Variation in Nuclear-Matter Binding Energies with Phase-Shift-Equivalent Two-Body Potentials*

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Three-body-correlation corrections to the saturation curves of two phase-shift-equivalent potentials have been computed. The corrections for both curves are of the same sign. They are very small at normal densities but increase rapidly with increasing density and defect integral.

In an earlier paper[1] (hereafter cited as I) we have shown that reasonable phase-shift-equivalent potentials can produce substantially different saturation curves in the Brueckner approximation. The question remained to what extent the results are affected by higher-order correlation corrections. Rough estimates of the three-body correlation effects by Wong and Sawada[2] indicate that they should not change the qualitative picture. We have recently computed the three-body-correlation correction for two representative saturation curves, namely, cases Nos. 1 and 10 of Fig. 2 in I. The purpose of this note is to present the results of that computation.

The hole-line expansion of Bethe and Brandow[3] for the binding energy per particle is equivalent to the iterative solution of a coupled set of nonlinear integral equations.[4] The iteration procedure linearizes the equations. To each order (number of hole lines) the contribution to the potential energy per particle is obtained by solving a linear integral equation.[5] We have solved the integral equation for the three-body-correlation corrections numerically.

In order to write the equations that we have solved in a compact form, it is useful to introduce the partially antisymmetrized three-body wave functions

$$\psi(q_1, q_2; q_3) = -\psi(q_2, q_1; q_3) \tag{1}$$

with the scalar product

$$(\psi, \psi) = \tfrac{1}{2}\int d^3q_1 \int d^3q_2 \int d^3q_3 |\psi(q_1, q_2; q_3)|^2 . \tag{2}$$

The q's are momenta. Spin and isospin variables are suppressed for the sake of brevity. Summation over these variables is implied whenever there is a momentum integral. The kernel of the unit operator is then

$$\langle q_1', q_2', q_3'|1|q_3; q_2 q_1\rangle = \delta(q_3 - q_3')[\delta(q_2 - q_2')\delta(q_1 - q_1') - \delta(q_2 - q_1')\delta(q_1 - q_2')] . \tag{3}$$

The permutation operator X is defined by

$$\langle q_1', q_2'; q_3'|X|q_3; q_2, q_1\rangle = \langle q_1', q_2'; q_3'|1|q_2; q_1, q_3\rangle + \langle q_1', q_2'; q_3'|1|q_1; q_3, q_2\rangle . \tag{4}$$

The three-body correction to the energy is obtained by evaluating the expression

$$\mathcal{E}_3 = \frac{1}{2(2\pi)^3\rho_0}\int_{|p_1'|\le k_F} d^3p_1' \int_{|p_2'|\le k_F} d^3p_2' \int_{|p_1|\le k_F} d^3p_1 \int_{|p_2|<k_F} d^3p_2 \int_{|p_3|<k_F} d^3p_3\, \delta(\tfrac{1}{2}(p_1' - p_2' - p_1 + p_2))$$
$$\times\langle p_1', p_2'; p_3'|\tilde{G}(\omega)\frac{Q}{e}XF(\omega)(1+X)|p_3; p_2, p_1\rangle , \tag{5}$$

where ω is the sum of the self-consistent single-particle energies, i.e.,

$$\omega = \epsilon(p_1) + \epsilon(p_2) + \epsilon(p_3), \tag{6}$$

and the operator F is the solution of the integral equation

$$F(\omega) = \tilde{G}(\omega)X\frac{Q}{e}\tilde{G}(\omega) - \tilde{G}(\omega)X\frac{Q}{e}F(\omega) . \tag{7}$$

The operators $\tilde{G}(\omega)$, Q, and e are defined by

$$Q\psi(q_1, q_2; q_3) = \begin{cases} \psi(q_1, q_2; q_3) & \text{if } |q_1| > k_F \text{ and } |q_2| > k_F \\ 0 & \text{otherwise,} \end{cases} \tag{8}$$

$$e\psi(q_1, q_2; q_3) = [\epsilon(q_1) + \epsilon(q_2) + \epsilon(q_3) - \omega]\,\psi(q_1, q_2; q_3), \tag{9}$$

$$\langle q_1', q_2'; q_3' | \bar{G}(\omega) | q_3; q_2, q_1 \rangle = \delta(q_3' - q_3) \langle q_1', q_2' | G(\omega - \epsilon(q_3)) | q_2, q_1 \rangle, \tag{10}$$

where G is the two-body reaction matrix. The operator F is related to the operator W_3 of Ref. 4 by

$$\langle q_1, q_2; k_3 | W | p_3, p_2, p_1 \rangle = \langle q_1, q_2; k_3 | F(1 + X) | p_3; p_2, p_1 \rangle . \tag{11}$$

The numerical procedures for solving Eq. (7) and evaluating the expression (5) were first developed for the Reid soft-core potential and will be described elsewhere in detail. The main features of the approximation are as follows.

Momenta above the Fermi level are cut off at 8 fm^{-1}. We verified that the results are insensitive to this cutoff by increasing it to 12 fm^{-1}. Values of the orbital angular momentum L of the spectator are restricted to $L \leq 2$. The decrease of the contributions for $L = 0$, 1, 2 indicates that cutoff creates an error of, at most, a few percent. The angle-average approximation was used for the projection operator Q. In order to keep the size of the linear system manageable, we reduced the rank of the two-body reaction matrices by truncating the spectral representation of the operator

$$G = \sum_\beta e|\beta\rangle \lambda_\beta \langle\beta|e, \tag{12}$$

where $|\beta\rangle$ and λ_β are the eigenvalues and eigenvectors of the operator $(1/e)G(1/e)$. In case No. 1, we retained the largest positive and the two largest negative eigenvalues in this sum. In case No. 10, one repulsive and three attractive eigenvalues were retained. The approximation was tested both by comparing the truncated sum with the full matrix and by sample calculations for increased rank.

The results of the calculation are given in Fig. 1. The dashed curves are the same as in I; the solid curves include the three-body corrections. The corrections increase rapidly with the density and the defect integral.[6] They alter both saturation curves in the same direction. We see that the large difference in binding energies computed from different phase-shift-equivalent potentials is not reduced by the inclusion of the three-body term.

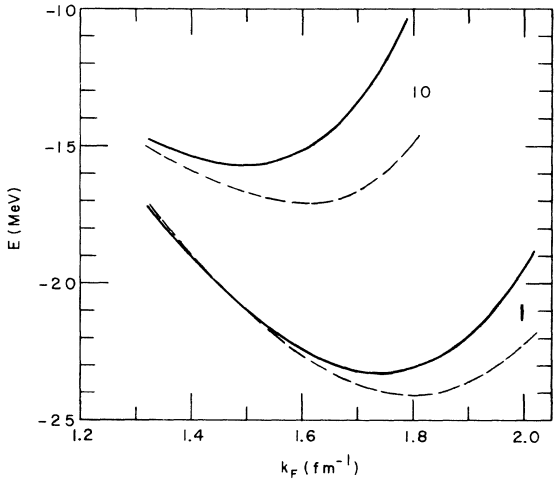

FIG. 1. Saturation curves for potentials Nos. 1 and 10 of I. The dashed curves are the same as in I. The solid curves include the three-body-correlation corrections.

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[5]See Eqs. (V.6), (V.26), and (V.27) of Ref. 4. There is a trivial misprint in Eq. (V.26). Two hole-creation operators $b^\dagger(p_1)b^\dagger(p_2)$ should be inserted at the end of the first line.

[6]See Fig. 4 of Ref. 1.