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Modified Tamm-Dancoff Approximation for Pairing Vibrations*

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The modified Tamm-Dancoff approximation (MTDA) for pairing vibrations is studied. It is shown that the previously reported success of the method is connected with the fact that seniority is a fairly good quantum number for nuclear ground states. It is pointed out that the MTDA may be useful for the treatment of pairing correlations in deformed nuclei. Numerical calculations are presented for Mg^{24}. The random-phase-approximation extension of the method is carried out and used to calculate pairing correlations in ground and excited states of the Pb isotopes.

1. INTRODUCTION

The concept of pairing vibration has been introduced by Bohr[1] and explored by Bes and Broglia[2] and Ripka[3] among others. This concept has been found useful for the treatment of pairing correlations in and near closed-shell nuclei. In particular the 4.8-MeV low-lying 0^+ state of Pb^{208} has been attributed to a two-phonon pairing-vibrational state. Similar calculations have been carried out for other closed-shell nuclear regions. Goswami and Nalcioglu[4] have recently shown that the conventional Tamm-Dancoff approximation (TDA) method for the treatment of vibration can be suitably modified [modified Tamm-Dancoff approximation (MTDA)] for the pairing-vibrational case and the results agree very well with the exact solutions even for states of three phonons.

In this paper, we shall study, first of all, the reasons for the success of this simple MTDA theory. Second, we shall carry out further extensions of the theory that are necessary for its use in important physical problems.

Bohr has emphasized the presence of N^2-type anharmonicity in the energy of the pairing-vibrational states. A simple group theoretic argument[5] can be given to show this for the pairing Hamiltonian if TDA is used for describing the vibrations. For MTDA, this N^2 dependence will be shown to cancel out, so that the MTDA value gives the correct ground-state energy. This result, however, depends on the approximate validity of seniority in the nuclear ground state.

One expects the pairing-vibration concept to be important for the treatment of pairing correlations in deformed nuclei. This is because for the deformed nuclei, the average degeneracy of levels is small and the BCS method is known not to be accurate.[6] On the other hand, the single-particle spectra often show a considerable energy gap resembling the shell effect. One can then treat the pairing correlations in neighboring nuclei by describing them in terms of a few quanta of pairing vibrations. We apply these ideas to the description of the isospin pairing correlations in Mg^{24} taking Ne^{20} as the core nucleus. A comparison with BCS and FBCS calculations already reported[7] will then prove the validity of this approach.

The extension of the MTDA to the modified random-phase approximation (MRPA) is then carried out. After a brief discussion of the MRPA method, we apply it to the description of the pairing vibrations in Pb isotopes, both in ground and excited states. The results are compared with earlier results,[2] and with experimental data.

In Sec. 2 we present the group theoretic arguments in favor of the MTDA method. The extension of the MTDA to deformed nuclei with the calculations for Mg^{24} is given in Sec. 3. Section 4 gives the extension to MRPA. The application to Pb isotopes is given in Sec. 5. Finally, a summary and the conclusions are given in Sec. 6.

2. MTDA METHOD

A. Basic Equations of MTDA

The basic equations of the MTDA have been derived in Ref. 4 and will be sketched very briefly.

We start with the Hamiltonian

$$H=\sum_{\alpha}\epsilon_{\alpha}C_{\alpha}^{\dagger}C_{\alpha}-\tfrac{1}{4}|G|\sum_{a,b}\sqrt{\Omega_a}\,\sqrt{\Omega_b}\,A_a^{\dagger}A_b\,. \tag{1}$$

Here α denotes the single-particle quantum numbers $n_{\alpha}l_{\alpha}j_{\alpha}m_{\alpha}$. We restrict ourselves to one type of charge (single-closed-shell nuclei). Also we shall use the corresponding latin letter a to denote the same quantum numbers except the projection m_{α}. Furthermore,

$$A_a^{\dagger}=\frac{1}{\sqrt{\Omega_a}}\sum_{m_{\alpha}}S_{\alpha}C_{\alpha}^{\dagger}C_{-\alpha}^{\dagger}\,,\quad A_a=(A_a^{\dagger})^{\dagger}\,,$$

$$\Omega_a=2j_a+1\,,\quad S_{\alpha}=(-)^{j_a-m_{\alpha}}\,.$$

Following Ref. 4, we assume that the ground state of a $(K+N)$-particle even-even nucleus can be obtained approximately from the previous $(K+N-2)$-particle nucleus by means of the pairing phonon creation operator (K denotes the number of nucleons in the double-closed-shell core)

$$B^{\dagger}(K+N)=\frac{1}{2}\sum_{a}\frac{X_a(K+N)}{\sqrt{\Omega_a}}A_a^{\dagger}, \tag{2a}$$

with

$$X_a(K+N)=\sqrt{\Omega_a}\,\langle K+N|\,A_a^{\dagger}\,|K+N-2\rangle$$

$$=\frac{\Omega_a[1-2N_a(K+N-2)]\,\Delta(K+N)}{2\epsilon_a-\omega_0^M}\,, \tag{2b}$$

where

$$\Delta(K+N)=\frac{|G|}{2}\sum_{a}X_a(K+N)\,, \tag{2c}$$

$$N_a(K+N-2)=\langle K+N-2|\,\hat{N}_a\,|K+N-2\rangle$$

$$\hat{N}_a=\frac{1}{\Omega_a}\sum_{m_{\alpha}}C_{\alpha}^{\dagger}C_{\alpha}\,. \tag{3}$$

ω_0^M is the energy of the ground state of the $(K+N)$-particle nucleus with respect to the ground state of a $(K+N-2)$-particle one and is the solution of MTDA secular equation[8]

$$\frac{2}{|G|}=\sum_{a}\frac{\Omega_a[1-2N_a(K+N-2)]}{2\epsilon_a-\omega_0^M}\,. \tag{4a}$$

Note that putting all $N_a=0$ in Eq. (4a) we get the TDA secular equation

$$\frac{2}{|G|}=\sum_{a}\frac{\Omega_a}{2\epsilon_a-\omega_0}\,, \tag{4b}$$

where ω_0 denotes the TDA energy of the pairing phonon. The key point in the derivation of Eqs. (2)–(4) is the commutation rule for the A operators

$$[A_a,A_b^{\dagger}]=2\delta_{ab}(1-2\langle\hat{N}_a\rangle)\,. \tag{5a}$$

The notation $\langle\hat{N}_a\rangle$ denotes expectation value of $\hat{N}_a$ in the ground state of the appropriate nucleus.

The "gap parameter" Δ and hence the X_a's are determined from an equation derived from the approximate boson character of the B operators:

$$2=\sum_{a}\frac{X_a^2(K+N)}{\Omega_a}[1-2N_a(K+N-2)]\,. \tag{5b}$$

Finally, to determine $\langle\hat{N}_a\rangle$, we note that the state $|K+N\rangle$ can be written as

$$|K+N\rangle=\beta(K+N)\,B^{\dagger}(K+N)\cdots B^{\dagger}(K+2)|K\rangle$$

$$+\text{other components}\,. \tag{6}$$

Here $\beta(K+N)$ is a constant representing the fraction of the state $|K+N\rangle$ that can be written in the form $B^{\dagger}\cdots B^{\dagger}|K\rangle$. The part of the state $|K+N\rangle$ termed "other components" represents, for example, contributions of states of higher seniority. These other components can be expected to contribute insignificantly to $\langle\hat{N}_a\rangle$ which is then given as

$$N_a(K+N)=\langle\hat{N}_a\rangle=\beta^2(K+N)\langle K|\,B(K+2)\cdots B(K+N)\hat{N}_a\,B^{\dagger}(K+N)\cdots B^{\dagger}(K+2)|K\rangle\,. \tag{7a}$$

The right-hand side of Eq. (7a) can be evaluated by noting that for all $N>0$,

$$B(K+N)|K\rangle=0\,.$$

Then β can be determined from the requirement that

$$\sum_{a}\Omega_a N_a(K+N)=N\,, \tag{7b}$$

N being the total number of extracore particles.

B. Algebraic Treatment of the Pairing Hamiltonian and MTDA

An algebraic treatment of the pairing Hamiltonian can be developed based on the quasispin formalism. Define

$$S_{+}=\sum_{a}\sqrt{\Omega_a}\,A_a^{\dagger}=\sum_{a}S_{+}^{a}\,,\quad S_{-}=\sum_{a}\sqrt{\Omega_a}\,A_a=\sum_{a}S_{-}^{a}\,,$$

$$S_z=\sum_{a}S_z^{a}\,, \tag{8}$$

with

$$S_z^a = 2\hat{N}_a - \Omega_a .$$

We now write the Hamiltonian (1) in terms of the S operators,

$$H = \tfrac{1}{2}\sum_a \epsilon_a (S_z^a + \Omega_a) - \tfrac{1}{4}|G| S_+ S_- . \tag{9}$$

We have,

$$[H, S_+] = \tfrac{1}{2}\sum_a \epsilon_a [S_z^a, S_+] - \tfrac{1}{4}|G|[S_+ S_-, S_+] = 2\sum_a \epsilon_a S_+^a + \frac{|G|}{2} S_+ S_z \tag{10}$$

and

$$\langle K+N+2|[H, S_+]|K+N\rangle = 2\sum_a \epsilon_a \langle K+N+2|S_+^a|K+N\rangle + \frac{|G|}{2}\langle K+N+2|S_+ S_z|K+N\rangle . \tag{11}$$

The left-hand can be evaluated exactly assuming that $|K+N+2\rangle$ and $|K+N\rangle$ are eigenstates of H with eigenvalues $\omega(K+N+2)$ and $\omega(K+N)$, respectively.

In the second term of the right-hand side, we make a complete-set expansion. We thus obtain,

$$[\omega(K+N+2) - \omega(K+N)]\langle K+N+2|S_+|K+N\rangle = 2\sum_a \epsilon_a \langle K+N+2|S_+^a|K+N\rangle + \frac{|G|}{2}\sum_\xi \langle K+N+2|S_+|\psi^\xi(K+N)\rangle\langle\psi^\xi(K+N)|S_z|K+N\rangle . \tag{12a}$$

We retain only terms $|\psi^\xi(K+N)\rangle = |K+N\rangle$, based on the approximate validity of the seniority scheme for the nuclear ground states. This gives

$$[\omega(K+N+2) - \omega(K+N)]\langle K+N+2|S_+|K+N\rangle = 2\sum_a \epsilon_a \langle K+N+2|S_+^a|K+N\rangle + \frac{|G|}{2}\langle K+N+2|S_+|K+N\rangle\langle K+N|S_z|K+N\rangle . \tag{12b}$$

Now from Eq. (2c) and the definitions of S_+[Eq. (8)] and X_a [Eq. (2b)]

$$\langle K+N+2|S_+|K+N\rangle = \Delta(K+N+2)\frac{2}{|G|} , \tag{13}$$

$$[\omega(K+N+2) - \omega(K+N)]\Delta(K+N+2)\frac{2}{|G|} = 2\sum_a \epsilon_a \langle K+N+2|S_+^a|K+N\rangle + \Delta(K+N+2)\langle K+N|S_z|K+N\rangle . \tag{14a}$$

In the first term on the right-hand side if we use the TDA, we have

$$\langle K+N+2|S_+^a|K+N\rangle = \frac{\Omega_a}{2\epsilon_a - \omega_0}\Delta(K+N+2) .$$

Recall that ω_0 denotes the TDA energy of the pairing phonon. Thus, we get

$$[\omega(K+N+2) - \omega(K+N)]\frac{2}{|G|} = \sum_a \frac{2\epsilon_a \Omega_a}{2\epsilon_a - \omega_0} + \left[2\langle\hat{N}\rangle - \sum_a \Omega_a\right] . \tag{14b}$$

Here, $\langle\hat{N}\rangle = \langle K+N|\hat{N}|K+N\rangle$; and $\hat{N} = \sum_a \Omega_a \hat{N}_a$, the total number operator. From Eq. (14a), it follows easily that,

$$[\omega(K+N+2) - \omega(K+N)]\frac{2}{|G|} = \omega_0 \sum_a \frac{\Omega_a}{2\epsilon_a - \omega_0} + 2\langle\hat{N}\rangle .$$

Using the TDA secular equation (4b) we finally get

$$[\omega(K+N+2) - \omega(K+N)] = \omega_0 + |G|\langle\hat{N}\rangle . \tag{15}$$

Thus, for $N>2$, the ground-state energy is not given by the TDA energy and a N^2-type anharmonicity[9] is introduced. We will now prove that treating the first term in Eq. (14) by MTDA exactly cancels the N^2 dependence.

For MTDA, we have

$$\langle K+N+2|S_+^a|K+N\rangle = \frac{\Omega_a[1 - 2N_a(K+N)]}{2\epsilon_a - \omega_0^M}\Delta(K+N+2) ,$$

where ω_0^M denotes the MTDA energy solution of Eq. (4a) with N replaced by $N+2$. Then Eq. (14)

modifies to the form

$$[\omega(K+N+2)-\omega(K+N)]\frac{2}{|G|}$$
$$=\sum_a \frac{2\epsilon_a\Omega_a[1-2N_a(K+N)]}{2\epsilon_a-\omega_0^M}+\left[2\langle\hat{N}\rangle-\sum_a\Omega_a\right] \tag{16}$$
$$=\omega_0^M\sum_a\frac{\Omega_a[1-2N_a(K+N)]}{2\epsilon_a-\omega_0^M}$$
$$-2\sum_a\Omega_a N_a(K+N)+2\langle\hat{N}\rangle\,. \tag{17}$$

Using the secular Eq. (4a) in the first term and $\sum_a\Omega_a\langle\hat{N}_a\rangle=\langle\hat{N}\rangle$, we easily get

$$[\omega(K+N+2)-\omega(K+N)]=\omega_0^M\,, \tag{18}$$

i.e., the MTDA theory gives the exact ground-state energy. Notice that the validity of this result depends only on the validity of the truncation of the complete-set expansion in Eq. (12a) which is based on the goodness of the seniority quantum number for the nuclear ground states. Since this truncation is expected to be reasonably valid in most cases, the MTDA method can be expected to be fairly accurate as has been numerically shown in Ref. 4. It should be noted, however, that the arguments presented here prove only the validity of the energy secular Eq. (4a). The MTDA method of Ref. 4 extracts the $\langle\hat{N}_a\rangle$ appearing in the secular equation by using the pairing-vibration concept. This procedure becomes less and less accurate as N increases, and more involved methods[8] may have to be used for the calculation of $\langle\hat{N}_a\rangle$.

3. APPLICATION TO Mg^{24}

We have already pointed out in the Introduction that one of the applications of the MTDA method would be for the treatment of pairing correlations in deformed nuclei. In this section we shall illustrate this with the example of the treatment of $T=0$ pairing in Mg^{24}. This choice is dictated by the fact that Hartree-Fock-Bogoliubov calculations[7] on Mg^{24} have already shown that Ne^{20} can be assumed to be a good "core" nucleus. Also, BCS and FBCS results are available for Mg^{24} for comparison.

The $T=0$ pairing Hamiltonian in the deformed axially symmetric Hartree-Fock (HF) basis, is given as

$$H=\sum_{K\tau}\epsilon_K C_{K\tau}^\dagger C_{K\tau}$$
$$+\sum_{\substack{K,K'\\K>0\\K'>0}}\langle K\bar{K}\,K=0\;T=0|\,V_a|\,K'\bar{K}'\,K'=0\;T=0\rangle A_K^\dagger A_{K'}\,. \tag{19}$$

We define the pair creation operator

$$A_{K>0}^\dagger=\frac{1}{\sqrt{2}}\sum_\tau(-)^{1/2-\tau}C_{K\tau}^\dagger C_{\bar{K}-\tau}^\dagger\,. \tag{20}$$

The equation of motion for $A^\dagger$ can be written as

$$\langle N+1,Z+1|[H,A_K^\dagger]||N,Z\rangle$$
$$=[\omega(N+1,Z+1)-\omega(N,Z)]\langle N+1,Z+1|A_K^\dagger|N,Z\rangle$$
$$=2\epsilon_K\langle N+1,Z+1|A_K^\dagger|N,Z\rangle$$
$$+\sum_{K'>0}\langle K\bar{K}\,K=0\;T=0|\,V_a|\,K'\bar{K}'\,K'=0\;T=0\rangle$$
$$\times\langle N+1,Z+1|A_{K'}^\dagger|N,Z\rangle(1-2N_K)\,, \tag{21}$$

where

$$N_K=\tfrac{1}{2}\sum_\tau\langle N,Z|C_{K\tau}^\dagger C_{K\tau}|N,Z\rangle\,. \tag{22}$$

Define the amplitude

$$\tilde{X}_K=\frac{\langle N+1,Z+1|A_K^\dagger|N,Z\rangle}{(1-2N_K)^{1/2}}=\frac{X_K}{(1-2N_K)^{1/2}}\,, \tag{23}$$

and

$$\omega_0^M=\omega(N+1,Z+1)-\omega(N,Z)\,.$$

Then the equation of motion can be written as

$$\omega_0^M\tilde{X}_K=2\epsilon_K\tilde{X}_K+\sum_{K'}\langle K\bar{K}\,K=0\;T=0|\,V_a|K'\bar{K}'\,K=0\;T=0\rangle$$
$$\times\sqrt{1-2N_K}\,\sqrt{1-2N_{K'}}\,\tilde{X}_{K'}\,, \tag{24a}$$

with the normalization condition

$$1=\sum_K X_K{}^2(1-2N_K)=\sum_K\tilde{X}_K{}^2(1-2N_K)^2 \tag{24b}$$

as before, which can be solved for the energy and amplitudes $\tilde{X}_K$ of the pairing phonon by matrix diagonalization. The procedure for evaluating the N_K is exactly the same as outlined in Sec. 2 A. Note that the amplitudes $\tilde{X}_K$ are being used in order to obtain Hermitian matrices. Obviously, for the Mg^{24} calculation, we have to start with the Mg^{24} HF representation.[10] We then proceed to calculate first the state of the one $T=0$ phonon based on Ne^{20}, and subsequently, the state of two phonons corresponding to Mg^{24}. The force is taken to be the one used for generating the HF representation, namely Rosenfeld-Yukawa. O^{17} single-particle energies are used for the HF calculation. In Fig. 1 the results of the present calculation are compared with the BCS and FBCS calculations of Ref. 7. Clearly the accuracy of the MTDA method is intermediate between that of BCS and FBCS methods. This in itself is gratifying, since the arguments of the last section do

not apply to $T=0$ pairing. The important point is that finite-range forces can be used without the self-consistency problem, saving considerable computer time.

4. EXTENSION TO MRPA

Just as the ordinary TDA can be generalized to the RPA to include the effect of correlations of the ground state, the MTDA can also be generalized to the MRPA in a straightforward manner. Indeed, this considerably broadens the applicability and scope of the method, since small changes in the occupation numbers of the closed-shell "ground state" by pairing correlations can be taken into account. We generalize the Hamiltonian (1) so that the summation index runs both over unoccupied states (particles) as before, and also occupied states (holes).

The definition of the pairing phonon state is now extended to

$$B^{\dagger}(K+N)=\frac{1}{2}\sum_{a=\text{particles}}\frac{X_a(K+N)A_a^{\dagger}}{\sqrt{\Omega_a}} -\frac{1}{2}\sum_{m=\text{holes}}\frac{X_m(K+N)A_m^{\dagger}}{\sqrt{\Omega_m}}, \tag{25}$$

where X_a and X_m are given by Eq. (2b) with

$$\Delta(K+N)=\frac{|G|}{2}\left[\sum_a X_a(K+N)+\sum_m X_m(K+N)\right]. \tag{26}$$

The MRPA secular equation is given as

$$\frac{2}{|G|}=\sum_a\frac{\Omega_a[1-2N_a(K+N-2)]}{2\epsilon_a-\omega_0^M} +\sum_m\frac{\Omega_m[1-2N_m(K+N-2)]}{2\epsilon_m-\omega_0^M}. \tag{27}$$

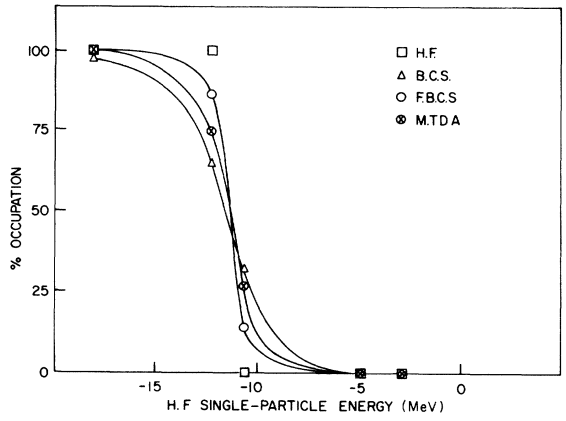

FIG. 1. Occupation of the fourfold degenerate single-particle states of Mg^{24} for the HF, BCS, FBCS, and the MTDA.

As before, the approximate boson character of the $B^{\dagger}$ operator gives the expression for evaluating $\Delta(K+N)$ from the relation

$$\sum_a\frac{X_a^2(K+N)[1-2N_a(K+N-2)]}{\Omega_a} +\sum_m\frac{X_m^2(K+N)[1-2N_m(K+N-2)]}{\Omega_m}=2. \tag{28}$$

Again the procedure for evaluating the $\langle\hat{N}_a\rangle$'s is the same as that outlined in Sec. 2 A. In numerical calculations, one may start assuming that in the case of the one-phonon calculations, the occupation numbers can be taken to have closed-shell values. Alternatively, one can put in occupation number corrections for the "closed-shell" nucleus with a "self-consistent" procedure, but the merit of such a method is dubious and will not be used here.

5. APPLICATION TO Pb ISOTOPES

We have calculated the relative ground-state energies $\omega_0^M=\omega(K\pm N)-\omega(K\pm N\mp 2)$ using the MRPA secular Eq. (27). The single-particle (hole) energies are taken from the work of W. W. True *et al.*[11] The strength of the pairing interaction is taken to be $G=0.0865$. The results are given in Table I, along with the values of Δ.

As pointed out in Refs. 2 and 12–14, it is particularly interesting to discuss the theoretical value of the ratio

$$\frac{[\Delta(K\pm N)]^2}{[\Delta(K\pm N\pm 2)]^2}.$$

This ratio can be determined with fair accuracy from the experimental ratio of two-nucleon-transfer cross sections.[12-14] For the "hole"-type Pb isotopes, we get

$$\frac{|\Delta(\mathrm{Pb}^{206})|^2}{|\Delta(\mathrm{Pb}^{204})|^2}=1:2.03,$$

which compares very well with the experimental value[12] of 1 : 1.7 (±20%). Note that our value is

TABLE I. Values of the ground-state energy w_0 and the gap parameter Δ for Pb isotopes.

	w_0 (MeV)			Δ (MeV)	
Nucleus	MTDA	MRPA	Bes and Broglia	MTDA	MRPA
Pb^{212}	−1.68	−2.50		0.430	0.596
Pb^{210}	−0.96	−1.40	−1.50	0.370	0.520
Pb^{206}	−0.21	−0.45	−0.40	0.197	0.381
Pb^{204}	0.04	−0.15		Divergent	0.540

also very close to that given by the harmonic phonon approximation, namely, 1:2.

On the other hand, for the "particle"-type Pb isotopes, we get

$$\frac{|\Delta(\mathrm{Pb}^{210})|^2}{|\Delta(\mathrm{Pb}^{212})|^2} = 1:1.32;$$

whereas, the experimental data[13] give 1:2.3. Reasons for this discrepancy are not clear at present.

The MRPA method can in particular be used for the calculation of excited 0′ pairing-vibrational states in Pb isotopes analogous to the 4.87-MeV state of Pb^{208}. We assume[2] that this latter state is a "two-phonon" pairing-vibrational state, where the two phonons are made of physical "particles" and "holes," respectively. One can first calculate the ground state of Pb^{210} as

$$|\mathrm{Pb}^{210}\rangle = |\Psi_{2\mathrm{p}}\rangle = B_{\mathrm{p}}^{\dagger}(K+2)|\Psi_K\rangle; \quad |\Psi_K\rangle = |\mathrm{Pb}^{208}\rangle, \tag{29a}$$

with

$$B_{\mathrm{p}}^{\dagger}(K+2) = \frac{1}{2}\left(\sum_a \frac{X_a(K+2)A_a^{\dagger}}{\sqrt{\Omega_a}} - \sum_m \frac{X_m(K+2)A_m^{\dagger}}{\sqrt{\Omega_m}}\right). \tag{29b}$$

The two-phonon pairing-vibrational state of Pb^{208} is then given as

$$|\mathrm{Pb}^{208\prime}\rangle = |\Psi'^{(0)}_{2\mathrm{p}-2\mathrm{h}}\rangle = B_{\mathrm{h}}(K+2)|\Psi_{2\mathrm{p}}\rangle, \tag{30a}$$

with

$$B_{\mathrm{h}}(K+2) = \frac{1}{2}\left(\sum_m \frac{X'_m(K+2)A_m}{\sqrt{\Omega_m}} - \sum_a \frac{X'_a(K+2)A_a}{\sqrt{\Omega_a}}\right). \tag{30b}$$

The one-phonon calculation is identical to that of Ref. 2. However, the inclusion of $\langle \hat{N}_a \rangle$ in MRPA introduces correlation correction in the two-phonon calculation. In particular, this improves the ratio[15] of

$$\frac{|\Delta(\mathrm{Pb}^{210})|^2}{|\Delta(\mathrm{Pb}^{208\prime})|^2}$$

from 1.85 to 2.1. The experimental value[14] is 2 ± 0.8 which agrees with our calculation. Clearly, the correlation corrections are important. Calculation of the excited pairing-vibrational states of other Pb isotopes can now be carried out using the state $|\Psi'^{(0)}_{2\mathrm{p}-2\mathrm{h}}\rangle$ of Pb^{208} as the starting point. Table II shows our predicted excitation energies of the "excited" pairing-vibrational state (relative to the respective ground state) analogous to the 4.87-MeV state of Pb^{208} for some Pb isotopes. We also give the predicted value of Δ for these states.

TABLE II. Excitation energy above nuclear ground state E_x and the gap parameter Δ for some Pb isotopes predicted by the present method.

Nucleus	E_x (MeV) MRPA	Expt.	Δ (MeV) MRPA
$Pb^{212\prime}$	5.72		0.678
$Pb^{210\prime}$	5.37		0.580
$Pb^{208\prime}$	5.06	4.87	0.358
$Pb^{206\prime}$	5.86		0.517
$Pb^{204\prime}$	5.43		Divergent

6. SUMMARY AND CONCLUSIONS

The important results of this paper are as follows: First, the success of the MTDA secular equation in predicting the ground-state energy is shown to be due to the approximate validity of seniority for nuclear ground states. Second, the accuracy of the MTDA method for deformed nuclei is shown to be significantly better than the BCS method. The advantage of the MTDA method for the calculation of pairing effects in deformed nuclei is that one can use realistic forces with considerably less numerical complication than FBCS calculations.[16] Third, the method is generalized to include ground-state correlations (MRPA). This enables us to calculate the pairing-vibrational states in or near closed-shell nuclei. Significant improvement is found for the theoretical predictions for the 4.87-MeV state of Pb^{208}.

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Levels of ^{86}Kr Fed in the Decay of ^{86}Br†

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The decay of ^{86}Br has been studied with high-resolution Ge(Li) detectors by applying on-line mass-separation techniques to ^{235}U fission products. 19 transitions were positively assigned to this decay through half-life comparisons, and for 10 of them direct measurements of the half-life were performed, yielding an average value of 59±4 sec. Levels for ^{86}Kr are proposed at (J^π within parentheses): 1564.62±0.09 keV (2^+), 2349.60±0.14 keV (1,2), 2850.2±0.4 keV, 2926.24±0.13 keV (1,2), 3099.20±0.16 keV (3^-), 4315.75±0.17 keV (2^-,3^-), 5406.6±0.5 keV (1,2), 5519.0±0.9 keV (1^-,2^-), and 6209.9±0.5 keV (1^-,2^-).

I. INTRODUCTION

The nucleus ^{86}Kr has four protons less than a filled doubly magic core. Both the feasibility of a shell-model description and the possibility of obtaining information about the new core through the ^{86}Kr spectrum makes this nucleus a very interesting object for nuclear physicists.

Although Stehney and Steinberg[1] discovered 54-sec ^{86}Br in 1962 through the ^{86}Kr(n,p) reaction, other authors [2,3] obtained it as a ^{235}U fission product and, from its decay, studied the level structure of ^{86}Kr. However, when the ^{86}Br activity is produced in this way, complicated fast chemical techniques are generally required to separate the Kr, I, and Xe radioactive isotopes. Once this is accomplished it is still difficult to avoid contamination due to other Br isotopes, especially ^{87}Br, which has almost the same half-life as ^{86}Br. The only successful attempt to separate ^{86}Br with this approach has been made by Williams and Coryell[2] who exploited the difference in the half-lives of the Se precursors, and performed singles and coincidence γ-ray spectroscopy with NaI(Tl) detectors. Recently, Lundán[3] studied the decay of the ^{86,87}Br mixture with Ge(Li) detectors. His results are in partial disagreement with those of Williams and Coryell, only six γ rays being assigned to mass 86 based on the identification given by these authors. Lundán interpreted an initial growth of the activity of some γ rays in his measurements as due to an isomeric state of 4.5 sec in ^{86}Br.

In recent years, from nuclear-reaction studies on ^{86}Kr targets, several levels have been identified[4,5] in this isotope. However, a definite spin-parity was assigned only to the first excited state.

The present work is devoted to clarifying the general features of the decay of ^{86}Br and removing the discrepancies between Refs. 2 and 3 by using on-line electromagnetic mass separation of ^{235}U fission products and high-resolution γ-ray detectors. Definite mass assignments were obtained for 19 γ rays, and a level scheme is proposed for ^{86}Kr based on our own, and on previously reported data. It contains three new levels and more than twice the number of γ rays of the only previously reported[2] level scheme. Spin-parity assignments are proposed on the basis of logft value and relative γ-ray intensities.