H. J. Lipkin, Phys. Rev. Letters 27, 432 (1972), and references therein.]

[25]J. Delorme and M. Rho, Phys. Letters 34B, 238 (1971).

[26]At present it is extremely difficult to obtain an angular experimental resolution capable of distinguishing (a) from (b); however, as will be clear in the following, the characteristics of (a) and (b) are radically different and rather interesting.

[27]J. D. Walecka, in *Proceedings of the Williamsburg Conference on Intermediate Energy Physics, February 1966* (College of William and Mary, Williamsburg, Virginia, 1966); see also J. M. Eisenberg and H. J. Weber, Phys. Letters 34B, 107 (1971).

[28]Quoted in Ref. 13.

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K^--Nucleus Optical Potential in K^--Mesonic Atoms*

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Direct numerical calculations in K^--mesonic atoms have shown that the perturbation theory commonly used is inadequate. The theory fails to describe the very strong repulsive (level-shift) effect due to the absorptive part of the optical potential. Because of this large repulsive effect, the optical potentials best fitted to the recent level-shift and width data are found to be attractive. The potential may be very attractive, but not enough to produce a K^--nucleus bound state by the strong interaction against the strong repulsive effect of the large imaginary part: The latter works as if it were a hard core. Taking the $3d$ state of the K^--S^{32} atom as a model, we also examine how well we can determine the nuclear-density parameters from the K^--mesonic-atom spectra within the optical-potential model.

I. INTRODUCTION

Until last year, data from yield measurements were the only information available about K^--mesonic-atom spectra. Very comprehensive measurements were reported in the K^--mesonic atoms of numerous nuclei by the Berkeley group,[1] and their data were examined theoretically by various authors.[2,3] Of major interest was to determine the nuclear-density distribution, particularly near the surface. The yield measurements revealed the absorptive nature of the K^--nuclear interaction. The absorption rates measured seem to be understood, within errors in the measurements, in terms of the lowest-order pseudopotential using a nuclear-density distribution of the same form as the charge distribution. Basically, these authors used perturbation theory to compute the absorption rates from their pseudopotentials utilizing known K^--nucleon s-wave scattering amplitudes.

Since last year, energy-level shifts and widths have been measured in K^--S^{32} [4] and K^--Cl^{35} [5] atoms. Contrary to the case of the yield data alone, it is possible to determine the real part of the optical potential as well as its imaginary (absorptive) part from the level-shift and width data. From a purely computational point of view, the yield or width data alone are insufficient to determine accurately even the imaginary part because of an effect on the absorption from the real part of the potential. It is usually regarded to be a small second-order perturbative effect; however, even in π^--mesonic atoms the effect has been observed to be not at all small.[6] Small deviations in mesonic-atom spectra do not guarantee that the perturbation theory can be applied successfully, just as in scattering problems, small phase shifts do not guarantee validity of the Born approximation.

In view of the above point, we have made a phenomenological study of the shift and width data in order to examine the role of the optical potential in the K^--mesonic-atom spectra. The entire study was done by solving the Klein-Gordon equation numerically. Two forms of the equation, one as a function of radial variable r and the other as a function of $1/r$, were integrated using the fourth-order Runge-Kutta method from $r=0$ and $r\to\infty$, respectively. The best parameter search was then made for the potential parameter $\bar{A}$, described in Sec. II, in order to match the logarithmic derivatives of two wave functions near a location where the magnitude of the wave function becomes maximum. This method is similar to the method used in π^--mesonic-atom calculations.[7] We have used no approximation such as perturbation theory except when we have examined its limitations.

As a result of the calculations, we have found that the effect of the imaginary part of the poten-

tial is repulsive and very strong and that perturbation theory is inadequate for the atomic levels which have shifts and widths directly observable. The effect of the real part of the potential on level widths is very appreciable when the interaction is repulsive, but the effect is not very appreciable when the interaction is attractive. In Sec. II, we show that the observed level shifts yield attractive optical potentials as a result of the strong repulsive effect of the imaginary part. Accordingly, the the real parts of these attractive optical potentials do not influence appreciably calculations of the widths: For these potentials perturbation theory gives reasonable absorption rates, although it gives quite unreasonable shifts. This assertion would change, of course, if the potential should turn out to be repulsive by analyses of more accurate data in the future. The limitations of perturbation theory are discussed in Sec. III.

One of the major interests in the investigation of the K-nucleus interactions lies in the investigation of the nuclear-surface structure.[8] By taking the $3d$ state of the K^--S^{32} atom as an example, we examine, numerically, in Sec. IV how well we could determine the nuclear-structure parameters solely from the knowledge of the K^--mesonic-atom spectra.

If the K^--nucleus interaction is indeed attractive, an interesting question then arises whether a K^--*nucleus* bound state of the strong interaction exists. The question is examined in Sec. V by use of the K^--nucleus optical potential. In Sec. VI we list the conclusions obtained in this work.

After completion of our work,[9] we learned that Krell obtained conclusions somewhat similar to ours by also performing numerical calculations.[10] Some of our results confirm his results and vice versa.

II. K^--NUCLEUS OPTICAL POTENTIAL

The form of the optical potential[2] used in our calculation is

$$V(r)=\frac{4\pi\hbar^2}{2\mu_K}\left(1+\frac{M_K}{M_N}\right)\bar{A}\rho(r), \tag{1}$$

where M_K and M_N are the masses of the K^- meson and the nucleon, respectively, and μ_K is the reduced mass of the K^--mesonic atom. The nuclear-density function $\rho(r)$ is written as a function of the density parameters (c,t)

$$\rho(r)=\rho_0\{1+\exp[(r-c)4\ln 3/t]\}^{-1}, \tag{2}$$

and $\rho(r)$ is normalized to the mass number of the nucleus. If the lowest-order pseudopotential is adopted, the parameter $\bar{A}$ is the K^--nucleon scattering lengths averaged over the nucleons in the nucleus[11]:

$$\bar{A}(\text{pseudopot})=0.50\pm0.02-i(0.59\pm0.02)\ \text{F}. \tag{3}$$

In our analysis, $\bar{A}$ is to be determined phenomenologically. Equation (1) assumes the same form for the proton and neutron distributions: The assumption does not imply our intention to discard some recent attractive suggestions concerning the nuclear distribution,[12] but is merely necessary to keep the number of free parameters at a minimum, since only a few experimental data are currently available. As discussed in Sec. IV, it would perhaps be better to examine the question of the detailed distribution more closely when more data are accumulated.

The $4f\rightarrow 3d$ transition energy observed in the K^--S^{32} $(N=Z)$ atom is $161.76\pm0.25-i(1.1\pm0.3)$ keV.[4] The transition energy calculated for all electromagnetic effects is 162.13 keV, which includes the Klein-Gordon transition energy for the point charge, 161.59 keV,[13] the vacuum-polarization effect obtained by a standard perturbation calculation,[14] 0.983 keV, the finite-charge-distribution effect, −0.014 keV, and the electron-screening effect, ~0.0002 keV.[15] The effect of electromagnetic nuclear excitation is believed to be negligible for light nuclei.[16] By numerically solving the Klein-Gordon equation, we find the potential parameter $\bar{A}$ yielding the difference in the observed and electromagnetic ones to be[17]

$$\bar{A}(S^{32})=-0.75\pm0.57-i(0.85\pm0.42)\ \text{F} \tag{4}$$

for $(c,t)=(2.93, 2.45)$ F, which gives an rms radius of 3.07 F.[18]

After the completion of our work, the measurement of the $4f\rightarrow 3d$ transition energy in the K^--Cl^{35} atom has been reported[5] to be $182.41\pm0.40-i(1.96\pm0.50)$ keV. The transition energy calculated for all electromagnetic effects is 183.35 keV, which includes the Klein-Gordon energy, 182.23 keV, the vacuum-polarization effect, 1.14 keV, and the finite-charge-distribution effect, −0.02 keV. The electron-screening and nuclear-excitation effects are considered to be negligible as in the K^--S^{32} atom. The potential parameter $\bar{A}$ yielding the difference in the transition energies is found to be

$$\bar{A}(Cl^{35})=-0.76\pm0.72-i(1.06\pm0.48)\ \text{F} \tag{5}$$

for $(c,t)=(3.07, 2.60)$ F, which gives an rms radius of 3.24 F.

The values of $\bar{A}$ for K^--S^{32} and K^--Cl^{35} atoms are essentially equal within their errors that originated from the experiments: The nuclear isospin dependence in $\bar{A}$ is difficult to evaluate from these values. Perhaps the best way to summarize our

results would be to take an arithmetic average of Eqs. (4) and (5) to regard the Cl^{35} nucleus as a nucleus of $N=Z=17.5$ The result is

$$\bar{A}(\text{av}) = -0.75 \pm 0.45 - i(0.94 \pm 0.32)\ \text{F}. \quad (6)$$

When a square-well potential of radius $1.1A^{1/3}$ F is used, the potential equivalent to $\bar{A}$(av) above has a depth

$$V = -103 \pm 62 - i(130 \pm 44)\ \text{MeV}. \quad (7)$$

The imaginary part of $\bar{A}$ in Eqs. (4)–(6) obtained from K^--mesonic data is somewhat larger than that of the average K^--nucleon scattering lengths in Eq. (3). This may be the effect of K^--absorption by two nucleons discussed in the past[19] and, as will be seen below, may also be that of the multiple-scattering contribution in the pseudopotential.

In contrast to the imaginary part, the real part has a large discrepancy: Re$\bar{A}$ in Eqs. (4)–(6) is negative, whereas ReA(pseudopot) is positive. In general, if an interaction is weak, the positive scattering length in our convention[17] means a repulsive interaction potential. If it is not weak, the sign of the scattering length alone does not determine whether the potential is attractive or repulsive. It is well known that the K^--nucleon interaction below the elastic threshold is dominated by the bound state $\Lambda(1405)$. Therefore, the average K^--nucleon scattering length, Re$\bar{A}$(pseudopot) is positive, not because the K^--nucleon interaction is repulsive, but very likely because it is attractive.[20] This implies that the averaged K^--nucleon potential is very much stronger than we might expect from the magnitude of the K^--nucleon scattering lengths and that the lowest-order pseudopotential may well be inadequate. In fact, a multiple-scattering expansion of the K^--nucleus t matrix in terms of K^--nucleus t matrices is found to diverge upon an approximate evaluation.[21] What we have described so far may give the impression that the problem is rather intricate. In fact it is not: It could be summarized crudely, but simply, as a collection of attractive K^--nucleon potentials yielding an attractive K^--nucleon interaction. This would be a reasonable description, since it is the basic ansatz in formal multiple-scattering theory[22] that the K^--nucleus potential operator is a sum of K^--nucleon potential operators. The relatively strong attractive interaction may raise a question whether a strange bound state of K^--nucleus exists owing to the strong interaction. As seen in Sec. IV, this is very unlikely because of the large value of $|\text{Im}\bar{A}|$ that works effectively as a repulsive hard core. Our result of a negative Re$\bar{A}$ appears to be reasonable.

Strictly speaking, multiple-scattering theory tells us that Re$\bar{A}$ in the pseudopotential is an average of the K-nucleon amplitude evaluated at the energy where the K-nucleon interaction actually occurs in the nucleus, not in free space. That is, a correction due to the binding effect of the nucleons in the nucleus ought to be included. This correction has been considered to be very important in the K^--nucleus interaction[23] because of the fact that the K-nucleon amplitude varies very rapidly below the elastic threshold because of the existence of $\Lambda(1405)$.[24] In fact, the rapid variation makes the proper evaluation of the binding effect very difficult and the pseudopotential method may well be questionable.[25] Recently, however, a model calculation on this problem has been reported[26] using a soluble Faddeev three-body equation. It was found that the K-nucleon amplitude with a bound nucleon at the elastic threshold varies little with variation of the nuclear binding energy. That is, the effective K-nucleon amplitude in the nucleus is influenced only slightly by the existence of $\Lambda(1405)$. This is a very important result also in connection with higher-order contributions in the pseudopotential. This question is under further study.

It is unfortunate that no very low-energy K^- scattering data are available for reasonably heavy nuclei. The only data available at very low energy are those of K^--He^4 scatterings. The data, however, appear not to be accurate enough to determine the sign of the interaction, although the data do seem to show that the K^--He^4 interaction is very absorptive and has a negligible nonlocal part.[27] Nuclear-emulsion data are also available in the somewhat high energy range of $T_K = 95$–125 MeV.[28] The result of the analysis in Ref. 28, converted into our convention, is found to yield

$$\bar{A}(\text{Emulsion}) = -0.24^{+0.20}_{-0.16} - i0.35\ \text{F}. \quad (8)$$

This value is smaller than our results, but does show that the K^--nucleus potential is likely to be attractive.

III. ATOMIC SPECTRA DESCRIBED BY THE OPTICAL POTENTIAL

In Sec. II we found the local optical potentials that properly describe the observed $3d$ energy levels of K^--S^{32} and K^--Cl^{35} atoms. If the local form of the potential is a correct description of the K^--nucleus interaction, our potentials should describe properly other states of the K^--mesonic atoms considered and also various states of other K^--mesonic atoms. In this section, we compute the various atomic spectra using the local potentials and examine their characteristics.

First, we do a model calculation on the energy-

TABLE I. A model calculation of the level shifts and widths in the K^--S^{16} atom for different values of the potential parameter $\bar{A}$. ΔE (exact) are the results of the numerical calculation and ΔE (perturbation) are those of the perturbation theory.

State	$\bar{A}$ (F)	ΔE (exact)	ΔE (perturbation)
$3d$	$-0.6-i0.8$	$0.355-i1.012$ keV	$-0.959-i1.279$ keV
	$0-i0.8$	$0.477-i0.674$ keV	$0-i1.279$ keV
	$+0.6-i0.8$	$0.721-i0.440$ keV	$+0.959-i1.279$ keV
$4f$	$-0.6-i0.8$	$-0.389-i1.798$ eV	$-1.114-i1.485$ keV
	$0-i0.8$	$0.343-i1.293$ eV	$0-i1.485$ eV
	$+0.6-i0.8$	$1.084-i1.106$ eV	$+1.114-i1.485$ keV
$5g$	$-0.6-i0.8$	$(-0.594-i1.071)\times10^{-3}$ eV	$(-0.701-i0.934)\times10^{-3}$ eV
	$0-i0.8$	$(0.081-i0.914)\times10^{-3}$ eV	$(0-i0.934)\times10^{-3}$ eV
	$0.6-i0.8$	$(0.707-i0.822)\times10^{-3}$ eV	$(+0.707-i0.934)\times10^{-3}$ eV

level shifts and widths of $3d$, $4f$, and $5g$ states in the K^--S^{32} atom by two computational methods: exact numerical calculation and perturbation theory. Since the latter is commonly used, it is perhaps worthwhile to examine whether the theory is adequate in the K^--mesonic-atom problems. As seen in Sec. II, the real part of the potential is rather difficult to determine. We calculate the shifts and widths for three values of $\mathrm{Re}\bar{A}$, ±0.6 and 0.0 F, with the same value of $\mathrm{Im}\bar{A}=-0.8$ F. The results of the calculation are shown in Table I. In the table we see that the perturbation theory is inadequate in most cases for the lower levels. In particular, as pointed out in Sec. II, it fails to provide the correct sign of the $3d$ shift when the interaction is attractive. It also gives a magnitude too large by a factor of 3 for the $3d$ width when the interaction is repulsive. Clearly the interference between the real and imaginary parts of the potential is too strong to be handled by perturbation theory. In the $4f$ and $5g$ states,[29] we observe that the interference diminishes in higher states; thus the perturbation-theory predictions improve quite appreciably. As a consequence, we find the phenomenon that the signs of the level shifts are different for the $3d$ state and the higher ones. This phenomenon ought to be differentiated from the possible sign change in energy levels of π^--mesonic atoms[30] due to competition between local and nonlocal parts of the potential.

Second, using the optical potential with the parameter of Eq. (4), we calculate the energy-level shifts and widths in some other K^--mesonic atoms as well as those of the $3d$ state in the K^--S^{32} atom. From the results of this calculation, we can confirm that the preceding observations are also valid in other mesonic atoms. When other K^--mesonic-atom measurements become available, they could also be compared with our calculated results: If they should agree, it would be a justification of the local potential as well as a confirmation of the current measurements. Some of our calculated results are shown in Table II. In the table we see that the features observed in the model calculation for the K^--S^{32} atom are also valid for other K^--mesonic atoms. The $3d$ level in the K^--Cl^{35}

TABLE II. Some examples of the energy-level shifts and widths predicted by the nonlocal potential best fitted to the CERN data on the $3d$ state in the K^--S^{32} atom. ΔE (exact) are the results of the numerical calculation and ΔE (perturbation) are those of the perturbation theory.

Nucleus	(c, t) value rms radius	(State) Klein-Gordon energy (keV)	ΔE (exact)	ΔE (perturbation)
He^4	Gaussian 1.47 F	(1s) 24.873	$6.46-i2.40$ keV	$-25.33-i28.48$ keV
		$(2p)$ 6.456	$0.2-i0.9$ eV	$-0.6-i0.7$ eV
S^{32}	(2.93, 2.45) F 3.07 F	$(4f)$ 74.553	$-0.5-i2.0$ eV	$-1.4-i1.6$ eV
Cl^{35}	(3.07, 2.60) F 3.24 F	$(3d)$ 182.213	$0.75-i1.95$ keV	$-2.28-i2.57$ keV
		$(4f)$ 84.286	$-1.1-i4.8$ eV	$-3.4-i3.8$ eV
Ni^{58}	(3.96, 2.45) F 3.72 F	$(4f)$ 230.261	$0.100-i0.545$ keV	$-0.512-i0.575$ keV

atom agrees very well with the measured data used in Sec. II, although this is to be expected from the results in that section.

IV. DEPENDENCE ON NUCLEAR-DENSITY PARAMETERS

The K^--mesonic atoms have been considered to perhaps be a useful tool to investigate the nucleon distribution in the nuclear-surface region.[8] We performed a model calculation on the $3d$ state in the K^--S^{32} atom in order to examine how well the atomic spectra can determine the form of the nuclear distribution. We varied each nuclear-density parameter, c and t, by 5% from their central values $(c,t)=(2.93, 2.45)$ F. The variations are done in two different ways: First, we examined changes in $\bar{A}$ caused by the variations when the level shift and width (ΔE) were fixed to the central value of the measured data. Second, we examined changes in ΔE caused by the variations when $\bar{A}$ was fixed to the best-fit central value.

The results of the calculation are summarized in Table III. In the second column of the table we show how the rms radius of the nuclear-density distribution changes corresponding to the variations of the parameters (c,t). We see that the variations considered here amount to a little less than 0.1 F in the rms radius. In the third column we describe the changes in $\bar{A}$ when ΔE is fixed. We observe that Δt changes $\mathrm{Re}\bar{A}$ 3 to 7 times more than $\mathrm{Im}\bar{A}$. This shows that the determination of the t value requires an accurate knowledge of $\mathrm{Im}\bar{A}$ when ΔE is rather accurately known. In the fourth column we describe the changes in ΔE when $\bar{A}$ is fixed. We observe that Δt changes the shift about 6 times more than the width. This shows that the determination of the t value requires an accurate knowledge of the level shift when $\bar{A}$ is rather accurately known. In both methods of variation, the changes in the c value result in deviations evenly split between $\mathrm{Re}\bar{A}$ and $\mathrm{Im}\bar{A}$, and also between the shift and spread.

This model calculation shows an intricate relation among the density parameters (c,t), the potential parameter $\bar{A}$, and the level shift and width ΔE. In reality, the measured ΔE carries a fairly large experimental uncertainty in the current art. As discussed in Sec. II, the value of $\bar{A}$ is not yet determined well by theoretical means such as multiple-scattering theory. Keeping this in mind, let us look at the relation between the three quantities more closely: From Eq. (4) in Sec. II, we learn that, when (c,t) values are fixed to the central ones, an uncertainty of about 0.3 keV in both the shift and width results in an uncertainty of about about 0.4 ~ 0.6 F in $\bar{A}$. This 0.3 keV is larger, by an order of magnitude, than the smallest variation of ΔE (shown in Table III) as the result of the variations in the t value. From high-energy electron scattering and μ^--atom data we now have accurate knowledge of the (c,t) values of the nuclear charge distribution. With the ansatz that the basic nuclear-matter distribution is similar to the charge distribution, we could consider that accuracy in the determination of the t value up to 10 ~ 20% would be necessary to learn the nuclear surface structure. As seen above, to determine the t value to this accuracy, we would require a knowledge of $\bar{A}$ to within about 0.1 F. We could thus conclude that the determination of $\bar{A}$ has to be done very accurately to obtain information about the nuclear-surface structure from the K^--mesonic-atom spectra: The present knowledge of $\bar{A}$ is very poor. We note, however, that when an abundance of K^--mesonic-atom data becomes available in the future, it is possible that this problem could be reduced to simply finding the best χ^2 fit as proposed in π^--mesonic atoms.[31] Even in that situation some theoretical knowledge of $\bar{A}$ certainly would be required to ensure that the phenomenological approach is sound.

V. ABSORPTIVE OPTICAL POTENTIAL AND THE QUESTION OF A K^--NUCLEUS BOUND STATE

In Sec. II we observed that the large imaginary part of the optical potential obtained yields the repulsive effect as if it were a hard-core potential. In order to understand this property more clearly, we calculate the K^--S^{32} scattering length for vari-

TABLE III. The effects of variations in the nuclear-distribution parameters (c,t).

Variations in (c,t)	Corresponding variations in the rms radius (F)	Deviations in $\bar{A}$ (F)	Deviations in ΔE (keV)
c increases	+0.09	$+0.120 + i\,0.109$	$0.056 - i\,0.076$
c decreases	−0.08	$-0.135 - i\,0.110$	$-0.050 + i\,0.071$
t increases	+0.07	$+0.184 + i\,0.058$	$0.018 - i\,0.118$
t decreases	−0.06	$-0.232 - i\,0.034$	$-0.017 + i\,0.110$

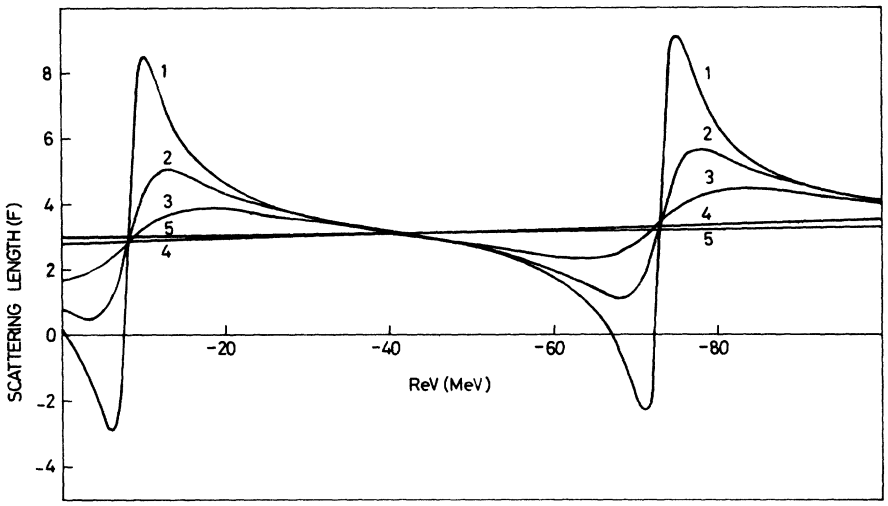

FIG. 1. K^--S^{32} s-wave scattering length calculated for various square wells $V = \mathrm{Re}V + i\,\mathrm{Im}V$. Curves (1)–(5) are for an ImV of (1) −2 MeV, (2) −5 MeV, (3) −10 MeV, (4) −40 MeV, and (5) −80 MeV, while ReV is varied continuously from 0 to −100 MeV.

ous strengths of the optical potential. To simplify the problem we choose the form of a complex square-well potential, V, to describe the strong interaction[32] with the well depth of ReV = 0 to −100 MeV and ImV = −2, −5, −10, −40, and −80 MeV. The scattering lengths calculated for these potentials are shown in Fig. 1. We note that in this calculation the Coulomb interaction is completely removed and that the scattering lengths are purely due to the strong interaction. In the figure we see that for $|\mathrm{Im}V| \geq 10$ MeV the scattering length becomes very close to the value of the potential radius regardless of the value of ReV. The large imaginary part of the potential indeed behaves as a hard-core potential.

To confirm this analogy more firmly, we search for the ranges of ReV and ImV which yield at least one s-wave bound state. The condition for this is that there exists at least one positive root κ satisfying the relation

$$1/a + \tfrac{1}{2}\kappa^2 r_0 - \kappa = 0\,, \tag{9}$$

where a and r_0 are the s-wave scattering length and effective range. The ranges of ReV and ImV are shown in Fig. 2. It is clear from the figure that unless $|\mathrm{Im}V| < 15$ MeV we can not have an s-wave bound state of the K^--nucleus system. It is well known that in Schrödinger's equation with a common local potential for all l we can generally[33] show that

$$\frac{\partial (l+1/2)^2}{\partial E} > 0\,.$$

Therefore, if there is no s-wave bound state, there will be no bound state for higher angular momentum states if the K^--nucleus interaction is local as has been assumed here. But, if there should be a bound state, it would be a composition of $\Lambda(1405)$ and other nucleons. This is analogous to a bound state of $\Delta(1236)$, the 3-3 resonance, and other nucleons whose existence has been proposed, for example, in π-nucleus scattering near the 3-3 resonance energy.[34] The bound state of $\Lambda(1405)$ and the nucleons would be an interesting phenomenon if it should exist, but the present K^--mesonic-atom data do not appear to support this possibility.

Despite this negative result, it still may be instructive to investigate how the K^--mesonic atom would behave if the bound state should become probable or ImV should be small. In order to do this, we first calculate the level shifts and widths (ΔE) for small values of Im$\bar{A}$ with various values of Re$\bar{A}$ by taking the $3d$ state of the K^--S^{32} atom again as a model. Figures 3 and 4 show the level shifts and widths, respectively, for Im$\bar{A}$ = −0.1, −0.2, −0.4, and −0.8 F when Re$\bar{A}$ is varied from

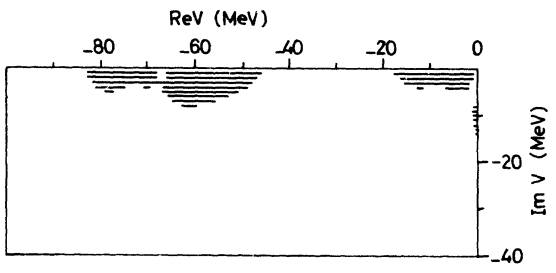

FIG. 2. Depth of the K^--S^{32} square-well potential which gives at least one bound state. It is shown by the shadowed area.

-0.6 to $+0.6$ F. In these figures, the results using perturbation theory are also displayed. Near $\mathrm{Re}\bar{A} = -0.3$ to ~ -0.5 F we observe dips appearing out of phase in the shifts and widths: They are deeper at smaller values of $|\mathrm{Im}\bar{A}|$. We observe that the dips practically disappear when $|\mathrm{Im}\bar{A}|$ is as large as 0.8 F which is close to the observed value.

In order to understand the physical nature of the dip, we draw the $3d$-state radial wave function (times the radius) $U(r)$ of the K^--S^{32} atom for $\mathrm{Re}\bar{A} = -0.40$, -0.45, and -0.50 F with $\mathrm{Im}\bar{A} = -0.10$, where the dip appears deeper than for other values in Figs. 3 and 4. In Fig. 5 we show the real and imaginary parts ($\mathrm{Re}U$ and $\mathrm{Im}U$, respectively) and the density distributions ($|U|^2$) of the wave functions. We observe that in addition to the usual bump of the Coulomb bound state, $\mathrm{Re}U$ and $|U|^2$ have a bump near $r = 3 \sim 4$ F and that beyond the bump, $\mathrm{Re}U$ changes its sign after $\mathrm{Re}A$ becomes -0.45 F.[35]

These results suggest the existence of two competing bound states; one due to the Coulomb interaction and the other due to the strong interaction. Krell has observed an oscillating structure in ΔE as a function of $\mathrm{Re}\bar{A}$ over a wider range of $\mathrm{Re}\bar{A}$ than we have considered. Our Figs. 3 and 4 show a part of this oscillating structure.[36] This structure has recently been investigated in terms of a model calculation.[37] It was found that the structure is indeed due to the competing process of the inner (Coulombic) and outer (of the strong interaction) states. At any rate, the phenomenon of this interesting structure seems to be somewhat academic if the observed level shifts and widths are indeed of correct magnitudes.

We have carefully examined the problem of the K^--nucleus bound state due to the strong interaction assuming $-\mathrm{Re}V$ is less than or in the neighborhood of 100 MeV. Judging from the recent experimental data,[4,5] this seems to be a reasonable range. We note, however, that if future experimental data should reveal that $-\mathrm{Re}V$ is exceedingly large, bound states would be feasible due to the attractive effect of $\mathrm{Re}V$ overcoming the repulsive effect of $\mathrm{Im}V$. In this case we would ob-

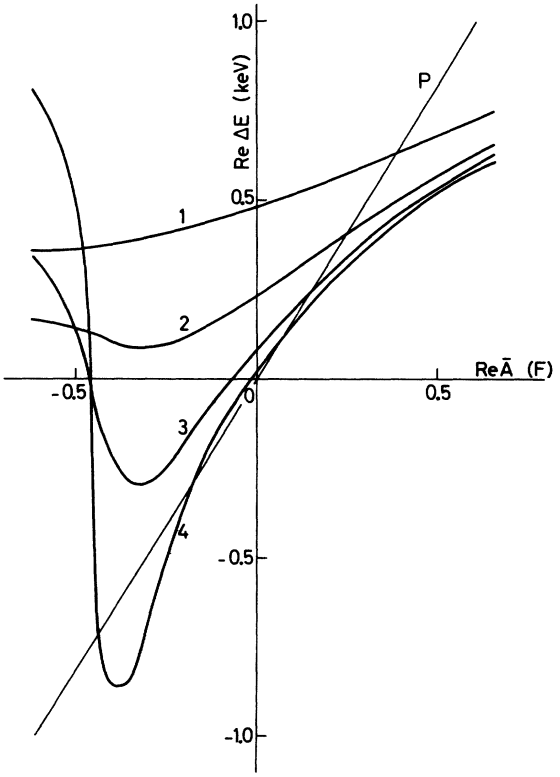

FIG. 3. The energy level shift $\mathrm{Re}\Delta E$ of the $3d$ state in the K^--S^{32} atom for various values of the potential parameter $\bar{A} = \mathrm{Re}\bar{A} + i\,\mathrm{Im}\bar{A}$. Curves (1)–(4) are $\mathrm{Re}\Delta E$ for an $\mathrm{Im}\bar{A}$ of (1) -0.8 F, (2) -0.4 F, (3) -0.2 F, and (4) -0.1 F, while $\mathrm{Re}A$ is varied continuously from -0.6 to 0.6 F. The thin line labeled P is $\mathrm{Re}\Delta E$ calculated with perturbation theory (the same for any value of $\mathrm{Im}\bar{A}$).

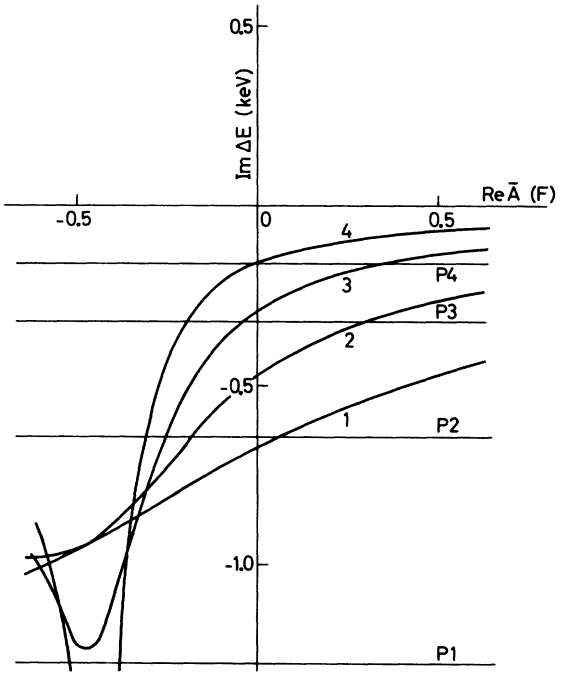

FIG. 4. One half of the energy level width, i.e., $\mathrm{Im}\Delta E$, of the $3d$ state in the K^--S^{32} atom for various values of the potential parameter $\bar{A} = \mathrm{Re}\bar{A} + i\,\mathrm{Im}\bar{A}$. Curves (1) –(4) are $\mathrm{Im}\Delta E$ for an $\mathrm{Im}\bar{A}$ of (1) -0.8 F, (2) -0.4 F, (3) -0.2 F, and (4) -0.1 F, while $\mathrm{Re}\bar{A}$ is varied continuously from -0.6 to 0.6 F. The thin lines P1–P4 are $\mathrm{Im}\Delta E$ calculated with perturbation theory for an $\mathrm{Im}\bar{A}$ of (P1) -0.8 F, (P2) -0.4 F, (P3) -0.2 F, and (P4) -0.1 F.

serve at least a few oscillating structures in the $\mathrm{Re}\bar{A}$-ΔE relation.

VI. CONCLUSIONS

In the following we summarize the results of our phenomenological study. We remind the reader that they are based upon the currently available K^--S^{32}- and K^--Cl^{35}-atom data under the assumption of a local K^--nucleus optical potential:

(1) Perturbation theory is inadequate for most of the low angular momentum states; in particular, it fails to describe the large repulsive effect of the imaginary part of the potential.

(2) With a large imaginary part, the potential is more likely to be attractive.

(3) The level shifts and widths depend appreciably upon the nuclear-density parameters in the optical potential; however, the strengths of the potential would have to be known fairly accurately in order to extract any detailed information of the parameters.

(4) Because of the strong repulsive nature of the

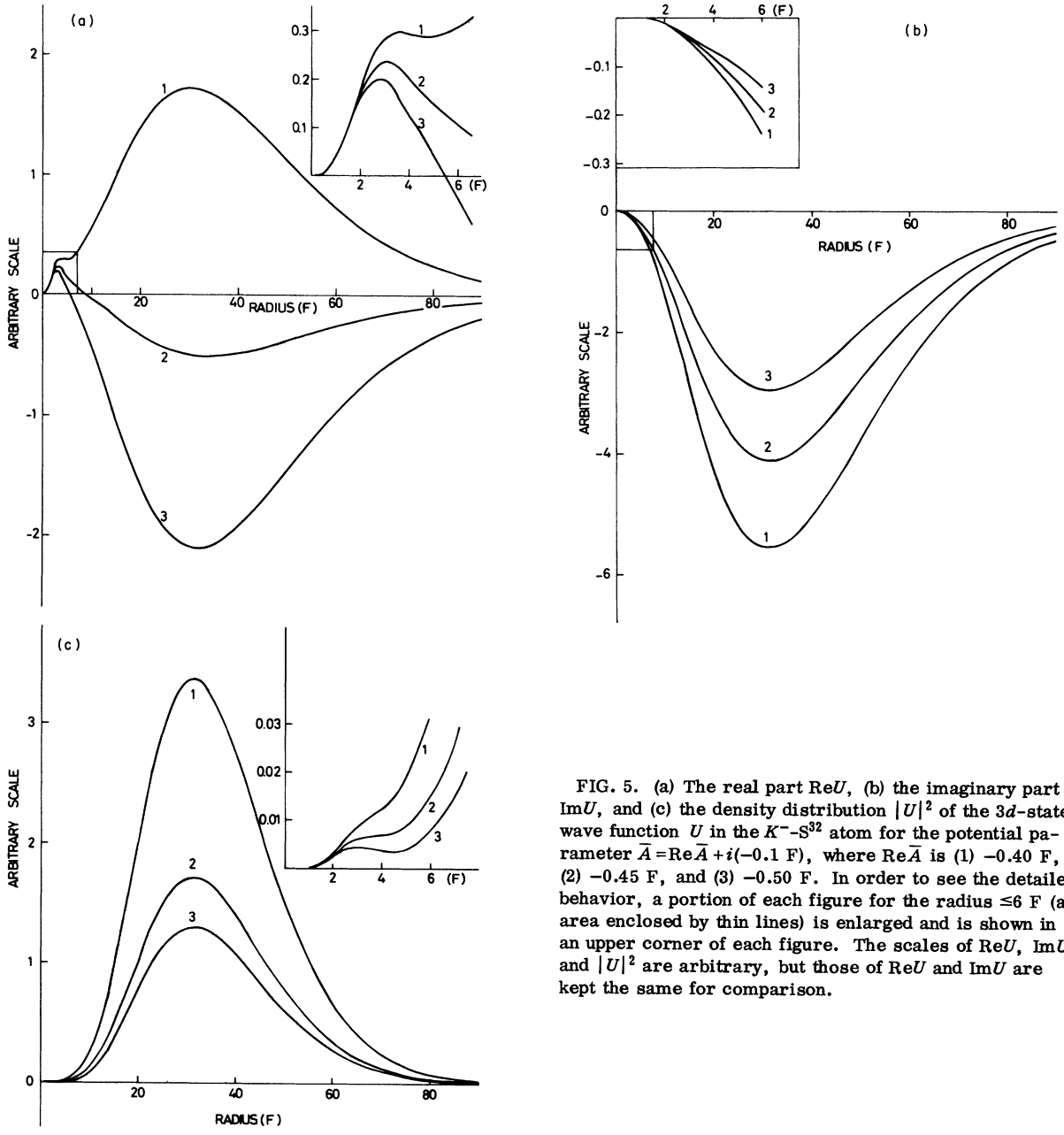

FIG. 5. (a) The real part $\mathrm{Re}U$, (b) the imaginary part $\mathrm{Im}U$, and (c) the density distribution $|U|^2$ of the $3d$-state wave function U in the K^--S^{32} atom for the potential parameter $\bar{A}=\mathrm{Re}\bar{A}+i(-0.1\ \mathrm{F})$, where $\mathrm{Re}\bar{A}$ is (1) -0.40 F, (2) -0.45 F, and (3) -0.50 F. In order to see the detailed behavior, a portion of each figure for the radius ≤ 6 F (an area enclosed by thin lines) is enlarged and is shown in an upper corner of each figure. The scales of $\mathrm{Re}U$, $\mathrm{Im}U$, and $|U|^2$ are arbitrary, but those of $\mathrm{Re}U$ and $\mathrm{Im}U$ are kept the same for comparison.

imaginary part of the potential, it seems unlikely that a bound state due to the K^--nucleus strong interaction exists.

ACKNOWLEDGMENTS

The major part of this work was done at the Lawrence Radiation Laboratory. I would like to express my appreciation to Professor E. Segrè and Professor C. Weigand for their hospitality and for many stimulating and instructive discussions on mesonic atoms and related subjects. The remainder of this paper was completed at CERN. I would like to thank Dr. T. E. O. Ericson for very informative and enlightening discussions and Dr. B. Zumino for a pleasant stay at the Theoretical Division in CERN.

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[18]As shown later, the value of $\bar{A}$ depends relatively strongly upon (c, t) values; a reference to the value without quoting (c, t) should be avoided. We emphasize a fact that these (c, t) values are of the nuclear-density distribution that could be considered, as a good estimate, the same as the nuclear charge distribution after removing the proton size. Our (c, t) values used in this work are obtained in this way taking the proton rms radius to be 0.8 F.

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Description of ^{4}He$(d, d)^4$He Polarization-Transfer Experiments*

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A convenient parametrization of experiments in which a polarized spin-1 particle bombards a target and a polarized spin-1 particle emerges is given. The parameters defined are a generalization of the "Wolfenstein parameters" long used in the description of spin-$\frac{1}{2}$ polarization-transfer experiments. A specific M matrix for a spin-0 target is used to find the relations between the various observables in several coordinate systems. The formalism is valid for the general $1+0 \rightarrow 1'+0'$ problem, where 1 and 1′ may indicate different spin-1 particles and 0 and 0′ may indicate different spin-0 particles, but where the product of initial channel and final channel parities is unchanged. The constraints imposed by time-reversal invariance when the reaction is specialized to elastic scattering are discussed. The prospects for a complete determination of the M matrix and for possible time-reversal tests are briefly touched upon.

I. INTRODUCTION

In this paper we will discuss the manner in which polarization-transfer experiments with a polarized spin-1 particle incoming and a polarized spin-1 particle outgoing may be parametrized. The parameters used have been given previously[1] and are analogous to the Wolfenstein parameters,[2] long used for the description of spin-$\frac{1}{2}$ to spin-$\frac{1}{2}$ polarization transfer. We restrict ourselves to the usual case where the product of initial intrinsic parities is the same as the product of the final intrinsic parities, which includes, of course, elastic scattering. We will then specialize to the case of an elastic scattering of spin-1 particles from a spin-0 target, as in $d+^4$He or in $\pi+d$ scattering. This study was motivated by the experimental studies of d-^{4}He polarization-transfer phenomena which are in progress at the Los Alamos Scientific Laboratory (LASL), by the general applicability of the formalism if target spin is neglected, as is done in most spin-1 optical-model studies, and by the intrinsic interest of this relatively simple spin system. A brief treatment of the related $1+1 \rightarrow 0+0$ problem has been given by Köhler and Fick.[3]

II. COORDINATE SYSTEMS

At the Third Polarization Symposium in 1970, certain conventions were adopted[4] for the parametrization of the simpler types of experiments involving spin-1 particles. The conventions did not include the polarization-transfer phenomena of interest in the present paper, but covered a special case of the more complex situation. In particular, for the rectangular tensor description of spin-1 polarization effects, which will be the basis of the present description, it was agreed that A's should be used for analyzing tensors and p's for polarizations of ensembles of particles. We will follow this notation as far as possible. We will use P's to indicate the polarization functions, that is, the polarization which would be produced by an unpolarized incident beam, and K's to indicate polarization-transfer coefficients of any rank.

The definition of the coordinate systems in terms of which the reaction is to be described is