Even-Odd Effects in Precompound Emission*

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The precompound theory of particle emissions predicts for the most energetic particles an especially simple dependence of the emission probability upon residual excitation energy. This paper examines this dependence, and compares it with experiment, to obtain the following results: (a) a distinction between processes involving even and odd nuclei; (b) values of single-particle densities, g, of plausible magnitude, whose uncertainties (due to experimental error in the data) are, however, too large to allow a firm conclusion about their energy dependence; and (c) the expectation that a reduction in experimental error by a factor of 2 or 3 might allow a test of a more detailed description of the residual nucleus's structure.

I. INTRODUCTION

Since the suggestion of precompound reactions was first formalized several years ago,[1-4] these processes have received a significant amount of theoretical and experimental attention. Some technical deficiencies of the first, somewhat oversimplified, theoretical description[1] have been overcome,[5-8] and connection has been made[9] between such simplified, but not inadequate, descriptions and more elaborate treatments of the full time-dependent problem.[10] Both approaches, in turn, describe in the phase space of quantum states essentially the same physics as earlier Monte Carlo analyses had described in terms of the classical position-momentum phase space.[11,12] Recently, new experiments designed specifically to study certain aspects of the precompound process have been launched and reported.[13] It seems fair to say that the subject is currently an area of lively interest still characterized by obvious questions more numerous than well-known answers.

In this paper[14] we point out a few of the implications which follow from the very simplest nuclear model [characterized by only one dimensional parameter, the single-particle level density, g, for the very simplest reaction, (p, n)], and we attempt to test this model against experimental data and to assess the quality of the answers. In this model the precompound theory predicts for reaction cross sections specific dependences upon the only dimensionless variables available in the model, namely, gE^* (the dimensionless excitation energy) and gU (the dimensionless excitation energy in the residual nucleus). The possible tests therefore are limited to three: (a) the dependence of cross sections upon gU; (b) their dependence upon gE^*; and (c) the magnitude implied by the data for the constant level density, g.

II. TEST OF SOME ASPECTS OF THE PROCESS

In this note the precompound description is tested against the data[15-17] from (p, n) reactions on Sn^{116} and Sn^{117}. The test is posed for the very simplest nuclear model, in which the excitation states of the nucleus are assumed to be well described in terms of particles and holes in a single-particle spectrum with constant level density g.

In such a model one obtains the prediction (apart from a slowly varying factor discussed below) that the cross section for small gU depends linearly upon gU for an even-even target, or for an odd-A target whose extra particle is assumed to remain inert during the particle-hole excitation process. If the extra particle is assumed to be scattered by the rearrangement processes accompanying the first particle-hole excitation, then a quadratic dependence upon gU is predicted. For this reason we discuss first the dependence upon residual excitation energy gU. The theoretical coefficient of gU or $(gU)^2$ in each case depends only upon the value of gE^*.

Thus, the consistency of the assumed model can be tested by inquiring whether the data indicate the expected dependence upon U, and if so, by determining g from the reaction data at various excitation energies E^* in a given nucleus, and checking whether g does in fact remain constant and whether its magnitude is reasonably consistent with single-particle level densities derived from other nuclear data. One would expect that a very stringent quantitative test of this kind (i.e., a test against very precise data) would provide evidence of the inadequacies of the present oversimplified model description of the low-energy excitations of the residual nucleus, and might suggest changes in the model to improve agreement with experiment,

a rewarding outcome, since just such evidence concerning nuclear structure is the ultimate goal of nuclear-reaction studies.

The discussion below will show that although the data utilized here are too imprecise to support such a detailed test, the data are adequate to resolve the question concerning the role of an odd particle in the target nucleus during the early particle-hole excitation processes.

III. DETAILS OF THE THEORETICAL ANALYSIS

A. Energy Distribution of Emitted Neutrons

We follow the philosophy of the original discussion of precompound reactions,[1] and assume that transitions occur at each of many stages of the reaction via residual interactions which can scatter particles (or holes), or create or annihilate one particle-hole pair. We also assume that the fractional neutron emission rate at a given stage, when π particles and λ holes are excited, is given by the ratio of the transition probability into all states with $\pi-1$ particles, λ holes (sharing the residual energy U), and one neutron particle with kinetic energy at infinity in the range dE_0 about E_0 and orbital angular momentum l to the transition probability into all possible π-particle-plus-λ-hole states. The corresponding level densities are $\rho_{\pi\lambda}(E^*)$, $\rho_{\pi,\lambda-1}(U)$, and $\rho_c^l(E_0)$. Finally, we retain the assumption of small fractional depletion at each stage, and utilize the second-order perturbation theory for the transistion rate. Then the fractional emission rate for neutrons of energy E_0 is

$$W(E_0)dE_0 = \sum_{\pi,\lambda}\left[\sum_l^{\infty}\frac{\rho_{\pi-1,\lambda}(U)\rho_c^l(E_0)\left|\langle b, c(l)\,|\,V\,|\,b, b\rangle\right|_{\mathrm{Av}}^2}{\rho_{\pi,\lambda}(E^*)\left|\langle b, b\,|\,V\,|\,b, b\rangle\right|_{\mathrm{Av}}^2}\right], \tag{1}$$

where $\left|\langle b, b\,|\,V\,|\,b, b\rangle\right|_{\mathrm{Av}}^2$ is an average squared matrix element between initial and final states both bound, and $\left|\langle b, c(l)\,|\,V\,|\,b, b\rangle\right|_{\mathrm{Av}}^2$ is the corresponding quantity for a final state with one continuum particle with orbital angular momentum l. The various level densities ρ in Eq. (1) are discussed in Sec. III D below.

B. Ratio of Average Squared Matrix Elements

We estimate the ratio

$$R_l^2 = \frac{\left|\langle b, c(l)\,|\,V\,|\,b, b\rangle\right|_{\mathrm{Av}}^2}{\left|\langle b, b\,|\,V\,|\,b, b\rangle\right|_{\mathrm{Av}}^2}, \tag{2}$$

by calculating the ratio of the average squared amplitudes inside the nucleus of the continuum wave function, $c(l)$, in the numerator and the bound state which replaces it in the denominator. We use a square well of radius $a = 1.2A^{1/3}\times10^{-13}$ cm to calculate R_l^2 by averaging over a broad range of internal wave numbers,

$$R_l^2 = a(k_ca)^2\left[j_l(k_ca)\cos\delta_l - \eta_l(k_ca)\sin\delta_l\right]_{\mathrm{Av}}^2, \tag{3}$$

for a continuum wave function for a neutron of kinetic energy $E_0 = \hbar^2k_c^2/2M$. Equation (3) and the phase shift δ_l in (3) follow from the two requirements that the value and first derivative of the external wave function match with the internal wave function at the edge of the potential, $r = a$. It also reflects the conventional choice that the continuum states have unit amplitude at infinity. Its product with $\rho_c^l(E_0)$ is, however, independent of this choice. Except for the very lowest energies, we assume that for $k_ca \geq l \geq 1$ the phase shifts δ_l assume a wide range of values, so that the right-hand side of (3) is given by the average over all δ_l from 0 to 2π:

$$\langle R_l^2\rangle_{\mathrm{Av}} = \tfrac{1}{2}a(k_ca)^2\left\{\left[j_l(k_ca)\right]^2 + \left[\eta_l(k_ca)\right]^2\right\}. \tag{4a}$$

Whence,

$$\langle R_l^2\rangle_{\mathrm{Av}} = \tfrac{1}{2}a, \quad \text{when } l \ll k_ca. \tag{4b}$$

For $l \gg k_ca$, the phase shift is small and (3) cannot be averaged over δ_l in this way. Then the low-energy approximation to the phase shift and the small-argument expansions of j_l and n_l lead to the estimate

$$\langle R_l^2\rangle_{\mathrm{Av}} = (k_ca)^{2l+2}\left[(2l+1)!!\right]^{-2}, \tag{4c}$$

which shows that $\langle R_l^2\rangle$ decreases rapidly for $l > k_ca$. The physical content of (4c) is the familiar statement that the angular momenta involved in a reaction between particles of momentum $\hbar k_c$ interacting with a range a are limited to values $\hbar l \lesssim \hbar k_ca$. We reproduce this feature of (3) by cutting off the l sum in (1) at the largest integer less than k_ca.

For $k_ca < 1$, the above discussion implies that the $l = 0$ term is the most important. For this term, however, (4c) implies $R_0^2 \propto (k_c)^2 \propto E_0$. Thus, we must take care not to apply (4b) at the very lowest energies, but instead to note that

$$\langle R_0^2\rangle \propto E_0. \tag{4d}$$

C. Evaluation and Smooth Averaging of l Sum

We can use the results (4b) or (4a), and the implication of (4c) and (4d) to evaluate the l sum in (1),

$$SdE_0 = \sum_l^{\infty} R_l^2\rho_c^l(E_0)dE_0, \tag{5a}$$

approximately, as follows:

$$S dE_0 \approx \sum_{l=0}^{\kappa} \langle R_l^2 \rangle_{\mathrm{Av}} 2 \frac{2l+1}{\pi} \left(\frac{2M}{\hbar^2 E_0}\right)^{1/2} dE_0, \qquad (5b)$$

where κ is the largest integer less than $k_c a$, and the explicit form of the continuum level density, $\rho_c^l(E_0)$, has been inserted.

The result is a function which is discontinuous at integral values of κ. In accordance with our philosophy of seeking a smooth, averaged description of the physical processes under discussion, we replace this discontinuous function S with a smooth function $\tilde{S}$, chosen to have the energy dependence which S would have were the integral quantity κ replaced by the continuous quantity $k_c a$, but adjusted in magnitude to give the best fit to the function (5b) for values of $k_c a$ in the range 2 to 5. This process results in

$$\tilde{S} = \frac{1.1}{\pi} \left(\frac{2Ma^2}{\hbar^2}\right)^{3/2} \left[E_0^{1/2} + 2\left(\frac{\hbar^2}{2Ma^2}\right)^{1/2} + T_3\right]. \qquad (6)$$

In this expression the coefficient has a numerical value of 1.1, instead of the value 1.0, as a result of the adjustment to give the best-fit magnitude. [The difference essentially accounts for a detail: (4b) is an underestimate of (4a) for the terms with $k_c a \approx l$.] Figure 1 shows a comparison of S and $\tilde{S}$.

The term T_3 in (6) is proportional to $(E_0)^{-1/2}$, and would describe a divergence for small E_0 if the behavior (4d) for S waves were overlooked. However, at very low energies where $k_c a < 1$, the expression (4d) for R_l^2 should of course replace (4b). Then no $(E_0)^{-1/2}$ divergence occurs. Moreover, for moderate values of E_0 such as those which actually occur in this paper, T_3 is small. We therefore assume $T_3 = 0$ in $\tilde{S}$.

The final result is therefore

$$\frac{W(E_0)}{\tilde{S}(E_0)} = \sum_{\pi,\lambda} \frac{\rho_{\pi-1,\lambda}(U)}{\rho_{\pi,\lambda}(E^*)}, \qquad (7)$$

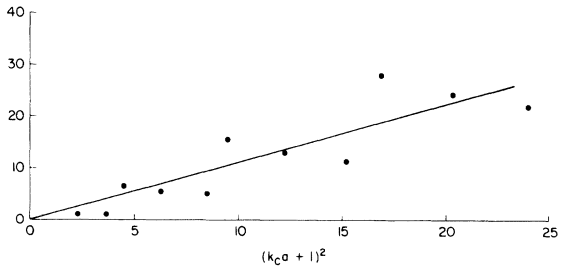

FIG. 1. The figure compares the dimensionless quantities $(h^2E_0/2Ma^2)^{1/2}S$ and $(h^2E_0/2Ma^2)^{1/2}\tilde{S}$, where S (dots) and $\tilde{S}$ (curve) are given by (5b) and (6), respectively. We conclude from this figure that the expression (6) for $\tilde{S}$ represents a reasonably good smooth approximation to S.

in which the distribution $W(E_0)$ has been divided by the slowly varying factor $\tilde{S}$ in order to facilitate the comparisons we shall undertake later.

D. Partial Level Densities

For nucleons of a specified type occupying single-particle states of a spectrum described by a constant level density g, the density of π-particle, λ-hole states at excitation energy E^* is given by[18-21]

$$\rho_{\pi,\lambda}(E^*) = g(gE^*)^{\pi+\lambda-1}[\pi!\lambda!(\pi+\lambda-1)!]^{-1}. \qquad (8)$$

This result can be obtained by induction from the corresponding result for $(\pi-1, \lambda)$ by means of a convolution of the densities $\rho_{10}(x) = g$ and $\rho_{\pi-1,\lambda}(E^*-x)$ and multiplication by the factor $(\pi-1)!/(\pi)!$ which accounts for the fact that there are now π identical particles instead of $\pi-1$. The result can also be generalized in the same manner to describe the density, $\rho_{\pi\lambda;\nu\mu}(E^*)$, of states of π neutrons, λ neutron holes, ν protons, and μ proton holes. The result is

$$\rho_{\pi\lambda;\nu\mu}(E^*) = g(gE^*)^{\pi+\lambda+\nu+\mu-1} \times [\pi!\lambda!\mu!\nu!(\pi+\nu+\mu+\lambda-1)!]^{-1}. \qquad (9)$$

For the case at hand we need only the formulas with $\nu = 1$, $\mu = 0$, because of our assumption that the protons in the $Z = 50$ closed shell of Sn are not available for excitation.

E. Predicted Dependence on Residual Excitation U

By means of the level densities (9), the right-hand side of (7) can be reduced to an explicit function of U, involving E^* and g as constants: one finds

$$\frac{\rho_{01;10}(U)}{\rho_{11;10}(E^*)} + \frac{\rho_{12;10}(U)}{\rho_{22;10}(E^*)} + \cdots = \frac{2}{(gE^*)}\left[\frac{U}{E^*} + 4\left(\frac{U}{E^*}\right)^3 + \cdots\right] \qquad (10)$$

for (p, n) reaction on the even-even target Sn^{116}; and

$$\frac{\rho_{11;10}(U)}{\rho_{21;10}(E^*)} + \frac{\rho_{22;10}(U)}{\rho_{32;10}(E^*)} + \cdots = \frac{6}{(gE^*)}\left[\frac{U}{E^{*2}} + \frac{5}{2}\left(\frac{U}{E^*}\right)^4 + \cdots\right] \qquad (11)$$

for the odd-N target Sn^{117} if the odd neutron is assumed to be scattered by the particle-hole excitation process. Omitted terms involve higher powers of U/E^*, and are therefore negligible for

small U. Indeed, even the terms in $(U/E^*)^3$ and $(U/E^*)^4$ included above are actually small corrections for the experimental circumstances considered herein.

IV. COMPARISON OF EXPERIMENT AND THEORY

A. Qualitative Features

In Figs. 2 and 3 the data[22] from Sn^{116} and Sn^{117} (p, n) reactions are plotted as U/E^* and $(U/E^*)^2$, respectively. It is obvious that the probability of the (p, n) reaction from the odd-A target exhibits a dependence better described by the (essentially) quadratic formula (11), while the (nearly) linear dependence (10) is exhibited by the data from the even-even target. One thus concludes that the loosely bound odd nucleon participates in the initial particle-hole excitation processes to a significant degree.

In addition, for the case of Sn^{116} the slopes of the (nearly) straight-line best-fitting curves generally decrease with increasing E^*, as would be implied by the proportionality of the theoretical coefficients to $(gE^*)^{-1}$, and the assumed constancy of g. The corresponding expectation is, however, not well realized in the odd-target (Sn^{117}) data of Fig. 2. We test this feature further by considering these coefficients quantitatively (see below).

B. Quantitative Comparison of Model and Data

By determining the values of the coefficients of (10) and (11) which provide the best fit to the data, we have extracted a value of g for each experimental situation considered. The results are tabulated in Table I, which also indicates the estimated standard derivation, σ_g^2, in g, given by the expression:

$$\sigma_g^2 = \sum_i^N [(\partial g/\partial X_i)\sigma_i]^2 , \tag{12}$$

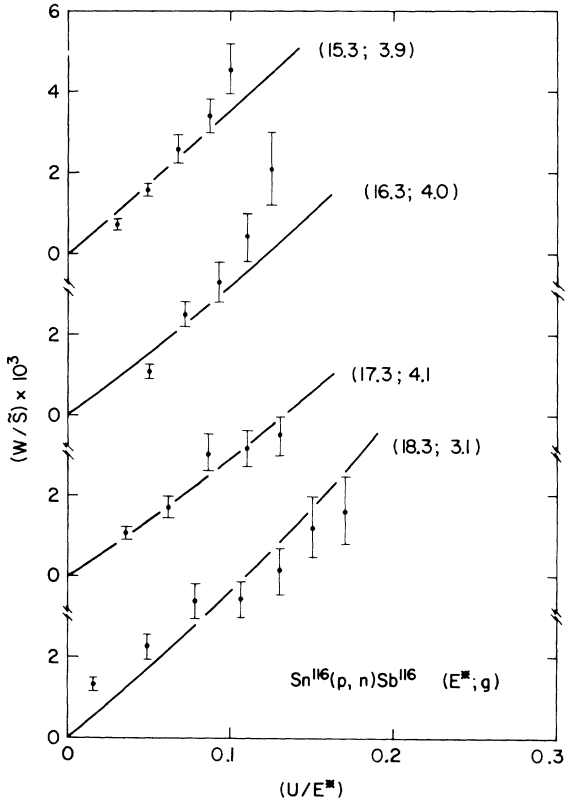

FIG. 2. Comparison of experimental and theoretical results [(7) and (10)]. The excitation energy E^* of the Sb^{117} nucleus is indicated for each data set, together with the best-fitting value of g which characterizes the theoretical curve. Clearly, no single g value could be chosen for the model at hand to provide from it a good description of these data.

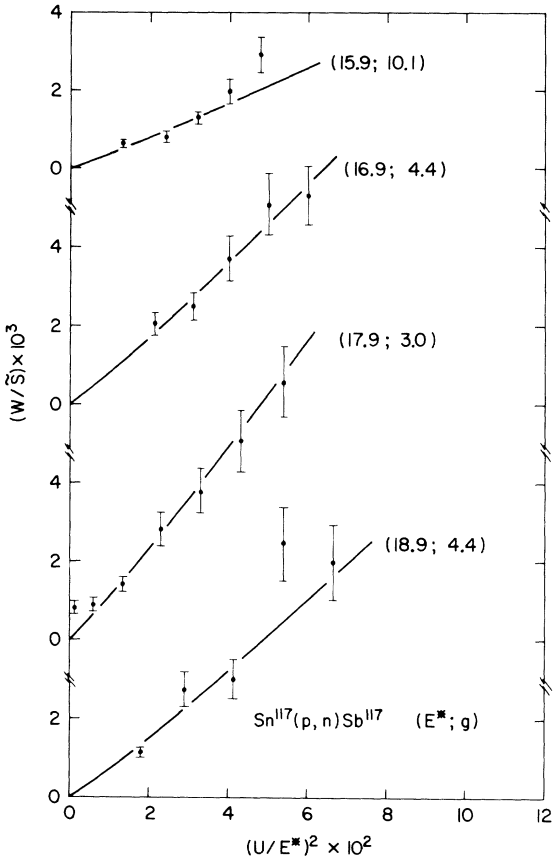

FIG. 3. Same as Fig. 2, except that the data involve the decay of Sb^{118}, and (11) is the appropriate theoretical expression. The best-fit values of g obtained here vary more among themselves than the corresponding results in Fig. 2. However, in contrast to Fig. 2, where the results were linear in U/E^*, these results give nearly straight lines when plotted against $(U/E^*)^2$.

TABLE I. This table exhibits for the two nuclei in question and for the various experimental values of the proton energy and excitation energy (columns 1 and 2) the values of g which characterize the best theoretical fit of the form given by (10) (for even-even targets) and (11) (for odd-even targets). Also listed is σ_g^2, the standard deviation of g estimated by (12). Finally, the quality of the best fit is indicated by the probability, $p(\chi^2)$, that χ^2 should exceed its value for the best fit. These quantities indicate in most cases that the assumed model is rather unlikely to be the appropriate description for the observed data. For comparison, we note that values of g in the range $3.5 < g < 4.5$ are indicated by level-density systematics [see A. Bohr and B. R. Mottleson, *Nuclear Structure* (Benjamin, New York, 1969), Vol. I, pp. 187, 188].

		Single-particle level densities from (p, n)		
E_p (MeV)	E^* (MeV)	$g \pm \sigma_g^2$ (MeV^{-1})	$\chi^2_N/(N-1)$	$p(\chi^2_{N-1})$
		Sn^{116}		
11	15.3	3.9 ± 0.3	8.4/4	0.08
12	16.3	4.0 ± 0.4	14.2/4	0.01
13	17.3	4.1 ± 0.3	1.12/4	0.86
14	18.3	3.1 ± 0.2	23.4/6	<0.01
		Sn^{117}		
11	15.9	10 ± 1.	7.2/4	0.13
12	16.9	4.4 ± 0.2	1.0/4	0.90
13	17.9	3.0 ± 0.2	34.2/6	<0.01
14	18.9	4.4 ± 0.3	8.4/4	0.08

where X_i are the N experimental data and σ_i the corresponding experimental errors. In addition, Table I gives values of χ^2 for each case fitted and the probability, p, that χ^2_{N-1} should exceed the value listed.

The results of Table I suggest that the adjustment of the parameter g in the constant-single-particle-level-density model is not able to produce highly probable agreement with that model. In view of well-established deviations from that model in nuclei at low excitation (e.g., pairing and collective levels), this result is not surprising. On the other hand Figs. 2 and 3, which display the same information as Table I in more detail, do not suggest any systematic alterations of the model which would improve the probability that the model describes the data.

We might reasonably conjecture that a reduction of the experimental error by a factor of 2 or 3 ought to increase the χ^2 values sufficiently to allow the decisive rejection of the present oversimplified model; hopefully, also, such data would exhibit systematic patterns which would suggest how the simple model ought to be altered.

V. CONCLUSIONS

The above results seem to substantiate the following conclusions:

(1) Within the framework of the constant-level-density model, odd target nucleons should be assumed to participate, at least partially, in the particle-hole excitation process.

(2) A constant-single-particle-level-density model provides an adequate qualitative description of data discussed here, but the dependence upon gE^* is not well predicted by a single constant g value.

(3) The extraction from precompound reaction data of detailed inferences concerning low-energy nuclear structure is not possible with the data used here. However, a reduction of experimental error by a factor of 2 or 3 ought to permit such a test of more detailed models.

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[21]Note that this expression differs from that for $n = \pi + \lambda$ identical Fermi particles which results if particles and holes are considered identical particles, as in Griffin, Ref. 1.
[22]The data used here are those of Wood, Borchers, and Holbrow, Ref. 16. The authors wish to gratefully acknowledge Dr. Borchers's kindness in supplying his data to us.