Effects of Two-Step Processes on the ^{40}Ca(t,d) and $(d,p)^{41}$Ca$(\frac{3}{2}^+, 2.02$ MeV) Reactions*

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An investigation is made of the effects of two-step processes on both the (t,d) and (d,p) reactions on ^{40}Ca, leading to the $\frac{3}{2}^+$ 2.02-MeV state in ^{41}Ca. The particular process considered is that in which the 3^- collective state in ^{40}Ca is excited first and then the stripping follows. The effects were found weak for both the (t,d) and (d,p) reactions.

The purpose of the present article is to discuss the effects of multistep processes in the (t,d) and (d,p) reactions on ^{40}Ca, leading to the $\frac{3}{2}^+$ 2.02-MeV state of ^{41}Ca. Using analyses based on the distorted-wave Born approximation (DWBA), the reactions were studied by many authors,[1,2] who extracted a spectroscopic factor for the neutron captured into the $d_{3/2}$ orbit. With the usual separation-energy prescription to obtain the bound-neutron form factor, this spectroscopic factor was found to be about 0.06, implying that the ^{40}Ca ground state has ~24% admixture of a core excitation amplitude of the $d_{3/2}{}^{-2}f_{7/2}{}^2$-type configuration. This is a significant departure from the closed-shell configuration.

It was found later, however, that if use is made of other prescriptions to obtain the form factor, quite different values are obtained for the spectroscopic factor. Actually, Philpott, Pinkston, and Satchler (PPS)[3] obtained form factors by solving an inhomogeneous equation instead of the homogeneous one that appears in the usual separation-energy method. The spectroscopic factor obtained in this way was found to be at least twice as large as 0.06 and, moreover, very sensitive to the particular type of two-body interaction which was needed in deriving the inhomogeneous term of the equation.[2,4] The use of the inhomogeneous equation certainly improves the degree of approximation in calculating the form factor, at least in principle, but the result's sensitivity to the two-body interaction makes it difficult to determine the spectroscopic factor unambiguously.

Besides this problem of the ambiguous form factor, it was suggested in Ref. 2 that a two-step process, in which the 3^- collective state of ^{40}Ca is excited first and then the stripping takes place, may play an important role, and thus, that it is quite desirable to make an estimate of this effect before any further elaboration is made in the DWBA analysis. In the present article we make such an estimate based on the coupled-channel Born approximation (CCBA).[5,6]

The form factor for a specified set of transferred orbital, spin, and total angular momentum, lsj, may be written as[7]

$$F_{lsj}(r) = N_0 B_{lsj} f_{lsj}(r). \tag{1}$$

In (1), N_0 is the over-all normalization constant [we use the values $N_0{}^2 = 1.5\times 3.371\times 10^4$ MeV^2 fm^3 for the (t,d) reaction[8] and 1.98×10^4 MeV^2 fm^3 for the (d,p) reaction,[9] respectively], while B_{lsj} is the spectroscopic amplitude given by

$$B_{lsj} = \frac{\langle \Psi_f \| a_{lj}^\dagger \| \Psi_i \rangle}{(2I_f+1)^{1/2}}, \tag{2}$$

where $|\Psi_i\rangle$ and $|\Psi_f\rangle$ are the initial- and final-state wave functions between which the stripping process takes place, and $a_{lj}^\dagger$ is the neutron creation operator in the single-particle state lj. Finally in (1), $f_{lsj}(r)$ is the normalized radial part of the captured-neutron wave function, and we evaluate it by using the usual separation-energy method. Since our main interest lies in the relative importance of the two-step process compared with the one-step process, we want to avoid any complication in obtaining the form factor.

In order to calculate the factor B_{lj} (we omit the suffix s, since in our present analysis $s=\frac{1}{2}$ always) for the stripping processes, we assume the following wave functions for the ^{40}Ca ground and ^{41}Ca $\frac{3}{2}^+$ states, respectively:

$$|{}^{40}\text{Ca},\ 0^+\rangle = \alpha_1 |0^+\rangle + \alpha_2 |[(d_{3/2})^{-2}{}_{TJ=10} \otimes (f_{7/2})^2{}_{TJ=10}]_{TJ=00}\rangle, \tag{3a}$$

$$|{}^{41}\text{Ca},\ \tfrac{3}{2}^+,\ T=\tfrac{1}{2}\rangle = |d_{3/2}{}^{-1} \otimes {}^{42}\text{Ca g.s.}\rangle = \beta_1 |d_{3/2}{}^{-1} \otimes (f_{7/2})^2{}_{TJ=10}\rangle + \beta_2 |d_{3/2}{}^{-1} \otimes (p_{3/2})^2{}_{TJ=10}\rangle, \tag{3b}$$

where $|0^+\rangle$ describes the doubly-closed-shell core. The actual values we use for the amplitudes are: $\alpha_1 = 0.89$, $\alpha_2 = 0.46$, $\beta_1 = 0.93$, and $\beta_2 = 0.20$. This

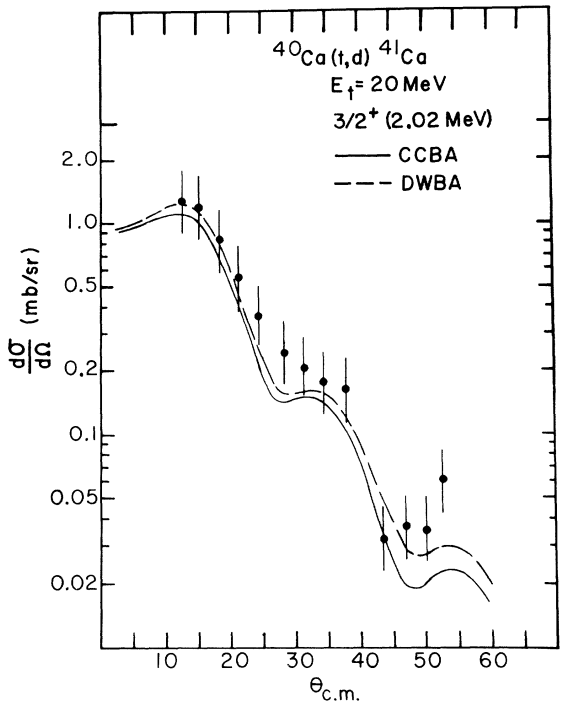

FIG. 1. Cross sections for the reaction ^{40}Ca(t,d)-^{41}Ca$(\frac{3}{2}^+$, 2.02 MeV) for E_t = 20 MeV. Experimental data were taken from Ref. 2, while the solid and dotted lines represent theoretical CCBA and DWBA cross sections, respectively. The optical-model parameters used were also taken from Ref. 2, the type AS1 being chosen for the triton potential.

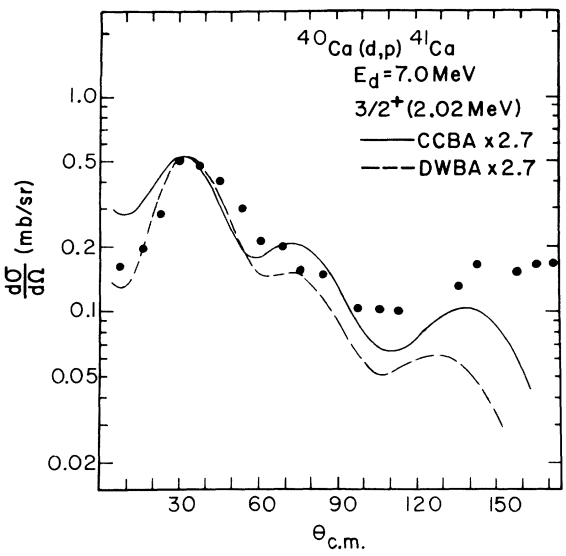

FIG. 2. Cross sections for the reaction ^{40}Ca(d,p)-^{41}Ca$(\frac{3}{2}^+$, 2.02 MeV) for E_d = 7 MeV. The data were taken from Ref. 1, and the solid and dotted lines are 2.7 times the theoretical CCBA and DWBA cross sections, respectively. The optical parameters were taken from Ref. 4.

means that the ^{40}Ca ground-state wave function is the same as that used by PPS,[4] while the ground-state wave function of ^{41}Ca is that of a $d_{3/2}$ hole coupled to the ^{42}Ca ground state. The ^{42}Ca ground-state wave function is taken from Kuo and Brown.[10] The use of these wave functions leads us to

$$B_{lj} = \tfrac{1}{2}\alpha_2\beta_1 = 0.21 . \tag{4}$$

Of course it is understood that $lj = d_{3/2}$.

To evaluate B_{lj} for the $3^- \rightarrow \frac{3}{2}^+$ stripping process, we assume that the 3^- wave function of ^{40}Ca consists of p-h (particle-hole) states of $(d_{3/2}^{-1} f_{7/2})$ and $(d_{3/2}^{-1} p_{3/2})$ configurations with a fifty-fifty mixture:

$$|^{40}\mathrm{Ca},\ 3^-\rangle = \gamma_1 |\, d_{3/2}^{-1} f_{7/2}\rangle + \gamma_2 d_{3/2}^{-1} p_{3/2}\rangle , \quad \text{with } \gamma_1 = -\gamma_2 = -\sqrt{\tfrac{1}{2}} . \tag{5}$$

The phases of the amplitudes were chosen such that the two components contribute coherently to the $E3$ transition from this 3^- to the ground state, and also that the amplitude of that $E3$ transition (and thus the value of β_3 in the sense of Tamura's work[11]) becomes positive. In the later CCBA calculations, we use $\beta_3 = 0.36$, a value obtained from the analysis of inelastic scattering data.[12]

With the wave functions (3b) and (5), the second step of our two-step process occurs as a capturing of a neutron into either the $f_{7/2}$ or the $p_{3/2}$ orbit. The β_{lj} corresponding to these capturing pro-

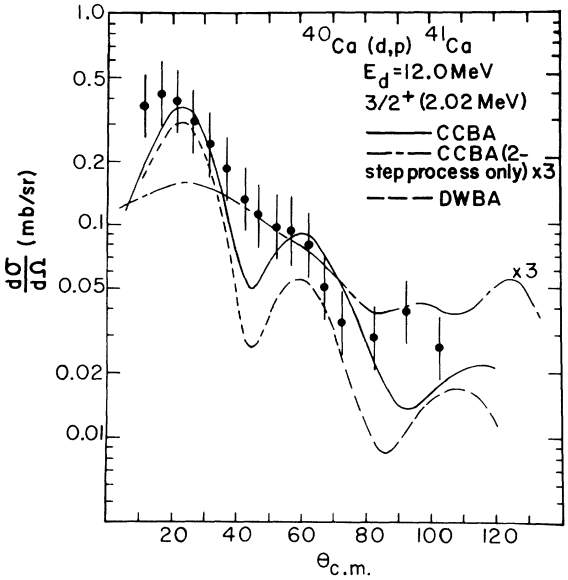

FIG. 3. Same as the caption of Fig. 2 except that E_d = 12 MeV, the data were taken from Ref. 13, the optical parameters from Ref. 4, and the dot-dash line represents the CCBA result obtained with the assumption of a pure two-step process. In the figure the CCBA and DWBA cross sections have not been scaled.

cesses are given as

$$B_{lj}=\left(\frac{7}{4(2j+1)}\right)^{1/2}\beta_i\gamma_i=\begin{cases}0.31, & i=1\ (l_j=f_{7/2})\\ -0.091, & i=2\ (l_j=p_{3/2}).\end{cases} \tag{6}$$

Using these B_{lj}, CCBA (t,d) calculations were performed for the incident energy $E_t=20$ MeV, the same as used in Ref. 2, and the results are compared in Fig. 1 with DWBA calculations, as well as with experiments. As is seen, the effect of the two-step process is rather small; the CCBA cross section is only slightly smaller, e.g. by 5~10%, than the DWBA cross section.

In order to see whether this conclusion might be changed if a different wave function is assumed for the 3^- state, we made a similar calculation assuming that the 3^- state consists entirely of the $(d_{3/2}^{-1}p_{3/2})$ configuration. This means putting B_{lj} of (6) equal to zero for $f_{7/2}$ and multiplying it by $\sqrt{2}$ for $p_{3/2}$. The change in B_{lj} values was thus rather large, but the resulting change in the CCBA cross section was found to be very small – less than 2%.

Such CCBA calculations were also made for (d,p) reactions at $E_d=7$ and 12 MeV, where experimental data are available,[1,13] and the results are given in Figs. 2 and 3, respectively. The corresponding DWBA results, as well as experimental cross sections, are also given there. As is seen, the CCBA results for both energies differ only very slightly (≈20%) from the DWBA results for angles between 20 and 40°. However, at larger angles the difference becomes larger, the CCBA cross section being nearly twice as large as the DWBA cross section and at the same time fitting the experiment somewhat better.[14]

In Fig. 3 another CCBA result (dot-dash curve) is also given which was obtained by assuming that there is no $d_{3/2}$ hole in the ground state of ^{40}Ca, and thus *only* the two-step process is contributing the excitation of the $\frac{3}{2}^+$ state of ^{41}Ca. As is seen, the angular distribution thus obtained agrees rather nicely with experiment for angles larger than 40°. However, the magnitude is about 3 times too small, and, further, the angular distribution differs significantly from experiment at smaller angles. It is thus seen that the pure two-step process is very unlikely to explain the experimental cross section completely. However, the flatness of the pure two-step cross section allows us to understand why the CCBA result, with a nonvanishing one-step contribution, is flatter than the DWBA result.

From these calculations we may draw the following conclusions. First of all, as far as the (t,d) reaction at $E_t=20$ MeV is concerned, the two-step process may safely be ignored. As for the (d,p) processes with $E_d=7$ and 12 MeV, it is advisable to use the CCBA if one intends to fit the full angular distribution. However, if one intends to extract the spectroscopic factor by fitting the angular distribution only around the first maximum, the DWBA is sufficient. In other words, problems such as the ambiguities in the choice of the form factors[3] can be discussed and should be solved within the usual framework of the DWBA. The failure to fit the data assuming a pure two-step process shows that the original conclusion, that there must be a sizable amplitude of the $d_{3/2}$ hole in the ground state of ^{40}Ca, is unchanged, even when the calculation is extended from the DWBA to the CCBA.

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Problems with Perturbation Treatments of Anharmonic Nuclear Oscillations

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Microscopic expressions are obtained for the coefficients in the angular momentum expansion $E_I = a_2(\frac{1}{2}I) + a_4(\frac{1}{2}I)^2 + \cdots$ of the energies of quasirotational members of quadrupole vibrational bands. Some limitations on the applicability of perturbation treatments of anharmonic corrections to the random-phase approximation are noted.

Recently, several workers have independently obtained explicit expressions for static quadrupole moments of vibrational states of nuclei assumed to have spherical ground states (in the Hartree-Bogoliubov sense).[1-5] Despite the wide variety of formal techniques, the expressions are entirely equivalent for a given choice of residual interaction. As previously shown by the authors,[4,5] this value of the quadrupole moment may be regarded as arising from the lowest-order perturbation treatment of anharmonic corrections to the harmonic or random-phase approximation (RPA) as provided by the Beliaev-Zelevinsky (BZ) boson expansion.[6] Included are not only effects of collective bosons but also all relevant couplings of the collective states to quasiparticle excitations. The purpose of this note is to briefly discuss some problems with perturbation treatments of nuclear anharmonicities previously hinted at.[5] We emphasize one difficulty, but also remove another ostensible one.

The problems are most easily seen using the semiclassical technique of either Meyer[3] or of the authors,[4,5] the relation being as follows. Meyer solves the nonlinear time-dependent Hartree-Fock (or, more generally, Hartree-Bogoliubov) equations of motion perturbatively, obtaining the density matrix as a multiply periodic series. Because of the degeneracy of a vibrational mode carrying nonzero angular momentum, one can find superpositions corresponding to simply periodic motion; in particular, rotations with constant angular velocity about some axis.[7] Such solutions are most conveniently studied as stationary ones by transforming to a rotating frame, and this just gives the cranking model for spherical nuclei used by the authors. In both approaches, the concern has been with the "stretched quasirotational subspace," i.e., the set of levels having maximum angular momentum for a given number of phonons. For definiteness, we shall limit ourselves to quadrupole phonons, although everything applies to vibrations of any multipolarity.

Both methods give for the energies of the quadrupole quasirotational band an expansion in powers of the angular momentum I:

$$E_I = \hbar\omega_2(\tfrac{1}{2}I) + a_4(\tfrac{1}{2}I)^2 + O(I^3) + \cdots , \tag{1}$$

and likewise for the static quadrupole moments:

$$\langle \mathfrak{M}(E2, 0) \rangle = q(\tfrac{1}{2}I) + O(I^2) + \cdots , \tag{2}$$

where $\hbar\omega_2$ is the RPA energy and q is the aforementioned value of the quadrupole moment of the $2+$ state. Perturbation theory applied to the anharmonicities in the BZ method also generates the forms (1) and (2), but ones in which each coefficient is an expansion in powers of the boson-expansion parameter.[4] It is important to note that the semiclassical expansion picks up the leading term in each coefficient, i.e., the only error lies in dropping small quantum-fluctuation terms [of order $(2j+1)^{-1}$].

There are two problems with (1) and (2) and related expansions, which concern us here. First, the coefficients beyond the lowest order are given by microscopic expressions having poles where a two-quasiparticle excitation energy is an integral multiple of $\hbar\omega_2$. Second, if the usual Hartree-