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Angular Distribution of Gamma Rays from the Reaction $C^{12}(Li^6, d)O^{16}$†

R. R. Carlson
Department of Physics and Astronomy, University of Iowa, Iowa City, Iowa 52240
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Angular distributions of the 6.92- and 7.12-MeV γ radiation from the reaction $C^{12}(Li^6,d)O^{16}$ were measured as functions of bombarding energy in order to obtain information about the reaction mechanism. Bombarding energies were at, and below, the Coulomb barrier for these low, negative-Q reactions. The tendency to proceed by a direct-reaction mechanism should be enhanced under these conditions. Results are compared with plane-wave calculations and are in reasonable agreement for the 6.92-MeV radiation but in strong disagreement for the 7.12-MeV radiation.

INTRODUCTION

Nuclear reactions resulting from lithium bombardment of light nuclei have been interpreted as proceeding by a direct-reaction mechanism at both high[1] and low bombarding energies.[2] For energies around the Coulomb barrier other mechanisms must come into play, since the forward-reaction cross sections show fairly rapid variations as functions of energy.[3,4] An understanding of the reaction mechanism is essential if these reactions are to be used to obtain spectroscopic information. Also the reaction mechanism is of interest in its own right, since one has two complex nuclei reacting. A proper interpretation might aid in the understanding of the interaction of even more complex nuclei.

Since lithium nuclei are very weakly bound, lithium-induced nuclear reactions generally have high Q values. This results in energetic particle groups for product nuclei in low excited states. It is these groups which have received the most study. However, product nuclei in high excited states, corresponding to low Q value, would be most likely to be made by a direct-reaction mechanism. This would be especially true at low bombarding energies where distances of closest approach are large. A large separation of the clusters making up the lithium nuclei is then possible at the time of interaction.

The argument is identical to that given by Wilkinson for dp stripping at low energies.[5] A low Q allows the reaction to proceed with the outgoing cluster having low momentum, and this condition corresponds to large separation of clusters in the lithium nucleus wave function. One cluster can then interact strongly with the target while the other is only weakly affected. For the (Li^6, d) reaction, α transfers with $l=0$, 1, and 2 would be quite possible under these conditions just as in the dp reaction.[5] The best bombarding energy for obtaining a direct reaction will be one where the outgoing deuteron carries away as much momentum as it had due to the bombarding energy of the lithium nucleus; for the reaction $C^{12}(Li^6,d)O^{16}$ this occurs for $E_{Li}=-2.4Q$.

The reaction $C^{12}(Li^6, d)O^{16}$ has a Q value of 5.689 MeV for the ground-state group.[6] It has been extensively studied but the reaction mechanism is still in question. The groups corresponding to the 6.92- and 7.12-MeV states have a stripping character at higher bombarding energies where they have been measured.[3] One would expect them to give better stripping distributions for lower bombarding energies on the basis of the arguments given above. This is apparently the case for a limited range of angles for bombarding energies from 4.5 to 5.5 MeV where measurements have been made.[7]

The 2^+, 6.92-MeV state[8] has a Q value of -1.23 MeV and could be formed by the transfer of an $l=2$ α particle in the reaction $C^{12}(Li^6, d)O^{16}$. The 1^-, 7.12-MeV state[8] has a Q value of -1.43 MeV and would have an $l=1$, α transfer. Both of these states are very near the threshold for α emission from O^{16} which occurs at 7.16 MeV. Both of these states will, therefore, have long-tailed wave functions making a direct- or stripping-reaction mechanism peculiarly appropriate.[5]

The very circumstances which make the reaction $C^{12}(Li^6, d)O^{16*}$ likely to proceed by a direct-reaction mechanism make the experimental measurement of the particle cross section difficult. Measurements for bombarding energies down to 9 MeV have been reported,[3] but this is still considerably above the Coulomb barrier (~6.0 MeV in the laboratory). Measurements at or below the Coulomb barrier are difficult because of the low energy of the outgoing deuterons.

The deexcitation γ rays can now be measured with reasonable efficiency with Ge(Li) detectors. The angular distribution of these γ rays depends on the reaction mechanism, since it will determine the population of the various magnetic substates of the residual nuclei. In the present experiment the angular distributions of the 6.92- and 7.12-MeV γ rays resulting from the reaction $C^{12}(Li^6, d)O^{16}$ were studied as functions of bombarding energy below and just above the Coulomb barrier. Simple direct-reaction calculations are compared with the data.

METHODS AND RESULTS

The reaction $C^{12}(Li^6, d)O^{16}$ was produced with a Li^{6+} beam provided by the University of Iowa Van de Graaff accelerator. Beam currents of about 0.3 μA were brought to the center of a 3-in.-diam cylindrical-shaped target chamber. The beam was brought in along a diameter and focused to a $\frac{1}{16}$-in.-diam spot at the target. Beam energy was known to 0.3%. The carbon target was 200 $\mu g/cm^2$ in thickness and 0.5 in. in diameter. 6-MeV particles lose 0.42 MeV in these targets.[9] A lightweight aluminum frame held the unbacked target at 45° to the beam direction. Frequent visual inspection of the beam spot was made. The chamber was kept evacuated to about 10^{-6} Torr and repeat runs showed no significant buildup of carbon in the chamber or changes in the target.

γ rays were detected with a 4-cm^3 Ge(Li) detector which was moved in a horizontal plane containing the beam direction. The front face of the active volume of the detector was 4.4 in. from the chamber center and had a radius of 0.6-in. The detector, therefore, subtended a full angle of 15° at the target. The detector dimensions were checked with a collimated source and were found to agree with the manufacturer's description.

The part of the chamber wall through which γ rays were observed consisted of $\frac{1}{16}$ in. aluminum and 0.010 in. copper. This has negligible absorption for 7-MeV γ rays. The chamber interposed the same amount of material between the source spot and the dectector from 135° on one side of the beam to 135° on the other. Chamber symmetry was checked by measurements of the angular distribution of the 3.09-MeV γ rays from the reaction $C^{12}(d,p)C^{13}$. Since the 3.09-MeV γ rays come from the spin-$\frac{1}{2}$ first excited state of C^{13}, their angular distribution should be isotropic. This was found to be the case within 2%.

A 3-in.-diam by 3-in.-long NaI(Tl) detector, at a distance of 30 in. was used as a monitor during angular-distribution measurements. A discriminator was set at about 3 MeV and pulses corresponding to energy deposition in the NaI(Tl) detector of an amount greater than this value were counted to give the monitor count. For a typical run of 15 min at the higher bombarding energies, the monitor counter would accumulate about 10^6 counts.

Pulses from the Ge(Li) detector were analyzed in a 4096-channel pulse-height analyzer and stored in a general purpose computer (CDC 160A). Figure 1 shows the γ-ray spectrum near the two-escape region for 7-MeV radiation as observed at 90° to the beam with 6.5-MeV bombarding energy. Programs were available which would display the spectrum or any portion of it and perform a background subtraction with a linear fit to the background under the peak of interest. Spectra were also recorded onto magnetic tape for more-detailed processing in the university computer center. In this processing higher-order polynomial fits were made to the background. It was found that a quadratic fit was all that was needed to properly describe the background.

The two-escape peaks of the 7.12- and 6.92-MeV γ rays from $C^{12}(Li^6, d)O^{16}$ are shown in Fig. 1. They are the highest-energy γ-ray peaks observed

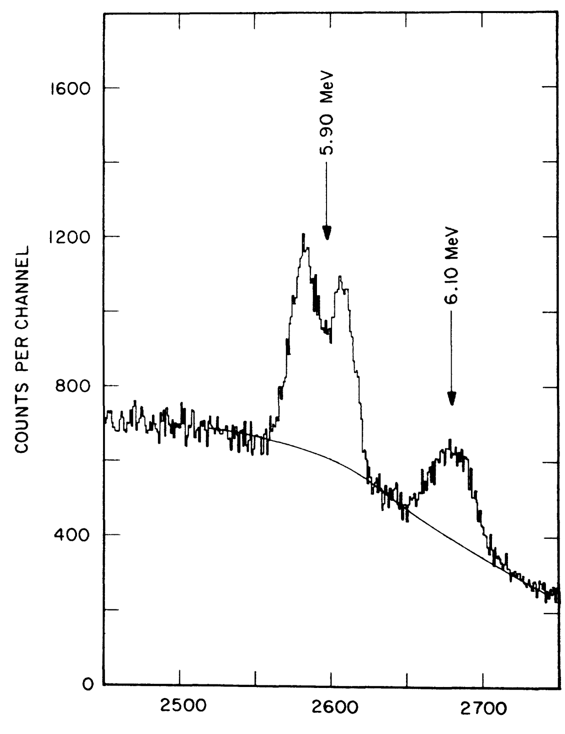

FIG. 1. Part of pulse-height distribution of the Ge(Li) detector at 90° to the beam direction for 6.5-MeV Li^6 bombardment of a C^{12} target. Two-escape peaks of the 6.92- and 7.12-MeV γ rays are shown. The smooth line is the least-squares-fitted background used to obtain the yield.

The two-escape peaks are superimposed on a Compton continuum due to these γ rays and the 6.44-MeV γ ray from $C^{12}(Li^6, \alpha)N^{14*}$. The 6.13-MeV γ ray from $C^{12}(Li^6, d)O^{16}$ also contributes to the background, since its Compton edge is just above the 6.92-MeV two-escape peak. Because of the small detector volume there were negligible amounts of total capture and one-escape peaks in this energy region. The background is shown as a smooth line in Fig. 1. It was obtained for each peak by a least-squares fit to the region on each side of the peak. The number of counts above the background and under the two-escape peaks of the 6.92- and 7.12-MeV γ rays were taken as measures of the yields of these two γ rays.

The energy of the two γ rays of interest changes with angle because of the Doppler shift. This gives a 1.3% increase in energy from 90 to 0° for 4.5-MeV bombardment. Since two-escape peaks are used as measures of yields, the increase in pair-production cross section with energy would result in a small change in efficiency with angle if the detector were large enough so that edge losses were not important. For the detector size used in this experiment there is evidence[10] that the change in edge losses just about cancels out the increase in cross section so no corrections were applied.

Approximately the full Doppler shift was observed for the 7.12-, 6.92-, and 6.44-MeV γ rays. This is expected in view of the short lives of the parent states. About half the full Doppler shift was observed for the 6.13-MeV γ ray. This arises, in spite of the long life of the parent state, because the target is sufficiently thin and unbacked. Since the 6.13-MeV γ-ray yield is rather small compared with the yield of the other γ rays in the present bombarding energy range, most of the background counts are contributed by recoil ions emitting at nearly full velocity. Errors due to background subtraction should, therefore, remain essentially the same at different angles.

Spectra were measured at 15° intervals from 0 to 90° with respect to the beam. Two-escape peaks of the 6.92- and 7.12-MeV γ rays were summed after background subtraction and normalized to give angular distributions of these γ rays. The results are shown in Figs. 2 and 3 for 4.5-MeV bombarding energy. Angular distributions like those shown were obtained for four bombarding energies in the range from 4.5 to 6.5 MeV. Nonisotropic angular distributions were observed for both γ rays throughout the range of bombarding energy with the deviation from isotropy greatest at the lower bombarding energies.

Since the γ radiation is emitted by an isolated state, the angular distribution must be symmetric about 90°. This was checked back to 120°. The

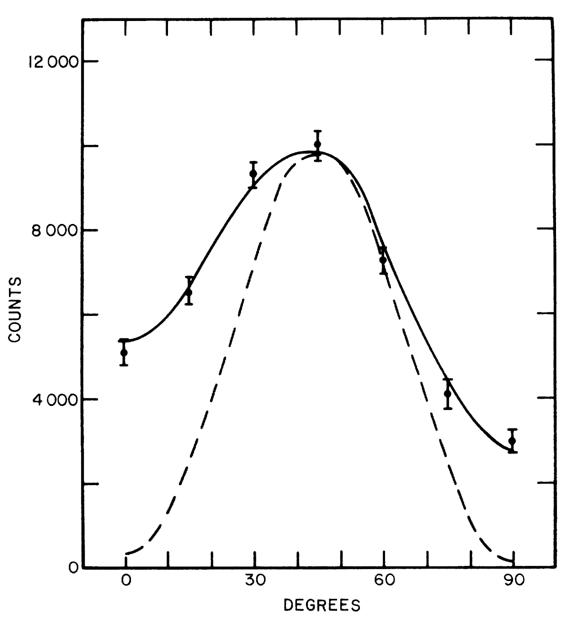

FIG. 2. Angular distribution of the 6.92-MeV radiation from the reaction $C^{12}(Li^6, d)O^{16}$ at 4.5-MeV bombarding energy. The solid curve is the least-squares fit to the data. The dashed curve is of the form $\sin^2 2\theta$, which is appropriate for population of the $m=0$ state alone.

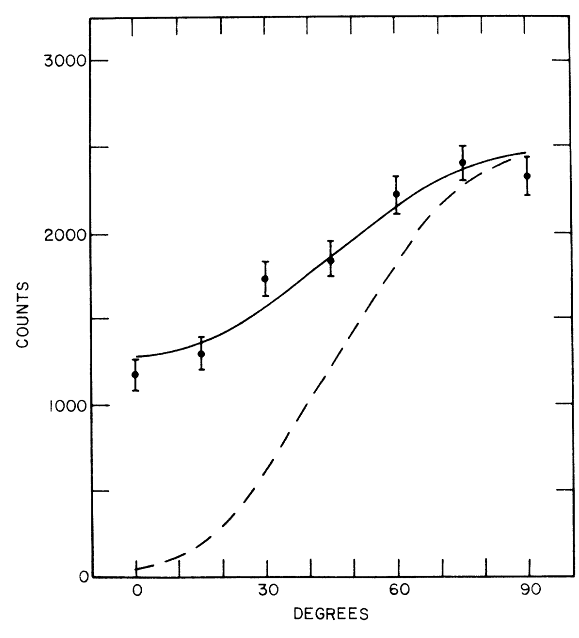

FIG. 3. Angular distribution of the 7.12-MeV radiation from the reaction $C^{12}(Li^6, d)O^{16}$ at 4.5-MeV bombarding energy. The solid curve is the least-squares fit to the data. The dashed curve is of the form $\sin^2\theta$, which is appropriate for population of the $m=0$ state alone.

6.92-MeV γ ray comes from a $2^+ \to 0^+$ transition and is, therefore, pure $E2$. Its angular distributions were least-square-fitted to the form:

$$W(0) = a_0 + a_2 Q_2 P_2(\cos\theta) + a_4 Q_4 P_4(\cos\theta),$$

where $W(\theta)$ is the normalized number of counts, Q_i are attenuation factors, and P_i are Legendre polynomials. The attenuation factors were calculated for a cylindrical detector with uniform efficiency across its face[11] and have the values: $Q_2 = 0.99$ and $Q_4 = 0.96$. The 7.12-MeV γ ray comes from a $1^- \to 0^+$ transition and is, therefore, pure $E1$, so the simpler form,

$$W(\theta) = a_0' + a_2' Q_2 P_2(\cos\theta),$$

could be used.

DISCUSSION

The γ-ray angular distributions can be thought of as the result of the superposition of radiation patterns resulting from the decay of the various magnetic substates of the O^{16} nuclear state. If the magnetic substates had equal populations, isotropic angular distributions would result. The reaction mechanism gives unequal populations, however, and nonisotropic distributions are the result.

For the 6.92-MeV 2^+ state three substate populations characterize the angular distribution, since no polarization measurements were made. The population parameters are given[12] in terms of the fitting constants as follows:

$$\omega_0 = \frac{1}{5} + \frac{2}{5}\frac{a_2}{a_0} - \frac{3}{10}\frac{a_4}{a_0},$$

$$\omega_1 = \omega_{-1} = \frac{1}{5} + \frac{1}{5}\frac{a_2}{a_0} + \frac{1}{5}\frac{a_4}{a_0},$$

$$\omega_2 = \omega_{-2} = \frac{1}{5} - \frac{2}{5}\frac{a_2}{a_0} - \frac{1}{20}\frac{a_4}{a_0}.$$

The subscript on the population parameters ω indicates the value of the magnetic quantum number, using the beam direction as the axis of quantization. Population parameters are normalized so that their sum is 1. The dominant population parameter is ω_0. This dominance decreases as bombarding energy increases. For the case where $\omega_0 = 1$, the angular dependence of the radiation is given by $\sin^2 2\theta$ and is drawn as a dashed line in Fig. 2 for comparison.[13] Figure 4 shows the measured values of ω_0, ω_1, and ω_2 as functions of bombarding energy.

Two population parameters characterize the

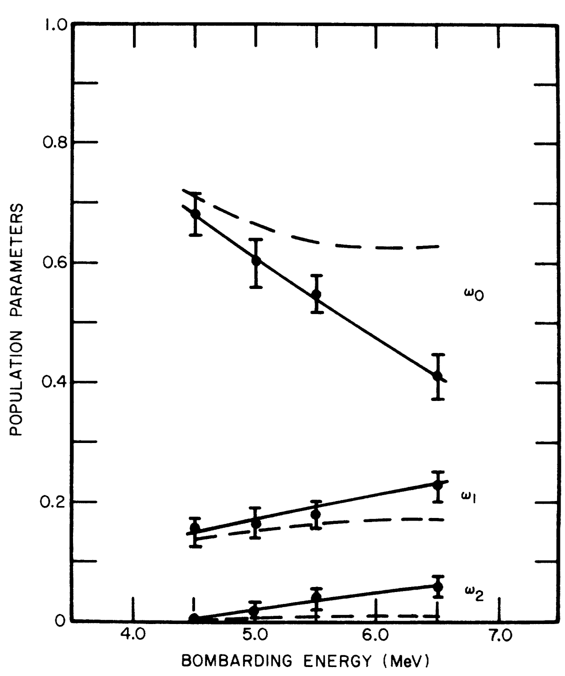

FIG. 4. Population parameters of the 6.92-MeV state of O^{16} as a function of bombarding energy for the reaction $C^{12}(Li^6, d)O^{16}$. Solid lines are drawn through the experimental points. Dashed lines show the population parameters calculated on the basis of a simple plane-wave theory. Labels on the lines denote the magnetic substate.

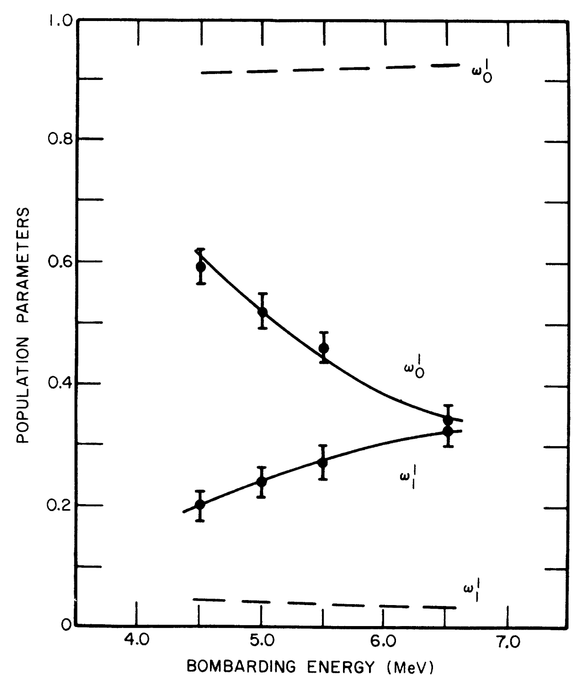

FIG. 5. Population parameters of the 7.12-MeV state of O^{16} as a function of bombarding energy for the reaction $C^{12}(Li^6, d)O^{16}$. Solid lines are drawn through the experimental points. Dashed lines show the population parameters calculated on the basis of a simple plane-wave theory. Labels on the lines denote the magnetic substate.

angular distribution of the 7.12-MeV γ radiation and they are given in terms of the fitting constants as follows:

$$\omega_0' = \frac{1}{3} - \frac{2}{3}\frac{a_2'}{a_0'},$$

$$\omega_1' = \omega_{-1}' = \frac{1}{3} + \frac{1}{3}\frac{a_2'}{a_0'}.$$

In this case, again, the dominant population parameter is ω_0' but the dominance is not nearly as pronounced. The angular distribution corresponding to $\omega_0'=1$, $\sin^2\theta$, is drawn in Fig. 3. The energy dependences of the population parameters are given in Fig. 5.

Plane-wave Born-approximation fits to the data were attempted to gain insight into the reaction mechanism. This approximation represents a simplification of the reaction mechanism, but since the fit to the data only depends on the shapes of calculated particle angular distributions the use of plane waves is, perhaps, justified. In addition, recoil effects are included in the plane-wave calculation.

Biedenharn, Boyer, and Charpie long ago showed[14] that, in the plane-wave Born approximation, the $(d, p\gamma)$ reaction could be thought of as if a neutron beam were absorbed by the target with the beam direction being that of the momentum transferred to the nucleus in the (d, p) reaction. Subsequent γ emission in coincidence with the proton then has the angular distribution arising from depopulation of the magnetic substate with orbital magnetic quantum number zero. The direction of quantization is that of the absorbed neutron or recoil nucleus. These arguments may be applied in an analogous manner to the reaction $C^{12}(Li^6, d\gamma)O^{16}$. In fact, the arguments are simplified because one has a spin-0 α beam captured and the subsequent radiation consists of γ rays of pure multipolarity. To obtain the calculated angular distributions, or what amounts to the same thing, the population parameters for quantization with respect to the beam direction, it is only necessary to use rotation matrix elements to calculate the contributions to ω_0, ω_1, and ω_2 at 0° coming from recoil at angle θ. These contributions are then weighted with the probability of having a recoil at angle θ, which is given by the differential cross section for deuteron production at the angle corresponding to the recoil angle θ.

The differential cross section for $C^{12}(Li^6, d)O^{16}$ was taken to be proportional to the square of the spherical Bessel function, $j_l^2(qR)$, where l is the angular momentum transfer, q is the linear momentum transferred by the α particle, and R is the interaction radius.[15] For the 6.92-MeV state, $l=2$; for the 7.12-MeV state, $l=1$. R is an adjustable parameter which was put equal to 5.0 F. Results were insensitive to the value of R. Because of the low bombarding energy and the low Q value for these reactions, the recoil oxygen nuclei have little energy in the center of mass and are carried forward with it. The maximum lab angle for the recoil particles is less than 30° and corresponds to deuterons at about 60°. As the bombarding energy is lowered toward the threshold, the recoil nuclei are more and more concentrated in the forward direction so that at threshold (1.84 MeV for the 6.92-MeV state) the shape of the cross section would have no effect and all recoil nuclei would be produced with population $\omega_0=1$. Away from threshold the cross-section shape weights some angles more heavily than others.

A computer program was written to do the calculations described above. Population parameters were calculated as functions of bombarding energy and the results are drawn in Figs. 4 and 5 for the 6.92- and 7.12-MeV γ rays, respectively.

In the case of the 6.92-MeV γ radiation, there is reasonable agreement between the calculated and measured values of the population parameters at the lowest energies where one would expect the best agreement. On the other hand, there is no agreement at all in the case of the 7.12-MeV γ radiation. This could be attributed to the difference in character of the two states. The 6.92-MeV state has been described as a 4p-4h state which is a member of a rotational band.[16] Such a state could be readily formed in a direct α-particle transfer.[17] The yield of this state is greater than that of the 7.12-MeV state even though the formation of the 6.92-MeV state requires a transfer of two units of angular momentum. The relative yields of the two γ rays can be obtained from Figs. 2 and 3; the 6.92-MeV radiation is $4\frac{1}{2}$ times as intense as the 7.12-MeV radiation. This ratio is preserved over the entire energy range.

In summary, the angular distributions of two γ rays from the reaction $C^{12}(Li^6, d\gamma)O^{16}$ have been studied under conditions such that the reaction would be expected to proceed by a direct-reaction mechanism. One of the γ rays, the 6.92-MeV radiation, appears to result from a direct reaction while the other, the 7.12-MeV radiation, does not.

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Density-Matrix Expansion for an Effective Nuclear Hamiltonian*

J. W. Negele and D. Vautherin†

Laboratory for Nuclear Science and Department of Physics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139

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An expansion for the nuclear wave-function density matrix in relative and c.m. coordinates is developed such that the leading term is the corresponding nuclear-matter density matrix at the local neutron and proton density. Truncation of all derivatives beyond second order yields an extremely simple form for the energy density which retains all the computational simplicity of the modified δ interaction and the Skyrme force, while maintaining contact with nuclear-matter theory based on a realistic interaction and reproducing the results of more-complicated density-dependent Hartree-Fock calculations.

I. INTRODUCTION

Attempts thus far to approximate the nuclear many-body ground state by a Slater determinant of single-particle wave functions determined self-consistently from the average field created by the other nucleons differ greatly in their computational complexity and the degree to which they retain contact with the phenomenological force between two free nucleons. The purpose of this present work is to relate the extremely simplified effective interaction used in a previous paper[1] to a more fundamental and thus computationally cumbersome theory derived directly from the nucleon-nucleon force.[2]

Because a comprehensive review of various approaches is now available,[3] we shall only briefly review several references of direct relevance to this present work. Several calculations[2,4-6] have now demonstrated that a local-density approximation to the reaction matrix derived from a realistic potential such as the Reid potential[7] yields satisfactory results for the gross properties of finite nuclei. We wish to retain the essential physical ideas of a two-body reaction matrix influenced by the presence of other particles via the Pauli operator and self-consistent energy denominators as discussed in Ref. 2, while eliminating, as much as possible, the complications of successive intermediate definitions of various effective interactions. The final form which is sought is a theory similar to Ref. 1 in which the potential energy density may be simply related to the nuclear density, the gradient of the density, and the kinetic energy density.

As a result of the form of the desired theory, it closely resembles previous work by Skyrme,[8] Köhler,[9] and Moszkowski.[10] Skyrme appears to have been the first to propose truncating an expansion of the forward scattering amplitude, or reaction matrix, in momentum space. He explicitly emphasized that the strength of the scattering changes with density, which was to be accounted for by letting the parameters depend upon