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Beta Decay of the ^{26}Al Ground State*

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The decay scheme of the ground state of ^{26}Al has been reexamined by measurement of its γ-ray spectrum with Ge(Li) detectors. The γ-ray spectrum shows very strong positron annihilation radiation, a strong peak at 1808.65 ± 0.07 keV, a weak peak at 1129.67 ± 0.10 keV, and a very weak peak at 2938 keV. These γ-ray energies correspond to transitions from the first and second excited states of ^{26}Mg, which have been determined more accurately in this experiment. The ^{26}Al branching ratios, calculated from the γ-ray relative intensities, are (82.1 ± 2.5)% positron emission to the 1809-keV level, (15.2 ± 2.5)% electron capture to that same level, and (2.7 ± 0.2)% electron capture to the 2938-keV level of ^{26}Mg. The 2938-keV state decays (10.2 ± 2.0)% directly to the ground state and (89.8 ± 2.0)% by cascade. These branching ratios combine with previous results to give a half-life of (7.16 ± 0.32) × 10^5 yr for the ground state of ^{26}Al.

I. INTRODUCTION

The decay of the ground state of ^{26}Al to ^{26}Mg has not been investigated since 1958–59. At that time it was established that the 5^+ ^{26}Al ground state decays to two 2^+ states of ^{26}Mg at 1.81 and 2.97 MeV with branching ratios of (96.0 ± 0.3) and (4.0 ± 0.3)%, respectively.[1] It was determined that the major branch was composed of (84.6 ± 1.8)% positron emission and (11.4 ± 1.9)% electron capture (EC).[1] The positron spectrum has a unique second-forbidden shape,[2,3] and an end point of 1160 ± 8 keV. Because the energy release is barely above $2m_0c^2$, the decay to the 2938-keV level proceeds by EC with a negligible positron-emission component. The partial half-life for positron emission to the 1.81-MeV level was measured with a 4π Geiger-Müller counter and mass-spectrometric analysis to be (8.73 ± 0.30) × 10^5 yr.[4] The half-life of the ^{26}Al ground state was thus determined to be (7.38 ± 0.29) × 10^5 yr.[1,4]

There are several reasons why a reinvestigation of the decay of ^{26}Al was believed worthwhile. First, earlier work[1,5] on the branching ratios was carried out using NaI(Tl) crystals to detect γ rays, and it was felt that more reliable measurements could be made with Ge(Li) detectors. Second, recent calculations[6] of the expected ratio of EC to positron emission (β^+) in the decay to the 1.81-MeV level revealed that the branching ratios of Rightmire, Simanton, and Kohman[1] differ from the expected result by 2.5 standard deviations. Since the EC-to-β^+ ratio is essentially determined by atomic and not nuclear configurations, it is something that should be readily and reliably calculable. Thus, if this discrepancy is real, it is quite serious indeed. For these reasons, then, an investigation of the relative decay modes of ^{26}Al was undertaken.

II. EXPERIMENTAL PROCEDURE AND RESULTS

The source[7] of ^{26}Al was a solution of $Al(NO_3)_3$ with a nominal activity of 10^5 disintegrations per min (0.045 μCi) which was evaporated to dryness as

Al_2O_3 under an infrared heat lamp. The powder residue weighed 150 mg, of which 16 mg could be the ^{26}Al oxide. The powder was mounted on a plastic disk and secured by a tightly fitting cover of plastic film.

The decay scheme of ^{26}Al is shown in Fig. 1, which summarizes all the present results. The data necessary to obtain the β and γ branching ratios and the ^{26}Mg excitation energies were all obtained from γ-ray singles spectra recorded using a 30-cm^3 Ge(Li) detector in conjunction with an 8192-channel analog-to-digital converter. The resolution of the system, as measured from the full width at half maximum (FWHM) for ^{60}Co γ rays, was 2.6 keV for the long counting times used. The branching ratios were obtained from measurements of the relative intensities of the three ^{26}Mg γ rays and annihilation radiation. A spectrum used for this purpose is shown in Fig. 2.

The relative intensity of 511- to 1809-keV radiation was measured with the ^{26}Al source encased in 2.5 mm of aluminum, more than sufficient thickness to stop 1.16-MeV positrons. In this mounting the intensity ratio remained constant with changes in the source-to-detector distance. To keep the natural background low throughout the spectrum and to reduce background contributions to the positron-annihilation intensity, the source and detector were surrounded by a shielding of lead bricks. The background spectrum was measured with the ^{26}Al source removed. The energies of the 1808.65 $\pm$ 0.07- and 1129.67 $\pm$ 0.10-keV γ rays were measured using a dispersion of 0.310 keV/channel and sources of ^{208}Tl, ^{137}Cs, ^{212}Bi, ^{54}Mn, ^{60}Co, ^{22}Na, ^{207}Bi, and ^{88}Y. Figure 3 shows a portion of a 16-h spectrum showing the 1809-keV photopeak region. To reduce the Compton background in the region of the much weaker 1130-keV peak, several spectra were recorded with the higher-energy ^{208}Tl, ^{207}Bi, and ^{88}Y sources removed. No attempt was made to determine the energy of the extremely weak 2938-keV peak with this accuracy. The energies quoted are the weighted means of three determinations for the 1129.67 $\pm$ 0.10-keV γ rays and four determinations of the full-energy, single-escape, and double-escape peaks of the 1808.65 $\pm$ 0.07-keV γ ray. The method used for obtaining the energies has been fully described.[8]

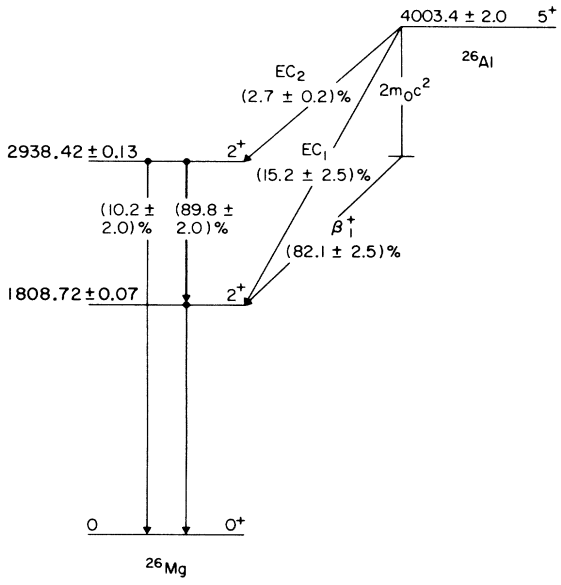

FIG. 1. Decay scheme for ^{26}Al. The β and γ branching ratios and the ^{26}Mg excitation energies are from the present results; the ^{26}Al-^{26}Mg mass difference is from Ref. 5. Energies are in keV.

The excitation energies of ^{26}Mg, corrected for recoil, are then 1808.72 $\pm$ 0.07 and 2938.42 $\pm$ 0.13 keV. Using the mass difference of ^{26}Al and ^{26}Mg of 4003.4 $\pm$ 2.0 keV,[5] the β^+ end-point energy for the main branch should be 1172.7 $\pm$ 2.0 keV, slightly higher than the measured[3] 1160 $\pm$ 8 keV.

The relative efficiencies of the Ge(Li) detectors for γ rays of different energies were measured by the method of Kane and Mariscotti[9] using ^{22}Na, ^{88}Y, and ^{208}Tl sources to span the energy range of interest. The fit to $\ln(\epsilon_1/\epsilon_2) = k \ln(E_2/E_1)$ was linear within statistical error. The slope of the line was found to be dependent on the source-to-detector distance. Relative efficiencies were measured at several distances and the ^{88}Y data from a calibrated Vienna source were used to establish the absolute efficiency of the detector for the sum-effect measurements. Attenuation of γ rays in the 2.5-mm aluminium housing was corrected for empirically by calibrating the relative efficiency of the detector using sources surrounded by that additional thickness of aluminium.

Relative intensities were determined by summing over photopeak areas corrected for a smoothed background in the neighborhood of the peak, correcting for (i) background contributions, (ii) sum effects, (iii) pair production in the detector, and (iv) annihilation of positrons in flight; and multiplying by the appropriate relative efficiencies. The results summarized in Table I include sta-

TABLE I. Results for the relative intensities of the four γ rays emitted by the ^{26}Al source.

γ-ray energy (keV)	Relative intensity
511	164.1 $\pm$3.2
1130	2.5 $\pm$0.2
1809	99.76
2938	0.24 $\pm$ 0.04

TABLE II. Summary of results for the decay of ^{26}Al.

Transition	Partial half-life (units of 10^{13} sec)	f_2 [a]	f_2t (sec)	$\langle G_2\rangle^2$ (F^4)
β^+ to 1809-keV level	2.75 ± 0.09 [b]	187.3 ± 2.2	(5.15 ± 0.18) × 10^{15}	0.78 ± 0.03
EC to 1809-keV level	14.9 ± 2.5	35.7 ± 0.5	(5.32 ± 0.90) × 10^{15}	0.76 ± 0.13
EC to 2938-keV level	83.6 ± 7.2	0.465 ± 0.007	(3.88 ± 0.34) × 10^{14}	10.4 ± 0.9

[a] Reference 6.

[b] Reference 4.

tistical errors only; the final results of Fig. 1 include our estimates of systematic errors as well. The EC branching ratio to the 2938-keV state is just the sum of 1130- and 2938-keV contributions to the total intensity, (2.7 ± 0.2)%. The positron branching ratio to the 1809-keV state is one half the 511-keV contribution to the total intensity (82.1 ± 2.5)%. The remaining (15.2 ± 2.5)% is the EC branching ratio to the 1809-keV state.

Corrections to the 511-keV intensity were due to several effects:

(i) The natural-background contribution, normalized to the counting time of the ^{26}Al source, was subtracted from the 511-keV photopeak area. This correction amounted to (0.8 + 0.3)%.

(ii) The sum peak at 2319.6 keV depleted the 511-keV peak. This peak depended on experimental conditions. At most, the sum correction added 0.08% to the 511-keV area and 0.62% to the 1809-keV area.

(iii) Pair production by γ rays with energy greater than $2m_0c^2$, in the Ge(Li) detector and in the sur-

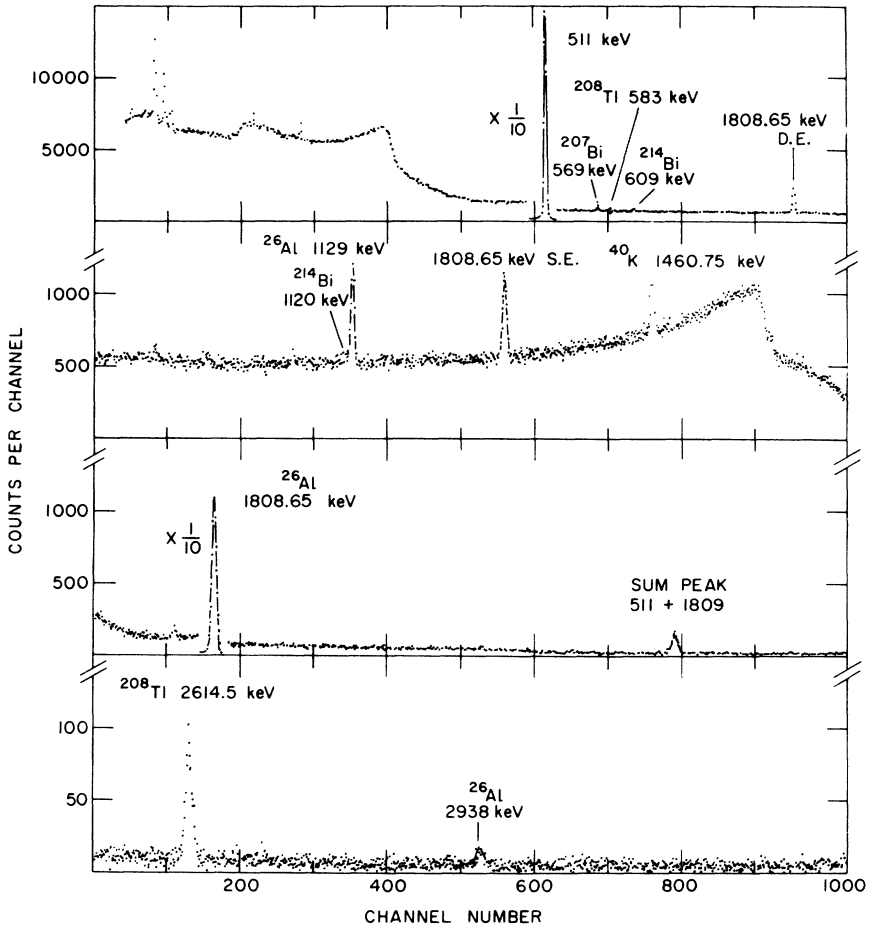

FIG. 2. γ-ray spectrum of ^{26}Al decay taken with a 30-cm^3 Ge(Li) detector. The ^{26}Al γ rays are identified; all other γ rays are also observed in the natural background.

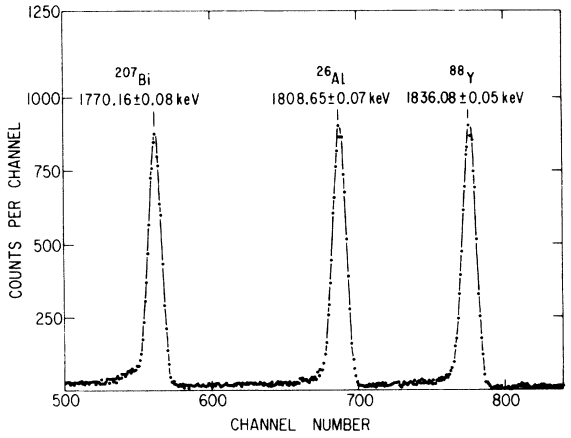

FIG. 3. Portion of a γ-ray spectrum used in obtaining accurate ^{26}Al γ-ray energies. The ^{26}Al peak energy was determined relative to the other two with energies as marked.

rounding lead shielding, added intensity to the 511-keV peak. It was estimated that, because of this effect, the γ rays of ^{26}Al contributed $(0.59 \pm 0.07)\%$ of the 511-keV intensity.

(iv) Annihilation in flight and three-quantum annihilation depleted the 511-keV peak area. The three-quantum annihilation is 0.27% of the two-quantum annihilation for positrons annihilating in solids and is independent of the stopping material.[10] The probability of annihilation in flight, however, depends on the positron kinetic energy and the average ionization potential of the stopping material.[11,12] The size of this effect was calculated by numerical integration of the positron spectrum with a total probability curve for annihilation in flight in lead[12] and then correcting to aluminum. This correction added $(2.65 \pm 0.20)\%$ to the 511-keV peak area.

(v) Contributions from positron-emitting contaminants in the ^{26}Al source itself could not be measured, but are estimated as negligible. No γ rays associated with positron-emitting contaminants could be found in the spectrum. The likely contaminants from chemical considerations have very short half-lives or are not positron emitters.

The natural background was low throughout the spectrum, with ^{40}K and ^{208}Tl as the strongest contributors. Their intensities were no greater in the ^{26}Al spectra normalized to the same counting times. A peak at 1120 keV, assignable to ^{214}Bi, was of much lower intensity than, and well resolved from, the 1130-keV peak. The peak at 510.7 keV due to ^{208}Tl could not be resolved from the 511-keV peak and was probably the main source of background in this region.

We now consider the decay of the ^{26}Mg 2938-keV level. γ-ray full-energy peak areas were measured at source-to-detector distances of 5, 15, and 25 mm to separate the small $2938 \rightarrow 1809 \rightarrow 0$ cascade sum contribution from the real 2938-keV intensity. The sum contribution, corrected for γ-γ angular-correlation effects using the $E2/M1$ mixing ratio of Häusser, Alexander, and Broude,[13] was calculated to be 11% of the total 2938-keV area at 5 mm and 5% at 25 mm. The branching ratio for decay from this state, $(10.2 \pm 2.0)\%$ to the ground state and $(89.8 \pm 2.0)\%$ via cascade, is the weighted mean of the independent determinations at the three distances. This results in good agreement with previous determinations.[13,14]

III. DISCUSSION

The partial half-life of the positron emission was previously measured as $(2.753 \pm 0.093) \times 10^{13}$ sec.[4] The positron branching ratio of $(82.1 \pm 2.5)\%$ results in a half-life of $(2.26 \pm 0.10) \times 10^{13}$ sec or $(7.16 \pm 0.32) \times 10^{5}$ yr. This is our recommended value for the ^{26}Al half-life.

In spite of a considerable effort with appreciably superior equipment, the branching ratios that we present in Fig. 1 are less accurately determined than the quoted results of 13 years ago.[1] The main reason for this, in our opinion, is that we now are more aware of the possible systematic errors which can influence measurements of this type. For instance, the results of Ref. 1 were not corrected for the annihilation in flight of positrons (at least no mention was made of such a correction) and this correction alone more than exceeds the quoted uncertainty.

The nuclear-structure information which can be extracted from the present results is summarized in Table II. The definitions of the $f_2 t$ values and the nuclear matrix elements, i.e., the $\langle G_2 \rangle^2$, are from Warburton, Garvey, and Towner,[6] where these quantities are compared with nuclear models. Our interest is in the relative $\langle G_2 \rangle^2$ for decay to the 1809-keV level. As opposed to the previous values[1,6] the $\langle G_2 \rangle^2$ determined by the β^+ and EC branches now agree. That is, the ratio of EC to β^+ is now in agreement with expectations.

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Surface Delta Interaction and Spherical Hartree-Fock-Bogoliubov Calculations

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A self-consistent procedure is outlined in which the interaction radius R of the surface δ interaction (SDI) is determined self-consistently from the spherical Hartree-Fock-Bogoliubov (HFB) field it produces. It is shown that if the surface R is determined this way, the Woods-Saxon single-particle wave functions are more or less equal in magnitude at R, as assumed in earlier works. The procedure is also shown to give numerical results for quasiparticle energies in Ni isotopes in agreement with experimental indications. Thus, the SDI model provides us with an example where the effective interaction is self-consistently determined with the field it produces.

The surface δ interaction (SDI), suggested by Moszkowski[1] as an effective interaction for nuclear-structure calculations, can be written as (for like particles)

$$V = -4\pi V_0 \frac{\delta(r_1 - R)\delta(r_2 - R)\delta(\theta_{12})}{r_1 r_2} . \quad (1)$$

The major physical argument in favor of such a simple effective interaction is that the residual interaction between nucleons (over and above the average single-particle potential) is most important at the nuclear surface R and weak in the nuclear interior. Additional justification comes from the Migdal theory[2] and the Perey effect.[3]

The radial part of the matrix element of the interaction gives the following integral:

$$g(n_1 l_1 j_1 n_2 l_2 j_2 n_1' l_1' j_1' n_2' l_2' j_2') = R^2 \psi^*_{n_1 l_1 j_1}(R)\psi^*_{n_2 l_2 j_2}(R)\psi_{n_1' l_1' j_1'}(R)\psi_{n_2' l_2' j_2'}(R) . \quad (2)$$

The way the SDI is usually employed in the literature is with the further assumption[1] that the wave functions are equal at the surface, so that g becomes a constant g_0. This assumption is approximately correct so far as absolute magnitudes are concerned, particularly if a Woods-Saxon well is used. On the other hand, as we have pointed out in a recent paper,[4] it is important to take account of the phase of the matrix functions at R, which can be taken as $(-)^n$. (n is the number of radial nodes, excluding those at $r = 0$ and $r = \infty$.) Accounting for these phases is essential for obtaining the observed coherence of the quadrupole transition matrix elements of 2^+ phonon states in spherical nuclei. The inclusion of these phases is also the point of departure from the old schematic model,[5] and one has to take the analytical form (1) seriously.

In this paper, we shall investigate the possibility of using the SDI as defined by Eq. (1) without *any* approximation. The major problem in doing this is the choice of the interaction radius R. It could approximately be taken to be the charge radius. However, more precisely, R is the radius of the extracore particles [for which the interaction (1) is the effective interaction] and should therefore be determined from the extracore-particle density