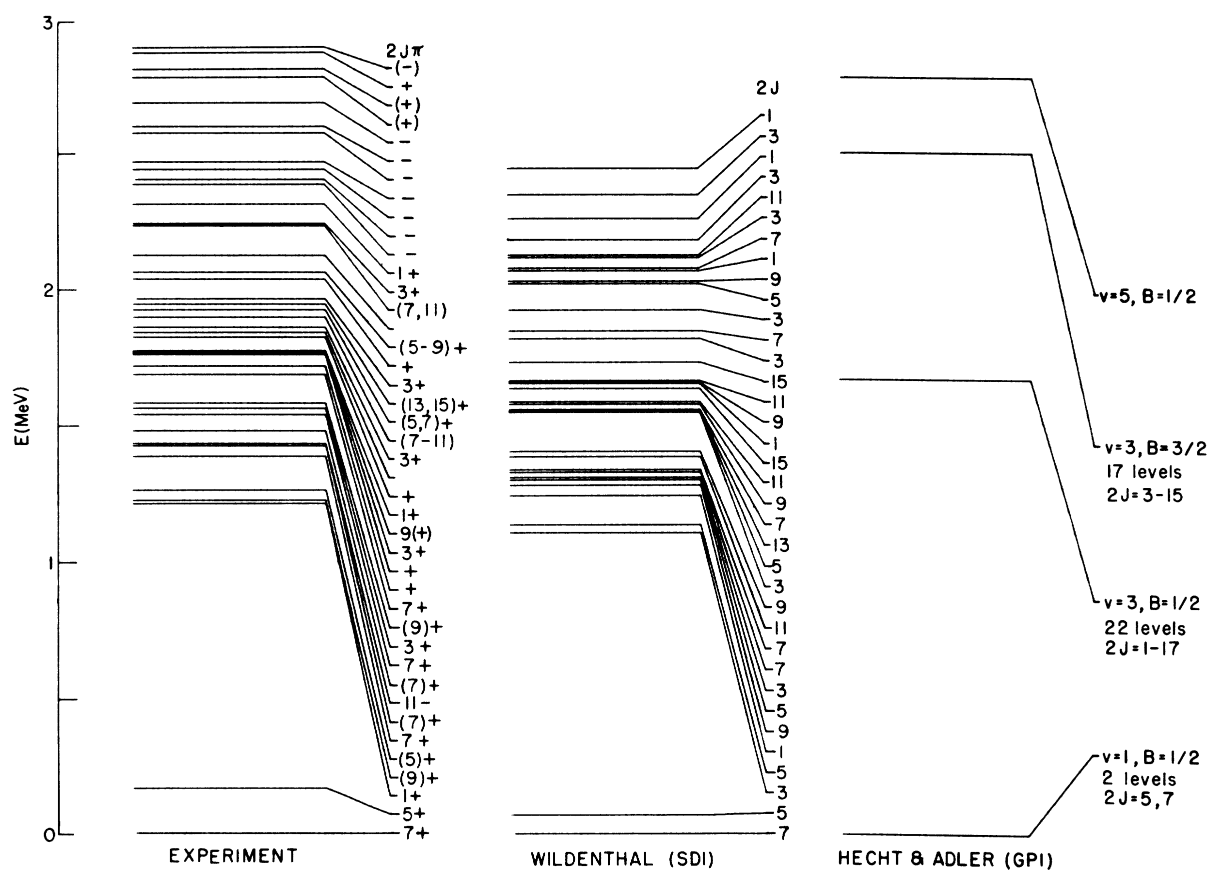

FIG. 16. Comparison of the experimentally observed spectrum of ^{139}La with the predictions of shell-model calculations.

served at 1.772 MeV is probably one or more of these states. The level at 1.756 MeV populated by the $(n, n'\gamma)$ reaction is probably still another level, since it and the 1.762-MeV level of the γ-decay work do not lie within the stated experimental uncertainties of each other; furthermore, the 1.762-MeV level observed by Berzins *et al.* decayed almost exclusively to the 166-keV first excited state, whereas this 1.756-MeV level was observed to decay only to the ground state and therefore probably has a high spin (and thus could not be the $\frac{1}{2}^+$ state). The large spectroscopic factor reported for the $(^3\mathrm{He}, d)$ reaction indicates that the $\frac{1}{2}^+$ level in this region of the spectrum is predominantly a $3s_{1/2}$ one-quasiparticle state.

The levels at 1.820 and 1.838 MeV are probably both high-spin states, since they undergo γ-decay only to the ground state. The $\frac{3}{2}^+$ level near 1.85 MeV which was excited by the proton-transfer experiments has previously been identified as the 1.857-MeV level observed by Berzins *et al.* This low spin assignment is consistent with the observed γ decay of this level to only the $\frac{5}{2}^+$ first excited state. Since this level was excited by the (α, α') reaction and since the $2d_{3/2}$ spectroscopic factor measured in the $(^3\mathrm{He}, d)$ reaction was comparatively small, it is possible that the dominant configuration of this level is $|\frac{3}{2}; 3(\frac{1}{2}1)\rangle$.

The level at 1.922 MeV has been tentatively assigned $(\frac{5}{2}, \frac{7}{2})^+$ by Hill and Wiedenbeck on the basis of its comparable γ-ray intensities for decay to the ground state and to the 0.166-MeV level. The level at 1.943 MeV has only been excited by the (α, α') experiment; since this level was primarily populated by an $L=4$ transition and since it was not observed in either the $(\beta\text{-}\gamma)$ decay studies or the (γ, γ') experiments, it is likely that it has a high spin, probably $(\frac{13}{2}, \frac{15}{2})^+$. The level at 1.963 MeV decays only to the $\frac{5}{2}^+$ level at 0.166 MeV and is therefore probably the same level observed in the proton-transfer experiments and assigned $\frac{3}{2}^+$.

The (α, α') angular distributions establish that the levels at 2.035 and 2.061 MeV have positive parity.

The negative-parity levels observed between 2.3 and 2.8 MeV, as discussed in Sec. VI B., are probably the result of coupling between the 3^- core state and one or more one-quasiparticle states.

IX. SUMMARY

Our results have shown that while no levels in ^{140}Ce are vibrational, the core evidently plays an important role in the structure of this nucleus. The core seemingly plays an equally important role in the structure of ^{139}La, but configuration mixing precludes an easy understanding of the (α, α') spectrum in terms of a weak-coupling model.

The B-C model appears to accurately predict the qualitative features of the (α, α') spectra: The $\Delta B = 0$ selection rule is reasonably well obeyed for both nuclei, and the predicted inhibition of the "pseudospin flip" qualitatively describes the features of the $^{139}\mathrm{La}(\alpha, \alpha')$ reaction. A major problem is the apparent lack of a good candidate for the $\frac{11}{2}^+$ level in ^{139}La, which is predicted to be one of the stronger transitions in the (α, α') reaction.

The theoretical developments of Hecht and Adler[5] which allow one to use single-component wave functions to describe these complicated many-proton states have proven extremely useful in elucidating the properties of these nuclei.

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APPENDIX

In order to evaluate the form factor for a microscopic calculation [Eq. (11)] one must evaluate the matrix element

$$\langle \Phi' | \sum_i i^L g_L(r_i, r) Y_L^{\mathfrak{M}}(\theta_i, \phi_i) | \Phi \rangle ,$$

where Φ and Φ' are the initial and final wave functions, assumed here to be the appropriate zero-order eigenfunctions of the B-C model:

$$\Phi = |J; v(BC)\alpha\rangle ,$$

$$\Phi' = |J'; v'(B'C')\alpha'\rangle .$$

This matrix element of the one-body operator

$$\sum_i i^L g_L(r_i, r) Y_L^{\mathfrak{M}}(\theta_i, \phi_i)$$

may be evaluated in terms of fractional-parentage expansions of the B-C wave functions. $A_{jj'}$ then takes the form [see Eqs. (11) and (12)]:

$$A_{jj'} = (2j+1)(2j'+1)\langle j'j -\tfrac{1}{2}\tfrac{1}{2}|L0\rangle[(2J+1)(2J'+1)(2B+1)(2B'+1)(2C+1)(2C'+1)]^{1/2}$$

$$\times\sum_{k_b=0}^{1}\sum_{k_c=|L-k_b|}^{L+k_b}\begin{Bmatrix} B & k_b & B' \\ C & k_c & C' \\ J & L & J' \end{Bmatrix}\begin{Bmatrix} \frac{1}{2} & 3 & j \\ \frac{1}{2} & 3 & j' \\ k_b & k_c & L \end{Bmatrix}(2k_b+1)(2k_c+1)$$

$$\times n \sum_{v''C''B''\alpha''}\langle n-1v''(B''C'')\alpha'';\tfrac{1}{2}3|\}nv(BC)\alpha\rangle\langle n-1v''(B''C'')\alpha''|\}nv'(B'C')\alpha'\rangle$$

$$\times(-1)^{B'+B''+k_b+k_c-C'+C''+j}\begin{Bmatrix} B & \frac{1}{2} & B'' \\ \frac{1}{2} & B' & k_b \end{Bmatrix}\begin{Bmatrix} C & 3 & C'' \\ 3 & C' & k_c \end{Bmatrix}.$$

The fractional-parentage coefficients needed have been tabulated by Racah.[10] The angular momentum subroutines contained in DWUCK[24] were most useful in evaluating the $A_{jj'}$ functions. It should also be noted here that $g_L(r_i, r)$, required for evaluation of the radial form factor $I_L^{jj'}(r)$ [see Eq. (13)], may be expressed as[25]

$$g_L(r_i, r) = e^{-\gamma(r_i^2+r^2)}\left(\frac{\pi}{2ix}\right)^{1/2} i^{-L} J_{L+1/2}(ix),$$

where

$$x = 2\gamma r_i r.$$

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