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Parity-Violating and Normal Multipole Mixing Ratios of the 57-keV Gamma Transition of ^{180}Hf†

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The parity-violating forward-backward asymmetry of the 57-keV γ radiation from low-temperature polarized ^{180m}Hf has been measured to be $-(19.4\pm9.2)\times10^{-4}$, indicating the lack of any large parity-violating effect for this very strongly hindered $E1$ transition, in agreement with our calculations. The $M2/E1$ and $E3/E1$ multipole mixing ratios of the 57-keV transition have been determined to be $\delta_1(M2/E1)=-(0.009^{+0.065}_{-0.085})$ and $\delta_2(E3/E1)=+0.013\pm0.020$.

I. INTRODUCTION

In a previous report[1] of an investigation of the parity-nonconserving asymmetry of the 501-keV γ transition emitted in the decay of ^{180m}Hf polarized at low temperatures, an effect of $-(1.66\pm0.18)\%$ was observed. This effect is produced by the small (order 10^{-7}) admixture in the nuclear Hamiltonian of the parity-violating weak interaction. Generally, this produces only small laboratory effects,[2,3] but in the case of the 501-keV transition of ^{180}Hf, the magnitude of the observed effect is enhanced by two nuclear-structure factors: (1) The "regular" (i.e., parity-conserving) part of the transition is sixfold K-forbidden and thus strongly hindered [hindrance relative to Weisskopf estimates $H_W(M2)=1\times10^{14}$, $H_W(E3)=2\times10^{9}$] allowing small irregular admixtures to compete favorably; (2) the proximity of the 8^- and 8^+ levels (Fig. 1) allows substantial mixing by the weak interaction. The effect measured previously is in good agreement with similarly large effects observed[4,5] in the circular polarization of the 501-keV transition emitted by unpolarized ^{180m}Hf.

Since the size of the laboratory effect produced by the weak interaction seems to depend somewhat on the retardation of the regular components of the γ transition, an even larger effect might be expected in the case of the 57-keV transition, which is sevenfold K-forbidden, and for which $H_W(E1)=3\times10^{16}$. The anomalous L-subshell conversion coefficients of the 57-keV transition were in excellent agreement with an admixture of 9.5% irregular $\tilde{M}1$ radiation into the $E1$ transition[6]; however, an alternate explanation in terms of nuclear penetration effects[6,7] was also suggested. The latter explanation was substantiated by measurement of the circular polarization[8] of the 57-keV radiation; the result $(6\pm6)\%$ was not consistent with the expected result of 56% if the 9.5% $\tilde{M}1$ admixture were correct. These results, however, do not exclude the possibility of an observable $\tilde{M}1$ admixture, and would be consistent with all irregular mixing ratios $\epsilon_1=\tilde{M}1/E1$ less than 0.06 in magnitude. In order to investigate this possibility we have observed the forward-backward asymmetry of the 57-keV γ radiation from low-temperature polarized ^{180m}Hf.

II. EXPERIMENTAL DETAILS

A. Sample Preparation

The 57-keV peak in the ^{180m}Hf γ spectrum contains substantial contributions from the $K\alpha$ x rays resulting from the highly converted low-energy transitions in ^{180}Hf and in ^{181}Ta (following the decay of ^{181}Hf). In order to reduce the dilution of the effect due to the presence of these unwanted ^{181}Hf transitions, a sample was produced from Hf enriched to 48% in ^{179}Hf.

The enriched Hf, obtained in the form of HfO_2, was mixed with Fe_2O_3 and reduced to $HfFe_{10}$, which was then melted with Fe and Zr to form the alloy $(Hf_{0.1}Zr_{0.9})Fe_2$. (This is the identical alloy to that used in the work described in Ref. 1; however, the previous alloy was made from nonenriched Hf.)

B. Experimental Procedure

The apparatus employed to cool the sample down to $T\sim22$ mK and polarize it in an external field $H\sim3$ kG has been described in detail in a number of previous publications.[1,9,10]

A ^{3}He-^{4}He dilution refrigerator was used to cool the sample, and two pairs of perpendicular Helmholtz coils provided the external polarizing magnetic fields. The external field direction was controlled by a minicomputer, which also analyzed the forward-backward asymmetries from the counting rates determined from data digitized by two 1024-channel analog-to-digital converters fed by two 40-cm^3 coaxial Ge(Li) detectors.

C. Data Analysis

The angular distribution of γ radiation emitted by polarized nuclei is described by

$$W(\theta)=\sum_k Q_k B_k U_k A_k P_k(\cos\theta), \tag{1}$$

where the Q_k correct for the angular resolution of the detectors, the orientation parameters B_k describe the orientation of the initial state, the deorientation coefficients U_k correct for the effects of unobserved intermediate decays, and the angular-distribution coefficients A_k describe the properties of the observed γ rays. The various coefficients are discussed further in Refs. 1 and 10. The P_k are the ordinary Legendre polynomials, and the angle θ is defined relative to the polarization direction. Equation (1) is normalized such that $Q_0=B_0=U_0=A_0=P_0=1$. Since there are no intermediate transitions in the case of the 57- and 501-keV γ rays, $U_k=1$.

The forward-backward asymmetry $\mathcal{A}$ is determined by the odd-k terms, and is given by

$$\mathcal{A}=\frac{W(0^\circ)-W(180^\circ)}{\overline{W}} \simeq \frac{2Q_1B_1A_1+2Q_3B_3A_3}{1+Q_2B_2A_2+Q_4B_4A_4}. \tag{2}$$

The direction of the external field is defined to be $\theta=0^\circ$.

The odd-k angular-distribution coefficients are given by

$$A_k=\{2\epsilon_1[F_k(1188)+\delta_1F_k(1288)+\delta_2F_k(1388)] +2\epsilon_2[F_k(1288)+\delta_1F_k(2288)+\delta_2F_k(2388)]\} \times[1+\delta_1^2+\delta_2^2+\epsilon_1^2+\epsilon_2^2]^{-1}, \tag{3}$$

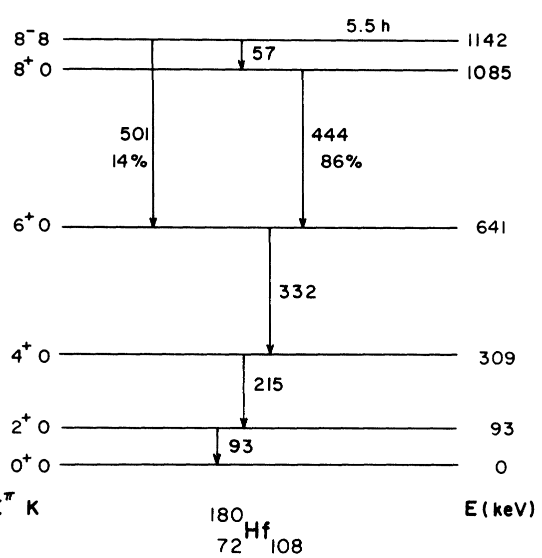

FIG. 1. Decay scheme of ^{180m}Hf.

where the mixing ratios are defined as

$$\epsilon_1=\frac{\langle I_f\|\tilde{M}1\|I_i\rangle}{\langle I_f\|E1\|I_i\rangle}, \tag{4a}$$

$$\epsilon_2=\frac{\langle I_f\|\tilde{E}2\|I_i\rangle}{\langle I_f\|E1\|I_i\rangle}, \tag{4b}$$

$$\delta_1=\frac{\langle I_f\|M2\|I_i\rangle}{\langle I_f\|E1\|I_i\rangle}, \tag{4c}$$

$$\delta_2=\frac{\langle I_f\|E3\|I_i\rangle}{\langle I_f\|E1\|I_i\rangle}. \tag{4d}$$

The F coefficients of Eq. (3) are defined and tabulated the work of Krane.[11] The multipole mixing ratios are defined using the phase convention of Krane and Steffen.[12]

The data were analyzed using the method described in Ref. 1, with the exception that five-minute counting periods were employed, and accordingly a linear (rather than logarithmic) averaging procedure was suitable.

As discussed previously, a correction was necessary to take into account the dilution of the measured effect by the Hf and Ta x rays. The presence of the ^{180}Hf x rays causes a reduction of the measured asymmetries by a factor 0.67, computed from the ^{180}Hf K conversion coefficients and branching intensities.[13] The contribution of the ^{181}Ta x rays (which increased with time relative to the short-lived ^{180}Hf activity) was determined by observing the half-life of the 57-keV peak as a function of time; this contribution generally amounted to 15% at the beginning of a measurement and 60% at the end. The correction for the presence of the Hf and Ta x rays was made by assuming the x-ray angular distribution to be isotropic, and a 10% uncertainty was introduced to take into account the possibility of small x-ray anisotropies, as well as uncertainties in the K conversion coefficients and branching intensities.

The statistical consistency of the data was evaluated by computing the normalized χ^2 value. Val-

TABLE I. Parity-violating forward-backward asymmetries of γ rays in the decay of ^{180m}Hf.

γ-ray energy (keV)	Asymmetry $\mathcal{A}$ (units of 10^{-4}) $T=22$ mK	$T=50$ mK
57	-19.4 ± 9.2	-12.5 ± 18.5
501	-140 ± 13	-68 ± 13
444	-0.6 ± 2.8 [a]	
Background (70 keV)	-2.2 ± 2.4 [a]	
Background (550 keV)	-6 ± 12 [a]	
Background (100–200 keV)	1.4 ± 0.6 [a]	

[a] Results of all cold data (22 mK as well as 50 mK).

ues close to unity were obtained within a series of approximately 50 five-minute runs, as well as between the 12 series.

III. RESULTS

A. Parity Mixing

The observed forward-backward asymmetries are summarized in Table I. The results quoted represent the average of measurements on three samples (one nonenriched), and have been corrected as previously described. The 22-mK results are derived from many independent measurements, while the 50-mK data result from only one or two measurements; the latter data were taken to investigate the possibility of cancellations between the B_1 and B_3 terms in the angular distribution.

The high- and low-energy background regions were observed in order to assess the possibility of bremsstrahlung contributing a spurious effect, and the large background region (comprising an interval of about 100 keV in the low-energy region) and 444-keV peak were employed to investigate the possibility of source motion or other effects which might give rise to spurious asymmetries. The vanishing asymmetries in all cases indicate the lack of such effects.

From an analysis of the 0-90° anisotropy of the 57-keV angular distribution described below, the regular $M2/E1$ and $E3/E1$ mixing ratios were determined. In terms of these mixing ratios, the forward-backward asymmetries may be written:

$$T=22\text{ mK: } A=(-0.69\pm 0.36)\epsilon_1+(4.2\pm 0.2)\epsilon_2, \quad (5a)$$

$$T=50\text{ mK: } A=(-0.37\pm 0.19)\epsilon_1+(2.6\pm 0.1)\epsilon_2. \quad (5b)$$

The coefficients of ϵ_1 in Eqs. (5a) and (5b) are sensitive to the regular $M2/E1$ mixing ratio, and the uncertainties in the coefficients are due primarily to the uncertainty in δ_1. We have taken $B_1=+1.2$ and $B_3=+0.35$ at 22 mK consistent with the observed hyperfine splitting[1] $\Delta=-7.9$ mK. (The signs of B_1, B_3, and Δ indicate that the nuclei are polarized in a direction opposite to the applied field.) Figure 2 illustrates the relationship between ϵ_1 and ϵ_2 determined from the measured asymmetry at 22 mK using Eq. (5a).

B. Multipole Mixing Ratios

Previous directional-correlation data[14] indicated a small or vanishing $M2$ admixture in the $E1$ 57-keV transition. This is rather surprising in view of the very large K-forbiddeness of the $E1$ multipole and its large hindrance. From our data we deduce for the anisotropy of the 57-keV angular distribution

$$\frac{W(0°)}{W(90°)}=0.637\pm 0.020.$$

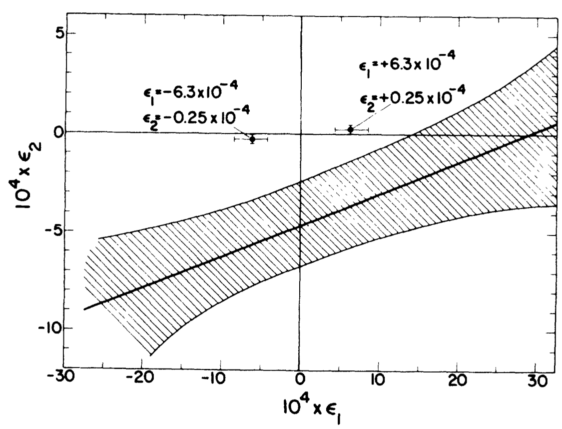

FIG. 2. Relationship between the irregular mixing ratios $\epsilon_1(\tilde{M}1/E1)$ and $\epsilon_2(\tilde{E}2/E1)$ based on the present results; the indicated uncertainty results from the approximately equal contributions of the uncertainty in the measured asymmetry $\mathcal{a}$ and the uncertainty in the deduced value of $\delta_1(M2/E1)$ used in the analysis [see Eq. (2a)]. The labeled points represent estimates computed in Sec. IV of the text, with uncertainties representative of the uncertainties in the values of the static electromagnetic moments and in the value of the parity mixing amplitude $\mathcal{G}$.

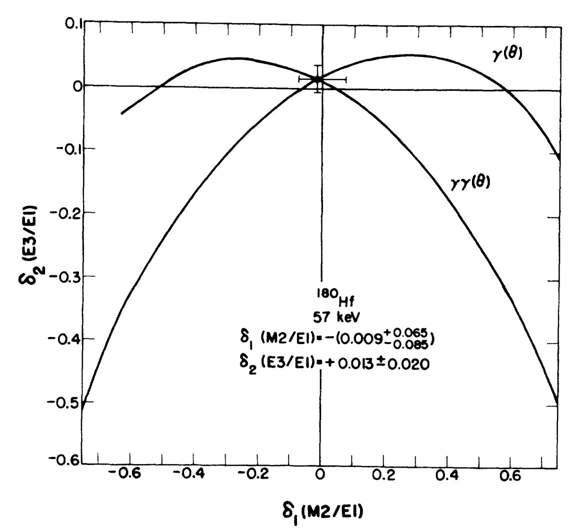

FIG. 3. Relationship between $\delta_1(M2/E1)$ and $\delta_2(E3/E1)$ of the 57-keV γ transition in ^{180}Hf determined by nuclear orientation, $\gamma(\theta)$, and directional-correlation, $\gamma\gamma(\theta)$ (Ref. 14), measurements. (The second roots derived for δ_1 and δ_2 from both measurements do not intersect and are not shown; both measurements yield δ_2 values of approximately -1.5 in the interval of δ_1 shown.)

The possible $M2$ (and $E3$) admixtures may be determined by comparing the present data with that derived from directional-correlation measurements[14]; Fig. 3 represents such a comparison. The present results, $\gamma(\theta)$, and directional-correlation results, $\gamma\gamma(\theta)$, intersect at a point at which

$$\delta_1(M2/E1) = -(0.009^{+0.065}_{-0.085}),$$

$$\delta_2(E3/E1) = +0.013 \pm 0.020\,.$$

[The formulation necessary for analyzing $M2$ and $E3$ admixtures is given in Eqs. (2) and (3) of Ref. 10; the mixing ratios are defined according to the phase of Krane and Steffen.[12]] These results indicate vanishingly small $M2$ and $E3$ admixtures in the 57-keV transition.

IV. DISCUSSION

The irregular mixing ratios ϵ_1 and ϵ_2 may be compared with estimates based on the static and dynamic electromagnetic multipole moments. The matrix elements are evaluated in terms of the electromagnetic multipole operators $\mathfrak{M}(\pi L)$ of Bohr and Mottelson[15]; the mixing ratios must be computed in terms of the operators $\vec{j}_N\vec{A}_L^{(\pi)}$ of Krane and Steffen.[12]
From the lifetime and branching ratio of the 1142-keV level, we estimate

$$|\langle 1085 \| \mathfrak{M}(E1) \| 1142\rangle| = 4.2\times 10^{-21}\ e\,\mathrm{cm}\,.$$

The $\tilde{M}1$ matrix element may be computed by assuming the 1142- and 1085-keV levels each have a small parity impurity due to the neighboring opposite-parity state, and thus

$$|1142\rangle = |8^-\rangle + \mathcal{G}\,|8^+\rangle\,, \tag{6a}$$

$$|1085\rangle = |8^+\rangle - \mathcal{G}^*|8^-\rangle\,. \tag{6b}$$

[From previous work[1] we find $|\mathcal{G}| = (1.9 \pm 0.3) \times 10^{-11}$.] The $\tilde{M}1$ matrix element then becomes

$$\langle 1142 \| \mathfrak{M}(\tilde{M}1) \| 1085\rangle = -\mathcal{G}\langle 1142 \| \mathfrak{M}(\tilde{M}1) \| 1142\rangle + \mathcal{G}\langle 1085 \| \mathfrak{M}(\tilde{M}1) \| 1085\rangle \tag{7}$$

$$= \mathcal{G}\left(\frac{3}{4\pi}\right)^{1/2} \frac{\sqrt{17}}{\langle 8810|88\rangle}[\mu(1085) - \mu(1142)]\,. \tag{8}$$

The magnetic moment of the 1085-keV level may be estimated by assuming $\mu(I) = Ig_R$, with $g_R = +0.35$[16]; $\mu(1142)$ has been determined to be $8.6\mu_N$,[17] and thus

$$\langle 1142 \| \mathfrak{M}(\tilde{M}1) \| 1085\rangle = -12.4\,\mathcal{G}\mu_N\,. \tag{9}$$

A similar calculation of the $\tilde{E}2$ matrix element involves estimating the quadrupole moments of the 1085- and 1142-keV levels. The former may be estimated from the $E2$ transition probabilities within a rotational band, and the latter by considering the 1142-keV level to be a two-quasiproton state consisting of the $\frac{7}{2}^+[404]$ and $\frac{9}{2}^-[514]$ Nilsson states.[17] The quadrupole moment of the ^{181}Ta $\frac{7}{2}^+[404]$ ground state has been determined to be 3.9 eb.[18] The quadrupole moment of the 6-keV $\frac{9}{2}^-[514]$ ^{181}Ta level is unknown; we set an upper limit on the $\tilde{E}2$ matrix element by assuming it to be as large as the ground-state moment and to combine with it such as to give the maximum possible quadrupole moment of the 1142-keV ^{180}Hf state. With these assumptions, we estimate

$$\langle 1142 \| \mathfrak{M}(E2) \| 1085\rangle = \mathcal{G}\left(\frac{5}{16\pi}\right)^{1/2} \frac{\sqrt{17}}{\langle 8820|88\rangle}[Q(1085) - Q(1142)] \tag{10}$$

$$\approx -10.8\ \mathcal{G}\ e\,\mathrm{b}\,. \tag{11}$$

We thus compute from the above matrix elements using natural units ($\hbar = m = c = 1$):

$$\epsilon_1 = \pm 6.3\times 10^{-4} \tag{12}$$

and

$$\epsilon_2 = \pm 0.25\times 10^{-4}\,. \tag{13}$$

The signs of the mixing ratios ϵ_1 and ϵ_2 are indeterminate due to the uncertainty in the sign of the $E1$ matrix element; however, Eqs. (9) and (11) indicate that ϵ_1 and ϵ_2 must have identical signs.

The estimates computed for ϵ_1 and ϵ_2 are indicated as points in Fig. 2, and do not lie far outside the experimental uncertainty of the present results. We thus conclude that the assumption of mixing of the $K = 0$ 8^+ state into the $K = 8$ 8^- state (assumed in the above calculation as well as in the extraction of $\mathcal{G}$ in Ref. 1) provides a reasonable first approximation in the computation of parity-mixing effects in ^{180}Hf.

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Qualitative Studies of the Angular Distribution for the (p, n) Charge-Exchange Reaction*

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For many nuclei the isobaric analog of the target ground state excited in the $X(p,n)Y$ charge-exchange reaction decays by the emission of a single proton ($\tilde{p}$) group. In this paper a relation is established for the angular dependence of the centroid energy, $\langle E_{\tilde{p}}(\theta)\rangle$, for this $\tilde{p}$ group. The measurement of $\langle E_{\tilde{p}}(\theta)\rangle$ determines the average momentum along the beam direction, $\langle P_{rz}\rangle$, given the recoiling nucleus Y in the (p,n) charge-exchange reaction. The value for $\langle P_{rz}\rangle$ determines the cross-section weighted average value of $\cos\theta_n$ for the (p,n) angular distribution $d\sigma/d\Omega\ (\theta_n)$; and thus provides a qualitative characterization of the (p,n) angular distribution. $\langle E_{\tilde{p}}(\theta)\rangle$ was measured for the ^{116}Sn$(p,n\tilde{p})$ process at 23.15 MeV; and a value for $\langle P_{rz}\rangle$ of 140 MeV/c was determined. The results of this experiment are compared with recent angular-distribution measurements of the (p,n) charge-exchange reaction on the tin isotopes at 23 MeV.

The (p, n) reaction preferentially excites the isobaric analog (IAS_{gs}) of the target ground state.[1,2] For nuclei with $A \gtrsim 60$ the IAS_{gs} often decays by proton ($\tilde{p}$) emission, giving the two-step process ${}^{A}_{Z}X_N(p,n){}^{A}_{Z+1}Y_{N-1}(\text{IAS}_{\text{gs}}) \rightarrow \tilde{p} + {}^{A-1}_{Z}X_{N-1}$. The $\tilde{p}$ decay of the IAS_{gs} was first observed by Yavin *et al.*[3] on the target nucleus ^{91}Zr, and subsequent experimental work has established $\tilde{p}$ groups from the $(p, n\tilde{p})$ process for target nuclei ranging from ^{67}Zn to ^{209}Bi.[4-6] For target nuclei with $A = 100 \pm 30$ it is usually true that the $\tilde{p}$ decay proceeds to a single level in the residual nucleus.

In this paper we would like to point out that the shift with laboratory angle of the centroid energy for the $\tilde{p}$ group, $\langle E_{\tilde{p}}(\theta)\rangle$, can give qualitative information concerning the angular distribution for the charge-exchange process, since:

$$\langle E_{\tilde{p}}(\theta)\rangle = E'_{\tilde{p}}\left[1 + \frac{2\langle v_{rz}\rangle}{v'_{\tilde{p}}}\cos\theta + \mathcal{O}(v_r^2/v'^2_{\tilde{p}}) + \cdots\right], \tag{1}$$

where $E'_{\tilde{p}}$ and $v'_{\tilde{p}}$ are the respective energy and velocity of the $\tilde{p}$ group in the rest frame of the IAS_{gs} and $\langle v_{rz}\rangle$ is the average laboratory velocity along the beam direction for the recoiling IAS_{gs} from the (p, n) charge-exchange reaction. (In this paper unprimed quantities are measured in the laboratory frame while primed quantities are measured in the rest frame of the IAS_{gs}.) Since $E'_{\tilde{p}}$ and $v'_{\tilde{p}}$ are known, the measurement of $\langle E_{\tilde{p}}(\theta)\rangle$ determines the average momentum, $\langle P_{rz}\rangle = m_r\langle v_{rz}\rangle$, of the recoiling IAS_{gs} along the beam direction. Qualitative information about the angular distribution for the (p, n) charge-exchange reaction may be obtained from the value of $\langle P_{rz}\rangle$, since it is determined from a cross-section weighted average over the (p, n) angular distribution. The value of $\langle P_{rz}\rangle$ may be used to determine the average value, $\langle\cos\theta_n\rangle$, of $\cos\theta_n$ for the (p, n) angular distribution, $d\sigma/d\Omega(\theta_n)$. We will first establish Eq. (1) and then present and discuss experimental data