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Study of Core Coupling in ^{87}Rb Through Isobaric-Analog Resonances. I*

C. L. Hollas,† H. R. Hiddleston,‡ V. D. Mistry,§ S. Sen, and P. J. Riley
University of Texas, Austin, Texas 78712
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Isobaric-analog resonances have been observed in proton elastic scattering from ^{86}Kr at proton energies from 4.76 to 10.10 MeV. The observed resonances in ^{87}Rb are identified as analogs of low-lying states of ^{87}Kr. The data have been analyzed using a shell-model description of isobaric-analog resonances. Orbital-angular-momentum-transfer values, resonance energies, total and partial widths, and spectroscopic factors for 14 of the observed resonances are presented. The results are shown to be in good over-all agreement with (d,p) studies on the target nucleus.

INTRODUCTION

Wave functions for states of nuclear systems composed of an even number of protons plus an additional unpaired neutron outside a closed neutron shell are of interest for experimental study due to their basic simplicity. Such states have been investigated in ^{87}Kr by Sass, Rosner, and Schneid,[1] and by Haravu, Hollas, Riley, and Coker[2] via the (d,p) stripping reaction on ^{86}Kr, which with the filling of the $g_{9/2}$ neutron orbital, has a closed $N=50$ neutron shell. Proton elastic scattering measurements at isobaric-analog resonances (IAR's) yield information similar to that obtained from (d,p) studies of the target nucleus, where the overlap between the final states and the states composed of neutron coupled to the ground state of the target is measured. In addition, by studying the proton decay of the IAR to excited states of the target, one can determine the extent that the compound nuclear wave function consists of single-particle states coupled to excited states of the target. In this and the following paper we report on the elastic and inelastic scattering of protons from ^{86}Kr. We have previously reported on preliminary measurements[3,4]; apart from these, and from the work of Refs. 1 and 2, the ^{86}Kr-^{87}Kr system has received little experimental or theoretical attention, since ^{86}Kr is a gas, and has been difficult to obtain in high isotopic purity.

The parent analog states of ^{87}Kr can be expressed by

$$|\Psi\rangle_J = b_{0J}|\Phi_0\otimes\nu_J\rangle_J + \sum_{Ij}{}' b_{Ij}|\Phi_I\otimes\nu_j\rangle_J .$$

Φ_0 represents the 0^+ ground state of the target, ^{86}Kr; Φ_I is the Ith excited state with spin I; and ν_j denotes the single-particle neutron states. $(b_{0J})^2$ is the spectroscopic factor and is measured directly in (d,p) stripping reactions. The b_{Ij} are essentially the coefficients of fractional parentage for the coupling of neutron single-particle states to the excited target states. These could be measured with (d,p) stripping only if the target could be prepared in the appropriate excited state.

The isobaric analog of the parent state is obtained formally by operation on $|\Psi\rangle_J$ with the isospin lowering operator T^-, which essentially converts each excess neutron into a proton:

$$\begin{aligned}|\Psi^A\rangle_J = T^-|\Psi\rangle_J &= T^-(b_{0J}|\Phi_0\otimes\nu_J\rangle_J + \sum_{Ij}{}' b_{Ij}|\Phi_I\otimes\nu_j\rangle_J)\\ &= b_{0J}(2T_0+1)^{-1/2}[|\Phi_0\otimes\rho_J\rangle_J + (2T_0)^{1/2}|\Phi_0^A\otimes\nu_J\rangle_J]\\ &\quad + \sum_j{}' b_{Ij}(2T_0+1)^{-1/2}[|\Phi_I\otimes\rho_j\rangle_J + (2T_0)^{1/2}|\Phi_I^A\otimes\nu_j\rangle_J].\end{aligned}$$

Proton decay from such an analog state can occur in several ways. The first term denotes the decay of the IAR back into the elastic channel; $(b_{0J})^2$ is related to the elastic proton partial width.[5] In the second term, $|\Phi_0^A\rangle$ is the isobaric-analog wave function of the ground state of the target. Proton decay arising from this term is to neutron particle-hole excited states of the target. This mode of decay has been observed in the $N=126$[6] and the $N=82$ regions,[7] but has not been seen in the $N=50$ region, primarily because the ground-state IAR occurs at a rather low energy (near 5.5 MeV), only slightly above the excitation energies of the neutron particle-hole states.[8] The third term represents proton decay to excited states of the target which are coupled to single-nucleon states in the compound system. Such decay has been reported for the $N=126$ region,[9] for the $N=82$ region,[10] as well as for the $N=50$ region.[11] The b_{IJ} coefficients are related to the inelastic proton partial widths in a similar manner as b_{0J} is related to the elastic proton partial width.[12] Thus a measurement of the inelastic proton partial widths provides a determination of the coefficients for the coupling of single-particle states to the excited core states. The last term denotes decay to neutron particle-hole states built on other excited states of the target, and should occur only at high excitation energies. The proton decay of the IAR to such states has not been reported.

This paper (paper I) contains a discussion of the observation and analysis of the resonances in the ^{86}Kr(p,p) elastic channel which are isobaric analogs of the states of ^{87}Kr. The extraction of the l values, the total widths, proton partial widths, the resonance energy of the IAR's, and the determination of the coupling coefficients $(b_{0J})^2$ (i.e., the spectroscopic factors S_{pp}) are discussed. In the following paper (paper II), we discuss the inelastic scattering of protons through IAR's in ^{87}Rb.

EXPERIMENTAL PROCEDURE AND DATA

Excitation functions for the elastic and inelastic scattering of protons from ^{86}Kr were measured simultaneously at the four laboratory angles of 95, 125, 145, and 165° over the energy range from 4.76 to 10.10 MeV in increments of approximately 16 keV, with finer increments of 8 keV in the neighborhood of resonances. At most energies, 400 μC of charge were collected at an average beam current of 400 nA, supplied by the University of Texas EN tandem Van de Graaff accelerator. The target gas, enriched to an isotopic purity of 96.3% ^{86}Kr, was contained within a 3-in.-diam thin-walled gas cell.[3]

The elastic scattering excitation-function data are shown in Fig. 1. The vertical lines at the bottom of the figure indicate the energy and l value of the elastic resonances, as deduced from the analysis of the data. The vertical lines just above the 165° data are used to indicate, for comparison purposes, the excitation energies and deduced l values of states in the corresponding (d,p) work,[2] with the ground-state (d,p) transition positioned at the energy of the lowest IAR $[E_R(\text{c.m.})=5.348$

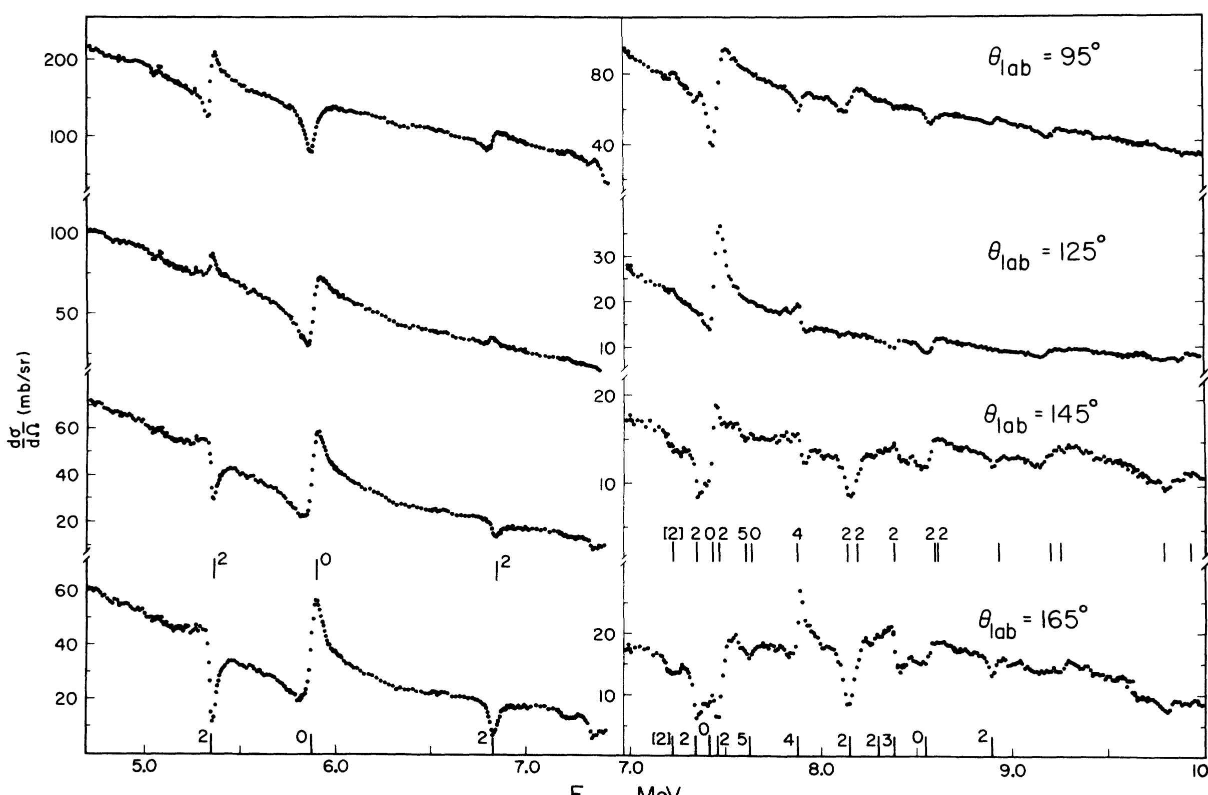

FIG. 1. The ^{86}Kr(p,p) elastic scattering excitation function for center-of-mass energies between 4.76 and 10.10 MeV. The data above 7 MeV are shown on the right on an expanded scale to better illustrate the resonant structure.

MeV].

The data above 7 MeV are drawn on an expanded scale to illustrate better the resonant structure. Below 7 MeV, three strong well-separated resonances appear. Above this energy the structure becomes more complex, as the resonances become more numerous and tend to overlap.

METHOD OF ANALYSIS

The analysis of the data is based on the energy-averaged scattering matrix of Zaidi and Dyer[13] for the description of the elastic scattering of protons from a spin-zero target:

$$S_{\lambda\lambda}=e^{2i\delta_\lambda}\left[\left|\frac{1-Y_\lambda+2i(\Delta/\Gamma^{sp}_{lj})Y_\lambda}{1+Y_\lambda+2i(\Delta/\Gamma^{sp}_{lj})Y_\lambda}\right| - e^{-i(\phi_\lambda+\chi_\lambda)}\frac{S_{pp}\Gamma^{sp}_{lj}|a_{\lambda\lambda}|^{-2}}{E-E_R+i\Gamma/2}\right]e^{-i(\phi_\lambda-\chi_\lambda)}\,.$$

E_R is the resonance energy, Γ is the total width, and δ_λ is the optical-model phase shift. S_{pp} is the elastic spectroscopic factor and is the square of the coupling coefficient, b_{0J}, for coupling to the target ground state. Γ^{sp}_{lj} is the theoretical single-particle width. All other quantities are as defined in Ref. 13.

The above scattering matrix was programmed into the computer code JULIUS[14] and used to generate a fit to the experimental data. The off-resonance background was first fitted by varying the optical-model potential parameters until the background trends were established at the four observation angles. From the background portion of the scattering matrix, the values for Y_λ and the phases χ_λ and ϕ_λ were extracted, and were retained to be used in the resonance portion of the scattering matrix.

TABLE I. Optical-model parameters used to fit the background part of the elastic scattering data.

$V=(67.5-0.8E)$ MeV	$r_{0r}=1.21$ fm	$a_r=0.63$ fm
$W_d=(5.3+0.7E)$ MeV	$r_{0i}=1.21$ fm	$a_i=0.49$ fm
$V_{so}=6.25$ MeV	$r_{so}=1.21$ fm	$a_{so}=0.63$ fm
	$r_C=1.24$ fm	

The quantity

$$\tilde{\Gamma}_{lj}=S_{pp}\Gamma_{sp}|a_{\lambda\lambda}|^{-2}$$

is the experimental proton partial width. $|a_{\lambda\lambda}|^2$ may be calculated from Y_λ and Γ_{sp}. Δ was used in the analysis as a free parameter, but its effect was small in all cases. The microscopic single-particle width Γ_{sp} was approximated by the phenomenological Lane-model expression[14]

$$\Gamma_{sp}\cong\Gamma^{sp}_{lj}=\frac{kT_0}{E}|\langle\chi^{(+)}_{pC}|V_1|\phi_{nA}\rangle|^2\,,$$

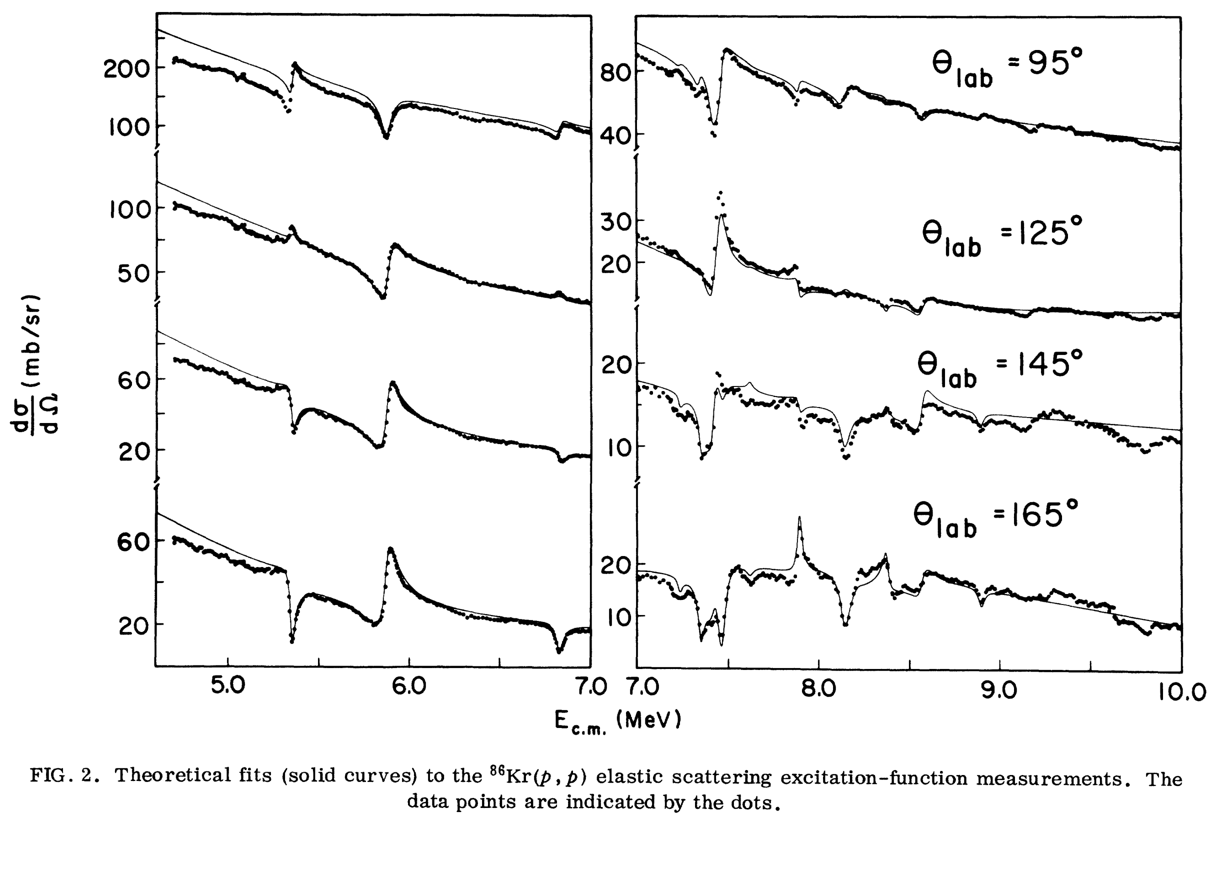

FIG. 2. Theoretical fits (solid curves) to the ^{86}Kr(p,p) elastic scattering excitation-function measurements. The data points are indicated by the dots.

where $|\chi_{pC}^{(+)}\rangle$ and $|\phi_{nA}\rangle$ are the radial portions of the solutions to the homogenous Lane equations, with the $|\chi_{pC}^{(+)}\rangle$ normalized to a δ function in the energy E. V_1 is the Lane symmetry potential, with the strength given by the relation

$$\tfrac{1}{2}(T_0 V_1) = 26(N-Z)/A\,,$$

in agreement with the value obtained from the analysis of (p,n) charge-exchange reactions.[15] The Lane-model widths, as defined above, were calculated with the computer code GPMAIN.[16]

The Γ_{lj} were then inserted into the scattering matrix, and the resonance energy, J^π, total width, and spectroscopic factors varied until a fit to the data was obtained simultaneously at the four angles of observation. A subprogram, CUANDO, was written, which displays the points generated by JULIUS, compared with the experimental data points, on the CDC 252 cathode-ray tube display console of the University of Texas CDC 6600 computer system, and allows immediate variations of all parameters by keyboard entries on the CDC 252 console.

RESULTS AND DISCUSSION

The background portion of the scattering matrix was calculated from a Woods-Saxon optical-model potential. To fit the data trends over the entire energy span, it was necessary that both the real and imaginary well depths be energy-dependent. The parameters used are listed in Table I. The fits generated with JULIUS are shown as the solid curves in Fig. 2. The first three resonances are well separated and are the IAR of the $d_{5/2}$ ground state, the 0.529-MeV $s_{1/2}$ state, and the 1.468-MeV, either $d_{5/2}$ or $d_{3/2}$ state of ^{87}Kr. The relative energies and the spectroscopic factors are in excellent agreement for these states, although the elastic spectroscopic factor for the $s_{1/2}$ state is somewhat larger than that obtained from the (d,p) analysis.[2] Above 7.0 MeV, the resonance structure is more complicated, as the density of the parent states, and thus the density of IAR's increases. The quality of the fit to the data is not as good as for the IAR's below 7.0 MeV. All resonances for which an l value could be determined

TABLE II. Summary of resonance parameters from the analysis of elastic proton scattering from ^{86}Kr and comparison with the results of ^{86}Kr(d,p) work. The (d,p) results are taken from Ref. 2.

		^{86}Kr$(p,p_0)^{86}$Kr						^{86}Kr$(d,p)^{87}$Kr			
E_R (c.m.) (MeV)	E_R(c.m.) −5.348 MeV	l	j	Γ (keV)	Γ_{lj}^{sp} (keV)	$\tilde{\Gamma}_{lj}$ (keV)	S_{pp}	E^* (MeV)	l	j	S_{dp}
5.348	0.000	2	$\frac{5}{2}$	36.0	12.0	7.3	0.62	0.000	2	$\frac{5}{2}$	0.56
5.870	0.522	0	$\frac{1}{2}$	77.0	75.3	40.9	0.62	0.529	0	$\frac{1}{2}$	0.46
6.826	1.478	2	$\frac{3}{2}$	49.6	45.3	9.4	0.22	1.468	2	$\frac{3}{2}$	0.23
			$\frac{5}{2}$		33.2	5.6	0.18			$\frac{5}{2}$	0.14
								1.570			
7.234	1.886	2	$\frac{3}{2}$	30.0	51.9	0.9	0.02	1.873	(2)	$\frac{3}{2}$	0.02
7.348	2.000	2	$\frac{3}{2}$	36.0	53.7	4.9	0.10	1.996	2	$\frac{3}{2}$	0.09
7.420	2.072	0	$\frac{1}{2}$	60.0	103.0	16.4	0.20	2.080	0	$\frac{1}{2}$	0.18
7.463	2.115	2	$\frac{3}{2}$	62.0	55.5	24.6	0.50	2.112	2	$\frac{3}{2}$	0.30
7.620	2.272	5	$\frac{11}{2}$	45.0		0.6		2.250	(5)	$\frac{11}{2}$	0.18
								2.277	(0)	$\frac{1}{2}$	0.03
7.892	2.544	4	$\frac{7}{2}$	30.0	4.2	2.2	0.60	2.515	4	$\frac{7}{2}$	0.49
8.142	2.794	2	$\frac{3}{2}$	65.0	66.1	10.3	0.18	2.775	2	$\frac{3}{2}$	0.10
								2.823	2	$\frac{3}{2}$	0.11
8.263	2.915	2	$\frac{3}{2}$	65.0	70.0	0.6	0.01	3.015	2	$\frac{3}{2}$	0.08
8.374	3.026	3	$\frac{7}{2}$	35.0	60.0	1.8	0.03				
								3.223	2	$\frac{3}{2}$	0.12
8.573	3.225	0	$\frac{1}{2}$	70.0	116.0	5.6	0.11				
								3.237	2	$\frac{3}{2}$	0.08
8.893	3.545	2	$\frac{3}{2}$	38.0	77.5	1.9	0.03	3.552			

were analyzed. No attempt was made to extract parameters for any resonance above 9.0 MeV, although there is structure in the data above this energy.

A total of 14 IAR's were analyzed. The results are tabulated in Table II, along with a comparison with the results of the ^{86}Kr$(d,p)^{87}$Kr analysis of Haravu *et al.*[2] In general, there is good agreement for the energies and spectroscopic factors obtained from the two analyses. The weak states seen at 1.570 and 2.277 MeV in the (d,p) work are not observed as resonances. The two states at 2.775 and 2.823 MeV are not resolved in this experiment, and were analyzed as a single $l=2$ resonance. A weak $l=3$ resonance is observed at 8.374 MeV, but was not seen in the stripping experiment. The spectroscopic factor for this state is rather uncertain; in calculating the Lane-model width, an unusually large well depth was required to provide the neutron bound-state function for an $f_{7/2}$ particle. Above 8.3 MeV, the correspondence of the levels is no longer good. A rather strong $s_{1/2}$ resonance is observed at 8.573 MeV, but no s state was seen in this energy region in the (d,p) work.

The IAR analysis supports the (d,p) analysis in that the single-particle strengths for the d and s states are spread over several states. Only for the ground-state IAR, the first s-state IAR, the d-state IAR at 7.463 MeV, and the g-state IAR is the spectroscopic factor for the coupling of the single-particle to the 0^+ ground state of ^{86}Kr larger than 0.3. Thus a large portion of the wave functions for these states lies in other configurations.

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† Present address: Department of Nuclear Physics, Australian National University, Box 4, P. O., Canberra, A.C.T., Australia.

‡ Present address: Department of Physics, University of Notre Dame, Notre Dame, Indiana 46556.

§ Present address: Radiation Therapy Department, Holy Cross Hospital, Austin, Texas 78702.

[1]R. E. Sass, B. Rosner, and E. J. Schneid, Phys. Rev. 138, B399 (1965).

[2]K. Haravu, C. L. Hollas, P. J. Riley, and W. R. Coker, Phys. Rev. C 1, 938 (1970).

[3]C. L. Hollas, H. R. Hiddleston, V. D. Mistry, and P. J. Riley, in *Nuclear Isospin,* edited by J. D. Anderson, D. Bloom, J. Cerny, and W. W. True (Academic, New York, 1969), p. 667.

[4]P. J. Riley, C. L. Hollas, H. R. Hiddleston, and V. D. Mistry, in *International Conference on Properties of Nuclear States* (Presses de l'Université de Montréal, Montréal, Canada, 1969), p. 326.

[5]D. Robson, Phys. Rev. 137, B535 (1965).

[6]C. O. Kavaloski, J. S. Liley, P. Richard, and N. Stein, Phys. Rev. Letters 16, 806 (1966); S. A. A. Zaidi, J. L. Parish, J. G. Kulleck, C. F. Moore, and P. von Brentano, Phys. Rev. 165, 1312 (1968); C. F. Moore, J. G. Kulleck, P. von Brentano, and F. Rickey, *ibid.* 164, 1559 (1968).

[7]G. C. Morrison, N. Williams, J. A. Nolen, and D. von Ehrenstein, Phys. Rev. Letters 19, 592 (1967); P. A. Moore, P. J. Riley, C. M. Jones, M. D. Mancusi, and J. L. Foster, Jr., *ibid.* 22, 356 (1969).

[8]J. B. Ball and C. B. Fulmer, Phys. Rev. 172, 1199 (1968); E. R. Cosman and D. C. Slater, *ibid.* 172, 1126 (1968).

[9]N. Stein, C. A. Whitten, and D. A. Bromley, Phys. Rev. Letters 20, 113 (1968).

[10]P. Beregi and I. Lovas, Nucl. Phys. A111, 289 (1968); P. A. Moore, P. J. Riley, C. M. Jones, M. D. Mancusi, and J. L. Foster, Jr., Phys. Rev. C 1, 1100 (1970); S. A. A. Zaidi, P. von Brentano, D. Rieck, and J. P. Wurm, Phys. Letters 19, 45 (1965); H. R. Hiddleston and P. J. Riley, *ibid.* 32B, 425 (1970).

[11]C. F. Moore, S. A. A. Zaidi, and J. J. Kent, Phys. Rev. Letters 18, 345 (1967); E. R. Cosman, R. Kalish, D. D. Armstrong, and H. C. Britt, Phys. Rev. C 1, 945 (1970).

[12]N. Stein, in *Nuclear Isospin,* edited by J. D. Anderson, D. Bloom, J. Cerny, and W. W. True (Academic, New York, 1969), p. 481.

[13]S. A. A. Zaidi and P. Dyer, Phys. Rev. 185, 1332 (1969).

[14]S. A. A. Zaidi and S. Darmodjo, Phys. Rev. Letters 19, 1446 (1967).

[15]G. R. Satchler, Nucl. Phys. 55, 1 (1965).

[16]S. Darmodjo, Ph.D. dissertation, University of Texas, 1968 (unpublished).