land, Nucl. Phys. A144, 161 (1970); J. B. McGrory, Phys. Letters 31B, 339 (1970).

[2]J. B. Marrion, T. W. Bonner, and C. F. Cook, Phys. Rev. 100, 91 (1955); W. W. Givens, R. C. Brease, G. C. Phillips, and A. A. Rollefson, Nucl. Phys. 43, 533 (1963); J. M. Freeman and D. West, *ibid.* 38, 89 (1962); J. J. Wesolowski, J. D. Anderson, L. F. Hanson, C. Wong, and J. W. McClure, *ibid.* 71, 586 (1965).

[3]J. W. Olness, A. R. Politti, and E. K. Warburton, Phys. Rev. 161, 1131 (1967).

[4]M. W. Green and E. B. Nelson, Phys. Rev. 153, 1068 (1967).

[5]K. Gull, B. H. Armitage, and B. W. Hooton, Nucl. Phys. A122, 81 (1968).

[6]R. D. Gill, K. Bharuth-Ram, K. P. Jackson, R. A. I. Bell, B. C. Robertson, J. L'Ecuyer, N. G. Chapman, and H. J. Rose, Nucl. Phys. A152, 369 (1970).

[7]J. D. Garret, R. Middleton, and H. T. Fortune, Phys. Rev. C 2, 1243 (1970).

[8]F. Ajzenberg-Selove and T. Lauritsen, Nucl. Phys. 11, 262 (1959); H. R. Weller and W. R. Ott, Bull. Am. Phys. Soc. 15, 1686 (1970).

[9]K. W. Kemper, D. S. Haynes, and N. R. Fletcher, Nucl. Instr. Methods 88, 289 (1970).

[10]P. M. Endt and C. Van der Leun, Nucl. Phys. A105, 1 (1967).

[11]J. H. Anderson and R. C. Ritter, Nucl. Phys. A128, 305 (1969).

[12]T. R. King, J. J. Kraushaar, R. A. Ristinen, R. Smythe, and D. M. Stupin, Nucl. Instr. Methods 88, 17 (1970).

[13]O. Haüsser, T. K. Alexander, A. B. McDonald, G. T. Ewan, and A. E. Litherland, Nucl. Phys. A168, 17 (1971).

[14]R. H. Bassel, R. M. Drisko, and G. R. Satchler, Oak Ridge National Laboratory Report No. ORNL-3240, 1962 (unpublished).

[15]A. R. Zander, M. Werby, K. W. Kemper, and N. R. Fletcher, Bull. Am. Phys. Soc. 15, 1694 (1970); A. R. Zander, K. W. Kemper, and N. R. Fletcher, Nucl. Phys. A173, 273 (1971).

[16]K. W. Kemper, D. S. Haynes, and N. R. Fletcher, Phys. Rev. C 4, 108 (1971).

[17]J. C. Hardy, H. Brunnader, J. Cerny, and J. Janecke, Phys. Rev. 183, 854 (1969).

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Photodisintegration of the Trinucleons

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We calculate the photodisintegration integrated ($\sigma_{\rm int}$) and bremsstrahlung-weighted ($\sigma_{\rm b}$) cross sections of the trinucleons (H^3 and He^3) by applying the sum rules of Levinger and Bethe. The ground state of the trinucleon is assumed to be a mixture of symmetric S state, the mixed symmetric S' state, and the D states; the radial dependence of all these states being (a) Gaussian and (b) Irving. We obtain the parameters of the radial part of these wave functions from a variational calculation of the binding energy of the triton using the velocity-dependent potential of Nestor *et al.* Our results for $\sigma_{\rm int}$ show good agreement with experiments.

A comparison with similar calculations using hard-core potentials shows the inability of $\sigma_{\rm int}$ calculations to distinguish between hard-core and velocity-dependent potentials. This feature of $\sigma_{\rm int}$ calculations, which is common at least to all the nuclei in the 1s shell, is explained in terms of the well-known range-depth relationship for the two-body potentials.

I. INTRODUCTION

Most of the earlier calculations[1-4] on the photodisintegration integrated $[\sigma_{\rm int}=\int_0^\infty \sigma(W)dW]$ and bremsstrahlung-weighted $\{\sigma_{\rm b}=\int_0^\infty [\sigma(W)/W]dW\}$ cross sections of the trinucleons are not realistic, because either the tensor component or hard core (or velocity dependence) in the two-body potential used in them is neglected. Only recently Lucas[5] and Davey and Valk[6] have employed hard-core realistic forces containing tensor components to calculate $\sigma_{\rm int}$ and $\sigma_{\rm b}$ for the three-nucleon systems. But so far no such calculation has been performed with a realistic velocity-dependent potential. In the present paper we apply the sum rules of Levinger and Bethe[7] to present the first such calculation of $\sigma_{\rm int}$ and $\sigma_{\rm b}$ of trinucleons using a velocity-dependent potential, viz., that of Nestor *et al.*[8](only singlet-even and triplet-even parts of the potential have been considered). The static central and tensor parts of the potential contain Majorana exchange force, while the velocity-dependent part is assumed to have Wigner character. We assume the ground-state wave function to be a mixture of the spherically symmetric S state, the mixed symmetric S' state, and the D states. The radial dependence of all these states is assumed to be (a) Gaussian and (b) Irving, whose parameters are determined by a variational calculation of the binding energy of the triton (H^3). On account of the charge symmetry of nuclear forces, the other trinucleon He^3 is also described by the same wave function and we expect

$$\sigma_{\rm int}(He^3)=\sigma_{\rm int}(H^3), \quad \sigma_b(He^3)=\sigma_b(H^3). \tag{1}$$

We compare our results for $\sigma_{\rm int}$ and σ_b with experiments[9] to examine to what extent our model for the trinucleons is satisfactory. We also compare our values for these cross sections with those of Lucas[5] and Davey and Valk[6] to study the equivalence of hard-core and velocity-dependent potentials in photoeffect calculations. Since our wave function is not very good – it has not been obtained from an elaborate variational calculation – the main result of our calculation is $\sigma_{\rm int}$ and not σ_b, which is sensitive to the wave function.

II. THE POTENTIAL AND THE GROUND-STATE WAVE FUNCTION

The explicit form of the potential of Nestor *et al.* is given in Ref. 8. Since all the odd-parity potentials and $\vec{L}\cdot\vec{S}$ potentials are expected to contribute very little to the triton binding energy and also to the integrated cross section,[5] we ignore these potentials and use the parameters for the singlet-even and triplet-even states of the Set B (Set B gives a better fit to the two-body data than other sets) in our calculations. We assume the effective two-body interaction in the ground state of the triton to be

$$\begin{aligned} V_{\rm eff}(r_{ij}) = \tfrac{1}{4}\{ &\sum_{i<j}[3V^t(r_{ij})+V^s(r_{ij})] \\ &+[V^t(r_{ij})-V^s(r_{ij})]\vec{\sigma}_i\cdot\vec{\sigma}_j\} \\ &+\sum_{i<j}V_T^t(r_{ij})S_{ij}\,. \end{aligned} \tag{2}$$

We follow Sachs's[10] formulation and notation for the ground-state wave function. Because in any variational calculation the P states and the antisymmetric S state are expected to have negligibly small probabilities and consequently are unimportant in sum-rule calculations,[5] we assume that the ground-state wave function of the triton is a mixture of the completely symmetric S state, the mixed symmetric S' state, and three D states and is given by

$$\psi=\sqrt{P_S}\,\psi_S+\sqrt{P_{S'}}\,\psi_{S'}+\sqrt{P_D}\psi_D\,, \tag{3}$$

in which ψ_D is a linear combination of the three D states, viz., ψ_6^m, ψ_7^m, and ψ_8^m in Sachs's[10] notation. That is

$$\psi_D=\psi_7^m+a\psi_6^m+b\psi_8^m\,. \tag{4}$$

The radial parts of the wave functions ψ_S, $\psi_{S'}$ and ψ_D are given by

$$f_i=N_i e^{-2^n\alpha_i(r^2+\rho^2)^n}\,, \quad i=S,S',D\,, \tag{5}$$

in which $n=\frac{1}{2}$ and $n=1$ give the Irving and Gaussian forms, respectively.

In our variational calculation with the Gaussian wave function we use all three D states, but in the case of the Irving wave function we use only the principal D state ψ_7^m, which is the most important D state in the binding-energy calculations.[10] We treat the α_i's, the probabilities P_S, $P_{S'}$, P_D, and a and b as variational parameters, and their best values obtained in our variational calculation of the binding energy of the triton are shown in Table I.

III. CALCULATION OF $\sigma_{\rm int}$ AND σ_b

The integrated cross section is proportional to the summed oscillator strength $\sum_n f_{0n}$ and is given by[11]

$$\sigma_{\rm int}=(2\pi^2e^2\hbar/Mc)\sum_n f_{0n}\,. \tag{6}$$

We follow the analysis of Srivastava[2] to evaluate $\sigma_{\rm int}$ for the triton in the dipole approximation and obtain

$$\begin{aligned}\sum_n f_{0n}=\frac{2}{3}-\frac{2}{3}\frac{M}{\hbar^2}\langle V^c(r_{12})r_{12}{}^2xP_{12}^M\rangle_{00} \\ -\frac{2}{3}\frac{M}{\hbar^2}\langle V_T^t(r_{12})r_{12}{}^2x'S_{12}P_{12}^M\rangle_{00}+\tfrac{8}{9}\langle\omega^c(r_{12})\rangle_{00}\,.\end{aligned} \tag{7}$$

In Eq. (7) x and x' denote, respectively, the fraction of the Majorana exchange force in the static central and tensor potentials.

We evaluate $\sum_n f_{0n}$ in Eq. (7) by using both the Gaussian and Irving form of wave function whose parameters are given in Table I. Finally, we eval-

TABLE I. Best values of the variational parameters of the wave function obtained in our variational calculation of the binding energy of the triton with the velocity-dependent potential of Nestor *et al*.

Trial wave function	α_S	$\alpha_{S'}$	α_D	a	b	Percentage probabilities of various states P_S	$P_{S'}$	P_D
Gaussian	0.08 fm^{-2}	0.11 fm^{-2}	0.13 fm^{-2}	0.2	0.08	98.0	0.5	1.5
Irving	0.57 fm^{-1}	1.20 fm^{-1}	1.02 fm^{-1}			96.0	1.0	3.0

uate σ_{int} from Eq. (6). We estimate the contribution of the $\vec{L}\cdot\vec{S}$ and $(\vec{L}\cdot\vec{S})^2$ parts and also of the odd-parity terms in the potentials of Nestor *et al.* to be about 3% and add it to the value of σ_{int} obtained in our calculation to get the value of σ_{int} shown in Table II, which also lists the experimental values of σ_{int}, as well as those obtained with hard-core potentials.

Foldy[12] has shown that the bremsstrahlung-weighted cross section is related to the mean-square charge radius $\langle r^2\rangle_{00}$ through the expression

$$\sigma_b = \tfrac{4}{3}\pi^2 \frac{e^2}{c\hbar}\frac{NZ}{A-1}\langle r^2\rangle_{00}, \tag{8}$$

where

$$\langle r^2\rangle_{00} = \frac{1}{Z}\langle \sum_p (\vec{r}_p - \vec{R})^2\rangle_{00} \tag{9}$$

$$= R_c^2 - R_p^2 . \tag{10}$$

In Eq. (9) $\vec{r}_p$ denotes the proton coordinates; and $\vec{R}$, the c.m. coordinates. The quantities R_c^2 and R_p^2 are the mean-square radii of the charge distribution in the nucleus and the proton, respectively.

For our Gaussian and Irving wave functions we use Eq. (9) to evaluate the rms radius of the triton. Equation (8) finally gives σ_b for the triton. Our values of $(\langle r^2\rangle_{00})^{1/2}$ and σ_b for H^3 along with those given by electron scattering[13] and photodisintegration experiments are shown in Table II, which also gives the experimental and theoretical values of the harmonic mean energy $W_H = \sigma_{int}/\sigma_b$, another quantity suitable for comparison with experiment. Table II also lists the values of these quantities obtained in calculations using hard-core potentials.

IV. RESULTS AND DISCUSSION

We see from Table II that both single-D-state Gaussian and Irving wave functions give nearly the same values for σ_{int}, so that σ_{int} is insensitive to the form of the wave function. The fact that the inclusion of additional D states in the Gaussian wave function causes a marked increase in the calculation of σ_{int} indicates the importance of D states in the calculation of σ_{int} of the trinucleons with a potential containing a tensor component. Also, in agreement with Lucas,[5] we find that additional D states decrease σ_b on account of their mixed symmetric nature.

Our values of the rms radius and σ_b of the triton for the Gaussian wave function are slightly lower than those given by the experiments of Fetisov, Gorbunov, and Varfolomeev,[9] and those for the Irving wave function are comparatively higher. Thus, unlike the integrated cross section the bremsstrahlung-weighted cross section is sensitive to the form of the wave function.

TABLE II. Values of σ_{int}, the rms radius, σ_b, and the harmonic mean energy.

	σ_{int} (MeV mb) Serber mixture	$\langle r^2\rangle^{1/2}_{He^3}$ (fm)	σ_b (mb)	$W_H = \sigma_{int}/\sigma_b$ (MeV)
Gaussian function with principal D state				
Present calculation	57.50	1.46	2.05	28.05
Gaussian function with all the D states				
Present calculation	65.00	1.44	2.01	32.33
Irving function with the principal D state				
Present calculation	56.55	1.88	3.41	16.58
Theory (Ref. 6)	65.00	1.59	2.36	27.50
Theory (Ref. 6)	64.40	1.60	2.40	26.80
Theory (Ref. 5)	63.64	1.75	2.92	21.57
Theory (Ref. 2)	56.00	1.92	3.50	16.00
Experiment: photodisintegration (Ref. 9)	62 ± 6	1.62	2.53 ± 0.19	24.50
Experiment: electron scattering (Ref. 13)		1.50		

Nice agreement of our results for $\sigma_{\rm int}$ with experiment shows that our wave functions and the velocity-dependent potential used by us (singlet-even and triplet-even potentials of Nestor *et al.*) provide a satisfactory model for the calculation of $\sigma_{\rm int}$ for the three-nucleon systems. The agreement with experiment is comparatively poor for σ_b because σ_b is sensitive to the wave function and our wave function is not very good.

A comparison of our results for $\sigma_{\rm int}$ with those of Lucas[5] and Davey and Valk[6] shows that it is not possible to differentiate between hard-core and velocity-dependent potentials on the basis of $\sigma_{\rm int}$ calculations for the trinucleons. This appears to be a general result, at least for nuclei in the 1s shell, and holds good not only for velocity-dependent and hard-core potentials but for any two potentials giving a good fit to the two-body data. Thus our calculations of $\sigma_{\rm int}$ for the deuteron[14] and the α particle,[15] as well as those of other authors[6, 16, 17] for these light nuclei with different potentials, justify this conclusion. In view of the fact that the expression for $\sigma_{\rm int}$ involves the potential directly, the inability of calculations of $\sigma_{\rm int}$ to distinguish between two potentials may seem surprising. However, this is a fact and finds its explanation, as shown in the Appendix, in the well-known range-depth relation, $V_0\alpha^2 = \text{constant}$, for the two-body potentials.

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APPENDIX. EQUIVALENCE OF TWO INTERNUCLEON POTENTIALS IN THE $\sigma_{\rm int}$ CALCULATIONS

The integrated cross section for any nucleus is given by[11]

$$\sigma_{\rm int} = \sigma_{\rm int}^T + \sigma_{\rm int}^{\rm ex}, \qquad \text{(A1)}$$

where

$$\sigma_{\rm int}^T = 60 \text{ MeV mb} \times NZ/A, \qquad \text{(A2)}$$

and

$$\sigma_{\rm int}^{\rm ex} = \text{Const}\langle \sum_i \sum_j r_{ij}^2\, V(r_{ij}) x P_{ij}^M \rangle. \qquad \text{(A3)}$$

In Eqs. (A1) and (A2) $\sigma_{\rm int}^T$ gives the kinetic energy contribution to $\sigma_{\rm int}$, and $\sigma_{\rm int}^{\rm ex}$ gives the contribution of the Majorana type of exchange force ($x = 0.5$) in the potential. The expression for $\sigma_{\rm int}^{\rm ex}$ involves the potential directly. Even then, the calculations of $\sigma_{\rm int}$ for nuclei in the 1s shell fail to distinguish between two different potentials.

In this connection it should be noted that a very good fit to the experimental data (see Table III) on $\sigma_{\rm int}$ for the trinucleons[9] and the α particle[18] is obtained if in the expression (A1) $\sigma_{\rm int}^T$ is calculated from the expression (A2) and the exchange contribution from the equation

$$\sigma_{\rm int}^{\rm ex}(\text{nucleus}) = (\text{number of } n\text{-}p \text{ pairs in the nucleus}) \times \sigma_{\rm int}^{\rm ex}(\text{deuteron}), \qquad \text{(A4)}$$

in which $\sigma_{\rm int}^{\rm ex}(\text{deuteron}) = 9.34$ MeV mb. [This is the value for $\sigma_{\rm int}^{\rm ex}$(deuteron) obtained by us in our calculation[14] of $\sigma_{\rm int}$ for the deuteron.] Thus each n-p pair in the trinucleons or the α particle acts independently and behaves like a quasideuteron in the photon absorption (so far as the calculation of $\sigma_{\rm int}^{\rm ex}$ is concerned). And if we can show that $\sigma_{\rm int}^{\rm ex}$(deuteron) is independent of the form of the potential, an obvious explanation results.

For the deuteron the exchange contribution is given by[14]

$$\sigma_{\rm int}^{\rm ex}(\text{deuteron}) = 15 \text{ MeV mb} \times \frac{4}{3}\frac{Mx}{\hbar^2}\langle Vr^2\rangle. \qquad \text{(A5)}$$

We next calculate the integral $\langle Vr^2\rangle$ in Eq. (A5) in the approximation of zero binding energy of the deuteron. We note that because the product Vr^2 occurs in the integrand, neither distances beyond the range of the potential (for which $V = 0$) nor too small values of r contribute to the integral, so that $\sigma_{\rm int}^{\rm ex}$(deuteron) is an intermediate-distance quantity. We assume a central square-well potential of depth V_0 and range α. In the evaluation of the

TABLE III. Values of $\sigma_{\rm int}$ for the 1s shell nuclei according to the model given by Eqs. (A1), (A2), and (A4).

Nucleus	Number of n-p pairs	$\sigma_{\rm int}$ (our model) $\sigma_{\rm int}^T$ (MeV mb)	$\sigma_{\rm int}^{\rm ex}$ (MeV mb)	$\sigma_{\rm int} = \sigma_{\rm int}^T + \sigma_{\rm int}^{\rm ex}$ (MeV mb)	$\sigma_{\rm int}$ (experimental) (MeV mb)
Deuteron	1	30.0	9.34	39.34	40.9 ± 3.4 [a]
Trinucleon	2	40.0	18.68	58.68	62 ± 6 [b]
α particle	4	60.0	37.36	97.36	95 ± 7 [c]

[a] See Ref. 16. [b] See Ref. 9. [c] See Ref. 18.

integral $\langle Vr^2 \rangle$ we have to consider only the inside wave function[10]

$$\psi = \left(\frac{2k_g}{1+k_g\alpha}\right)^{1/2} \sin\kappa_g r\,, \qquad \text{(A6)}$$

with $\kappa_g\alpha = \frac{1}{2}\pi$ and $k_g\alpha = 0.47$. We then obtain

$$\langle Vr^2 \rangle_{\text{central}} \approx 0.2\, V_0\alpha^2\,.$$

Inclusion of tensor and hard-core (or velocity-dependent) forces increases[14] $\sigma_{\text{int}}^{\text{ex}}$(deuteron), so in the presence of these forces we put

$$\langle Vr^2 \rangle = a\, V_0\alpha^2\,, \qquad \text{(A7)}$$

where $a > 0.2$ [we are in effect replacing the two-body central, tensor, and other forces in the deuteron by an effective central potential for the calculation of $\sigma_{\text{int}}^{\text{ex}}$(deuteron)]. Equation (A7) when combined with Eq. (A5) gives

$$\sigma_{\text{int}}^{\text{ex}}(\text{deuteron}) = 15\ \text{MeV mb} \times \frac{4}{3}\frac{Mx}{\hbar^2} a\, V_0\alpha^2\,. \qquad \text{(A8)}$$

Equation (A8) then explains why $\sigma_{\text{int}}^{\text{ex}}$(deuteron) is the same for any two potentials giving good fit to the deuteron properties – it is on account of the product $V_0\alpha^2$, the well-known range-depth relationship, which is a constant for any two-internucleon potentials. [Since most of the contribution to the integral $\langle Vr^2 \rangle$ comes from the plateau region ($\kappa_g\alpha = \frac{1}{2}\pi$), the fraction a is also almost constant for two potentials.] Further, Eq. (A8) combined with the expression (A4) shows why it is not possible to distinguish between two potentials on the basis of σ_{int} calculations for nuclei in the $1s$ shell.

With the known values of x and $V_0\alpha^2$ ($x = 0.5$, $V_0\alpha^2 = 1.02$ MeV b), we calculate a from Eq. (A8) for $\sigma_{\text{int}}^{\text{ex}}$(deuteron) = 9.34 MeV mb, the value required to give a good fit to the experimental value of σ_{int} for the deuteron and obtain $a = 0.38$. This value of a lies between 0.2 and 1.0 and therefore looks reasonable. Thus we conclude that the model based on Eqs. (A1), (A2), and (A4) works very well for nuclei in the $1s$ shell.

[1]M. L. Rustgi, Phys. Rev. 106, 1256 (1957).
[2]B. K. Srivastava, Phys. Rev. 137, B71 (1965).
[3]V. S. Mathur, S. N. Mukherjee, and M. L. Rustgi, Phys. Rev. 127, 1663 (1962).
[4]T. K. Lim, Nucl. Phys. A110, 417 (1968).
[5]C. Lucas, Nucl. Phys. A123, 173 (1969).
[6]P. O. Davey and H. S. Valk, Phys. Rev. 156, 1039 (1967).
[7]J. S. Levinger and H. A. Bethe, Phys. Rev. 78, 115 (1950).
[8]C. W. Nestor, Jr., K. T. R. Davies, S. J. Krieger, and M. Baranger, Nucl. Phys. A113, 14 (1968).
[9]V. N. Fetisov, A. N. Gorbunov, and A. T. Varfolomeev, Nucl. Phys. 71, 305 (1965).
[10]R. G. Sachs, *Nuclear Theory* (Addison-Wesley, Cambridge, Mass., 1955).
[11]J. S. Levinger, *Nuclear Photo-disintegration* (Oxford U. P., New York, 1960).
[12]L. L. Foldy, Phys. Rev. 107, 1303 (1957).
[13]H. Collard, R. Hofstadter, E. B. Hughes, A. Johansson, M. R. Yearian, R. B. Day, and R. T. Wagner, Phys. Rev. 138, B57 (1965).
[14]S. S. Raghavan and B. K. Srivastava, to be published.
[15]H. L. Yadav, D. Mahanti, and B. K. Srivastava, J. Phys. A 4, 49 (1971).
[16]J. G. Lucas and M. L. Rustgi, Nucl. Phys. A112, 503 (1968).
[17]M. L. Rustgi and S. N. Mukherjee, Phys. Rev. 129, 717 (1963).
[18]A. N. Gorbunov and V. Spiridonov, Zh. Eksperim. i Teor. Fiz. 33, 21 (1957) [transl.: Soviet Phys. – JETP 6, 16 (1958)]; Zh. Eksperim. i Teor. Fiz. 34, 862, 866 (1958) [transl.: Soviet Phys. – JETP 7, 596, 600 (1958)].