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Tests for T Invariance in Allowed Nuclear Beta Decay

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Various tests of T invariance involving allowed nuclear β decays are discussed. T-violating correlations are calculated to first order in recoil terms E/M and electromagnetic final-state scattering corrections are given.

I. INTRODUCTION

Since the discovery of CP violation by Christenson, Cronin, Fitch, and Turlay[1] many experiments have been done seeking the resultant T violation predicted by the CPT theorem.[2] Some of the most sensitive tests have been provided by searches for a correlation of the form $J\cdot p_e \times p_\nu$ in the β decay of Ne^{19} by Calaprice *et al.*[3] and the measurements, utilizing the Mössbauer effect, of the angular correlation of linearly polarized photons with nuclear orientation (i.e., a correlation of the form $J\cdot e_\gamma J\cdot k_\gamma J\cdot e_\gamma \times k_\gamma$) by Blume and Kistner[4] and by Atac *et al.*[5] The former seeks T violation in the weak interaction, while the latter experiments are sensitive to electromagnetic T nonconservation. In both cases the results are thus far negative.[6]

We consider in this note the possibility of observing T violation in nuclear β decay. Section II reviews the suggestion by Kim and Primakoff[7] that experiments on Ne^{19} alone may not answer the question as to whether the weak interaction is T invariant inasmuch as, being an analog decay, it is not able to reveal the presence of a T-violating second-class current.[8] Also, more general T-nonconserving correlations in allowed nuclear β decay are discussed.

In Sec. III the electromagnetic scattering correlations are given, as calculated for specific transitions by Callan and Treiman,[9] by Chen,[10] and by Brodine,[11] and the contribution of the induced tensor is included.

Finally, in Sec. IV we study the feasibility of seeking T violation in a β-γ process, wherein the need for experimental detection of the small nuclear recoil is obviated.[12] A general expression for such correlations and the electromagnetic scattering corrections are given.

II. T VIOLATION IN ALLOWED NUCLEAR β DECAY

β-decay experiments which might reveal possible time-reversal violation were suggested by Jackson, Treiman, and Wyld in 1957.[13] They pointed out that measurement of the correlation $J\cdot p_e \times p_\nu$,

$$\frac{d^5\Gamma}{dE\,d\Omega_{\hat{e}}\,d\Omega_{\hat{\nu}}} \propto \left(1 + D\frac{\vec{J}}{J}\cdot\frac{\vec{p}_e\times\vec{p}_\nu}{E_e E_\nu} + \cdots\right),$$

was sensitive to the relative phase between the Fermi and Gamow-Teller matrix elements

$$D = \pm\frac{2\,\mathrm{Im}\,g_V M_F g_A^* M_{GT}^*}{g_V^2|M_F|^2 + g_A^2|M_{GT}|^2}\left(\frac{J}{J+1}\right)^{1/2}, \qquad (1)$$

where M_F (M_{GT}) is the Fermi (Gamow-Teller) matrix element for the transition being studied, J is the spin of the parent nucleus, and the upper (lower) sign refers to electron (positron) decay.

Since then experiments measuring D have utilized neutron β decay[14] and $Ne^{19} \to F^{19} + e^+ + \nu_e$,[3] but no T violation has been found. Kim and Primakoff pointed out, however, that such experiments are primarily sensitive to the phase difference between M_F, M_{GT} so that when parent and daughter states are isotopic analogs, as is the case for the two decays which have been studied, only first-class currents can contribute and no limits are placed on second-class T-violating currents.

In order to examine this situation more carefully, we shall assume the validity of the usual current-current weak interaction and of the conserved-vector-current (CVC) hypothesis.[15] Then the β-decay amplitude is given by (for electron-decay; modifications appropriate to positron decay will be included at a later stage)

$$T_\beta(p_1,p_2,p,k)=\frac{G_V}{\sqrt{2}}\cos\theta_C\langle\beta_{p_2}|V_\mu(0)+A_\mu(0)|\alpha_{p_1}\rangle l^\mu\,, \tag{2}$$

where G_V ($\simeq 10^{-5}m_p^{-2}$) is the weak-coupling constant, θ_C is the Cabibbo angle, and l^μ is the matrix element of the lepton current,

$$l^\mu=\bar{u}(p)\gamma^\mu(1+\gamma_5)v(k)\,.$$

Here p_1, p_2, p, and k denote the four-momenta of parent nucleus, daughter nucleus, electron, and neutrino, respectively. If we let M_1, M_2 represent parent and daughter masses and define

$$P=p_1+p_2\,,\quad q=p_1-p_2=p+k\,,$$

$$M=\tfrac{1}{2}(M_1+M_2)\,,\quad \Delta=M_1-M_2\,,$$

then, correct to first order in recoil quantities, the decay spectrum is

$$d^5\Gamma=\frac{|T_\beta|^2}{(2\pi)^5}F_-(Z,E)\left(1+\frac{3E-E_0-3\vec{p}\cdot\hat{k}}{M}\right)\times(E_0-E)^2pEdEd\Omega_e d\Omega_\nu\,, \tag{3}$$

where $F_-(Z,E)$ is the Fermi function and accounts for dominant Coulomb effects, $E(\vec{p})$ is the electron energy (momentum), $\hat{k}$ is a unit vector in the direction of the neutrino momentum, and E_0 is the maximum electron energy,

$$E_0=\Delta\left(1+\frac{m_e^2}{2M\Delta}\right)\Big/\left(1+\frac{\Delta}{2M}\right).$$

The most general expression for the amplitude of an allowed ($\Delta J=0,\pm1$; "no") transition, correct to first order in recoil, is[16]

$$\langle\beta_{p_2}|V_\mu(0)+A_\mu(0)|\alpha_{p_1}\rangle=\frac{1}{2M}aP\cdot l\delta_{JJ'}\delta_{MM'}-\frac{i}{4M}\epsilon_{ijk}(J'M'1k|J'1JM)\times[2bl_iq_j+i\epsilon_{ij\lambda\eta}l^\lambda(cP^\eta-dq^\eta)]\,, \tag{4}$$

where J, J' are the spins of the parent and daughter nuclei, respectively, and M, M' represent the initial and final components of nuclear spin along some axis of quantization. Here repeated Latin indices are summed from 1 to 3, while repeated Greek indices imply a four-vector contraction with the metric $g_{00}=-g_{ii}=1$, and we are working in the rest frame of the parent. a, b, c, d represent reduced matrix elements. Using standard notation,

$$a=g_VM_{\rm F}\,,\quad c=g_AM_{\rm GT}\,,$$

while b is the so-called weak-magnetism contribution which, between nuclear analogs, would be given by[17]

$$b=A\left(\frac{J+1}{J}\right)^{1/2}M_{\rm F}\mu_V\,,$$

where A is the mass number and μ_V is the isovector magnetic moment measured in units of *nucleon* magnetons. The coefficient d, often called the induced tensor, is uniquely correlated with the existence of second-class currents if α, β are isotopic analogs.[18] On the other hand, if α, β are not members of a common isotopic multiplet, the existence of d is not forbidden by G-parity considerations and even receives a contribution from first-class currents in the nuclear impulse approximation.[19]

The T-violating component of the decay spectrum in terms of these coefficients is found to be[20]

$$d^5\Gamma\propto F_\mp(z,E)(E_0-E)^2pEdEd\Omega_e d\Omega_\nu\left[1+P\left(\frac{\hat{n}\cdot\vec{p}\times\hat{k}}{E}D_1(E)+\frac{\hat{n}\cdot\vec{p}\times\hat{k}}{E^2}\hat{k}\cdot\vec{p}D_2(E)\right)-\Lambda_J\left(\hat{n}\cdot\hat{k}\frac{\hat{n}\cdot\vec{p}\times\hat{k}}{E}D_3(E)+\frac{\hat{n}\cdot\vec{p}}{E}\frac{\hat{n}\cdot\vec{p}\times\hat{k}}{E}D_4(E)\right)+\cdots\right], \tag{5}$$

where $P=\langle m\rangle/J$ is the net polarization of the parent nucleus, $\Lambda_J=1-[3\langle m^2\rangle/J(J+1)]$ is a parameter which measures the degree of nuclear orientation, and

$$D_1^{\rm TRV}(E)=\frac{1}{|a|^2+|c|^2}\left\{\mp\delta_{J,J'}\left(\frac{J}{J+1}\right)^{1/2}\left[2\,{\rm Im}a^*c-\frac{E_0}{M}{\rm Im}a^*(c\pm d\pm b)+2\frac{E}{M}{\rm Im}a^*(3c\pm b)\right]\mp\frac{\gamma_{J,J'}}{J+1}\left[\frac{E_0}{2M}{\rm Im}c^*(d+b)-\frac{E}{M}{\rm Im}c^*d\right]\right\}, \tag{6}$$

$$D_2^{\rm TRV}(E)=\pm\delta_{J,J'}\left(\frac{J}{J+1}\right)^{1/2}\frac{1}{|a|^2+|c|^2}\frac{E}{M}6\,{\rm Im}a^*c\,,\qquad D_3^{\rm TRV}(E)=\theta_{J,J'}\frac{E_0-E}{2M}\frac{1}{|a|^2+|c|^2}{\rm Im}c^*(d+b)\,,$$

and

$$D_4^{\mathrm{TRV}}(E) = \theta_{J,J'} \frac{E}{2M} \frac{1}{|a|^2+|c|^2} \mathrm{Im}\, c^*(d-b),$$

with the upper (lower) sign referring to election (positron) decay and with

$$\gamma_{J,J'} = \begin{cases} (J+1), & J=J'+1 \\ 1, & J=J' \\ -J, & J=J'-1\,; \end{cases}$$

$$\theta_{J,J'} = \begin{cases} -(J+1)/(2J-1), & J=J'+1 \\ 1, & J=J' \\ -J/(2J+3), & J=J'-1\,. \end{cases} \tag{7}$$

The experiments which have been performed thus far have utilized spin-$\frac{1}{2}$ nuclei ($\Lambda_J=0$) and have measured the average value of $D_1^{\mathrm{TRV}}(E)$ over the electron energy spectrum

$$\bar{D}_1^{\mathrm{TRV}} \simeq \frac{1}{|a|^2+|c|^2}\left\{ \mp \sqrt{\tfrac{1}{3}}\left[2\,\mathrm{Im}\, a^*c + \frac{E_0}{M}\mathrm{Im}\, a^*(2c\mp d)\right] \mp \frac{E_0}{3M}\mathrm{Im}\, c^*b \right\}. \tag{8}$$

The most stringent limits are placed by the experiment of Calaprice *et al.*, who give

$$\bar{D}_1^{\mathrm{TRV}} \simeq (0.2 \pm 1.4)\times 10^{-2}\,.$$

Since we are assuming the CVC hypothesis, $\mathrm{Im}\, a^*c \ll 1$ requires $\mathrm{Im}\, b^*c \ll 1$ so that

$$\bar{D}_1^{\mathrm{TRV}} \simeq \frac{1}{|a|^2+|c|^2} \sqrt{\tfrac{1}{3}}\,\mathrm{Im}\, a^*\left(2c + \frac{E_0}{M} d\right). \tag{9}$$

Also, Ne^{19} β decay is an analog process, which implies that a, c are purely first class, while d is second class. We see that it is not completely correct that D_1^{TRV} provides *no* check on the presence of a second-class axial-vector current, although $\mathrm{Im}\, d$ could be fairly large without having much effect, since

$$AE_0/M \simeq 2\times 10^{-3}\,.$$

However, an experiment measuring only $\bar{D}_1^{\mathrm{TRV}}$ is ambiguous in that a cancellation could occur between a small phase difference between a, c and a sizable $\mathrm{Im}\, a^*d$. A way to resolve the ambiguity is to measure the energy dependence of the asymmetry. A nonzero value would indicate the presence of a T-violating axial-vector current. Also, study of D_3 or D_4 provides a direct measure of $\mathrm{Im}\, c^*d$. Finally, we note that, as suggested by Kim and Primakoff, studies of nonanalog β decays for which the Gamow-Teller matrix element is anomalously small may reveal T violation arising from meson-exchange corrections even if no T violation exists for neutron β decay. It is hoped that a number of these tests can be done.

III. ELECTROMAGNETIC FINAL-STATE INTERACTIONS

It is well known that a contribution to the correlation parameters $D_i(E)$ exists even if T invariance is valid due to electromagnetic final-state scattering. As suggested by Callan and Treiman,[9] one can calculate this contribution in lowest order by using unitarity, whereby the absorptive part of the weak-decay amplitude is determined by the product of the zeroth-electromagnetic-order β-decay amplitude and the amplitude for electron-(positron-) daughter-nucleus scattering, given by

$$T_\gamma(p_2', p_2, p', p) = \pm\frac{e^2}{k^2}\frac{1}{2M_2}\left[\delta_{M'',M'} Z(p_2'+p_2)\cdot L + i(J'M'1n|J'1J'M'')\epsilon_{ijn}\mu_2 L_i k_j\left(\frac{J'+1}{J'}\right)^{1/2}\right], \tag{10}$$

where

$$L_\mu = \bar{u}(p)\gamma_\mu u(p')\,.$$

$k=p_2-p_2'=p'-p$ is the four-momentum carried by the virtual photon, and M'' and M' are the projections of nuclear spin on some axis of quantization for the intermediate and final states. μ_2 is the total magnetic moment of the daughter nucleus, measured in nuclear magnetons.

The unitarity relation reads, symbolically,

$$\mathrm{Im}\, T_\beta(p_1, p_2, p, k) = \tfrac{1}{2}(2\pi)^4 \int \frac{d^3p'\, d^3p_2'}{(2\pi)^6}\frac{M_2}{E_2'}\frac{m_e}{E'}\delta^4(p'+p_2'-p-p_2)\sum_{\mathrm{spin}} T_\beta(p_1, p_2', p', k) T_\gamma(p_2', p_2, p', p)\,. \tag{11}$$

The contributions to the correlations $D_i(E)$ can now be evaluated, and we find, after tedious calculation,

$$D_1^{\mathrm{EM}}(E)=\frac{1}{|a|^2+|c|^2}\left(\pm\frac{Z\alpha E^2}{4Mp}\left\{\delta_{J,J'}\left(\frac{J}{J+1}\right)^{1/2}\mathrm{Re}\,a^*\left[(b\mp c)\left(1+3\frac{m_e^{\,2}}{E^2}\right)-d\left(1-\frac{m_e^{\,2}}{E^2}\right)\right]\right.\right.$$
$$\left.-\frac{1}{2}\frac{\gamma_{J,J'}}{J+1}\mathrm{Re}\,c^*(c\pm d\mp b)\left(3+\frac{m_e^{\,2}}{E^2}\right)\right\}$$
$$\left.\mp\frac{\mu_2\alpha E^2}{4Mp}3\delta_{J,J'}\left\{|a|^2-|c|^2\pm\frac{1}{[J(J+1)]^{1/2}}\mathrm{Re}\,a^*c\right\}\left(1-\frac{m_e^{\,2}}{E^2}\right)\right), \tag{12}$$

$$D_2^{\mathrm{EM}}(E)=D_3^{\mathrm{EM}}(E)=0\,,$$

$$D_4^{\mathrm{EM}}(E)=\frac{1}{|a|^2+|c|^2}\left\{-3\frac{Z\alpha E^2}{8Mp}\theta_{J,J'}[|c|^2\mp\mathrm{Re}\,c^*(b-d)]\right\},$$

where $\alpha=e^2/4\pi$ is the fine-structure constant.

The results for $D_1^{\mathrm{EM}}(E)$ are found to be identical to those given by Callan and Treiman,[9] Chen,[10] and Brodine[11] for the particular spin transitions treated by these authors. The expressions given in Eq. (12) are also, however, valid for an arbitrary allowed transition and include the contribution of the induced tensor term,[21] which may have an important effect even in the absence of second-class currents.[19]

These electromagnetic effects are small ($\sim 2\times10^{-4}$ for Ne^{19} not including the induced tensor term) but they may be detectable in certain decays[11] given the careful measurements already done in the T-violation experiments and in a test of the CVC hypothesis.[22] They are necessary in any case to assess the validity of any test of T violation and are interesting in their own right in the absence of T violation as a source of information about the induced tensor and/or weak-magnetism contributions.

IV. β-γ CORRELATIONS AND T INVARIANCE

One problem with T-violation tests as discussed in the previous sections is an experimental one: In order to determine the neutrino direction, the recoiling nucleus must be detected. This recoil is in general small and its detection difficult. A possible solution is to study nuclear β decays in which the daughters are γ unstable. The photons are considerably simpler to detect than nuclear recoil. However, for this case nuclear polarization is not sufficient, and such experiments require nuclear orientation.

Consider a process wherein a parent nucleus of spin J makes an allowed β transition to a daughter nucleus of spin J', which subsequently decays electromagnetically to a final nucleus of spin J''. Also, assume the T violation to be associated with the weak-interaction component of the transition. Then one finds for the T-violating correlations, with dipole radiation and the neutrino direction being unobserved,[23]

$$d^5\Gamma\propto F_\mp(Z,E)(E_0-E)^2pEdEd\Omega_e d\Omega_\gamma[1-\Lambda_J\beta_e\hat{n}\cdot\hat{K}\times\hat{p}\,\hat{n}\cdot\hat{K}E_1\Gamma^J_{J',J''}], \tag{13}$$

where

$$\beta_e=p/E\,,\quad E_1=2\,\mathrm{Im}\,a^*\left[c-\frac{1}{3}\frac{E_0}{M}(c\pm b\pm d)+\frac{1}{3}\frac{E}{M}(7c\pm b\pm d)\right],$$

and where we have omitted very small ($\sim\mathrm{Im}\,a^*cE/M$) kinematic corrections arising from the transformation to the laboratory frame (rest frame of the parent nucleus) from the center-of-mass frame of the daughter nucleus wherein the electromagnetic decay is most simply described. Here $\hat{K}$ is a unit vector in the direction of photon emission and[24]

$$\Gamma^J_{J',J''}(E1,M1)=-\delta_{J,J'}\frac{3}{4[J(J+1)]^{1/2}}\begin{cases}-(J+1)/(2J-1)\,, & J=J''+1\\ 1\,, & J=J''\\ -J/(2J+3)\,, & J=J''-1\,.\end{cases} \tag{14}$$

For quadrupole emission we find

$$d^5\Gamma \propto F_{\mp}(Z,E)(E_0-E)^2 pEdEd\Omega_e d\Omega_\gamma\left(1-E_1\{\Lambda_J\beta_e\hat{n}\cdot\hat{K}\times\hat{p}\,\hat{n}\cdot\hat{K}\Upsilon^J_{J',J''}\mp\tau_J\beta_e\hat{n}\cdot\hat{K}\times\hat{p}[(\hat{n}\cdot\hat{K})^3-\tfrac{3}{7}\hat{n}\cdot\hat{K}]\Delta^J_{J',J''}\}\right), \tag{15}$$

where

$$\tau_J = 1-\frac{5}{3}\langle M^2\rangle\frac{6J^2+6J-5}{(J-1)J(J+1)(J+2)}+\frac{35}{3}\langle M^4\rangle\frac{1}{(J-1)J(J+1)(J+2)},$$

is proportional to the statistical population tensor of rank four,[24]

$$\Upsilon^J_{J',J''}(E2,M2)=\delta_{J,J'}\frac{15}{28[J(J+1)]^{1/2}}\begin{cases}-2(J+1)/(2J-1), & J=J''+2\\ (J-5)/(2J-1), & J=J''+1\\ (2J-3)(2J+5)/(2J-1)(2J+3), & J=J''\\ (J+6)/(2J+3), & J=J''-1\\ -2J/(2J+3), & J=J''-2\end{cases} \tag{16}$$

and

$$\Delta^J_{J',J''}(E2,M2)=-\delta_{J,J'}\frac{15}{4[J(J+1)]^{1/2}}\begin{cases}(J+1)(J+2)/(2J-1)(2J-3), & J=J''+2\\ -2(J+2)/(2J-1), & J=J''+1\\ 6(J-1)(J+2)/(2J-1)(2J+3), & J=J''\\ -2(J-1)/(2J+3), & J=J''-1\\ J(J-1)/(2J+3)(2J+5), & J=J''-2.\end{cases} \tag{17}$$

Similar expressions obtain for higher multipolarities.[12]

We see that only analog decays may be utilized for such tests, and that a photon spectrum which is asymmetric under reflection in the J, p plane signifies a violation of T invariance. That is, if the nuclear orientation defines the z axis and the electron counter is placed along the x axis, any difference in the count rates for photon detectors placed in the directions $\cos\theta\,\hat{e}_z+\sin\theta\,\hat{e}_y$ and $\cos\theta\hat{e}_z-\sin\theta\hat{e}_y$ ($\theta\neq 0, \frac{1}{2}\pi$) violates T invariance.

As in the nonradiative case, this statement must be modified slightly because of the presence of electromagnetic corrections to the nuclear β decay. Using the unitarity relation as before we find[25]

$$E_1^{\text{EM}} = \mp\frac{Z\alpha E^2}{4Mp}2\,\text{Re}\,a^*\left[(c\mp b\pm d)-\frac{m_e^{\,2}}{E^2}(3c\mp b\mp d)\right], \tag{18}$$

which should be employed in order to assess the validity of any time-reversal experiment of this type.

Finally, we stress the importance of a measurement of the energy dependence of E^{TRV}. Since $\text{Im}\,a^*c$ is known to be small and $\text{Im}\,a^*b=0$ by the CVC hypothesis, we find

$$E_1^{\text{TRV}}(E)\cong 2\,\text{Im}\,a^*\left(c\mp\frac{E_0-E}{3M}d\right). \tag{19}$$

If E^{TRV} is averaged over the electron-energy spectrum, the experiment yields a measure of

$$\bar{E}_1^{\text{TRV}}\cong 2\,\text{Im}\,a^*\left(c\mp\frac{E_0}{6M}d\right), \tag{20}$$

for which a null result does not rule out a cancellation between the two terms. An evaluation of the slope is a direct way to seek the term $\text{Im}\,a^*d$ and thus provides a check on the suggestion of Kim and Primakoff.

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$$b=A[g_V\langle\beta\|\sum_i \tau_i{}^+L_i\|\alpha\rangle+g_M\langle\beta\|\sum_i\tau_i{}^+\sigma_i\|\alpha\rangle],$$

where $g_M=\mu_p-\mu_n=4.70$, while for d we find

$$d=A[g_A\langle\beta\|i\sum_i\tau_i{}^+\sigma_i\times L_i\|\alpha\rangle+g_{\text{II}}\langle\beta\|\sum_i\tau_i{}^+\sigma_i\|\alpha\rangle],$$

where g_{II} is the second-class axial form factor for neutron β decay. The second-class contribution is not unique, however, as pointed out by J. Delorme and M. Rho, Phys. Letters 34B, 238 (1971) and by L. Wolfenstein and E. Henley, *ibid.* 36B, 28 (1971).

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PHYSICAL REVIEW C VOLUME 5, NUMBER 5 MAY 1972

Simple Formula for the General Oscillator Brackets

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An explicit formula for Talmi-Moshinsky transformation brackets of unequal-mass particles is given which is the sum of simple expressions over five variables; it is especially suited for numerical calculations.

I. INTRODUCTION

Many papers[1-13] have been devoted to the study of the Talmi-Moshinsky transformation. Since the Talmi-Moshinsky brackets (TMB) find frequent and repeated application in programs for various model calculations of nuclear structure, one of the important aims of these studies was to derive as simple a formula for them as possible. Several excellent techniques have been developed for these purposes. Let us only mention the creation-operator technique for oscillator quanta which was introduced by Moshinsky[1,14] and then successfully applied in a number of works,[3,4,7] and