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Determination of inelastic scattering amplitudes from $(p,p'\gamma)$ reactions

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In view of the current experimental interest in utilizing ${}^{12}\mathrm{C}(p,p'\gamma){}^{12}\mathrm{C}$ spin observables to supplement the data on ${}^{12}\mathrm{C}(p,p'){}^{12}\mathrm{C}^*(1^+)$, we study the inelastic scattering from the point of view of the Goldstein-Moravcsik theorem, and identify 32 additional possible sets of $(p,p'\gamma)$ measurements. Compared to sets containing 16 observables [including eight on ${}^{12}\mathrm{C}(p,p'){}^{12}\mathrm{C}^*$], we find four different sets, each of which contain only a total of 14 observables and eighteen sets which contain only 15. Each set can lead to the unambiguous determination of the six inelastic amplitudes.

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Considerable interest has been evinced recently [1–3] in utilizing $(p,p'\gamma)$ coincidence observables [4–6] to determine empirically the inelastic scattering amplitudes for ${}^{12}\mathrm{C}(p,p'){}^{12}\mathrm{C}^*(1^+;15.11\ \mathrm{MeV})$, as it is known that the spin-transfer and cross-section measurements [7–15] restricted to the inelastic process itself are insufficient to effect an unambiguous determination of the inelastic amplitudes. Observing that the inelastic scattering has spin structure $0+\frac{1}{2}\rightarrow 1+\frac{1}{2}$ in which a spin-1 particle is produced in the final state, and recalling the Goldstein-Moravcsik theorem [16], we wish to point out here that measurement of observables associated with the vector polarization of ${}^{12}\mathrm{C}^*$ could be eschewed and the amplitude determination could be carried out elegantly through observables associated with the tensor polarization and avoiding incidentally the more difficult photon polarization measurements.

Recalling the generalization of the Goldstein-Moravcsik theorem [17,18] for arbitrary spin j and using the notation of Ref. [18], the Fano statistical tensors [19] t^k_q characterizing the state of polarization of ${}^{12}\mathrm{C}^*(1^+)$ are given by

$$t^k_q=(-1)^{k+1}3^{1/2}(A\otimes\rho_p A^\dagger)^k_q\ , \tag{1}$$

where ρ_p denotes the density matrix specifying the (initial) spin state of the proton beam and the amplitudes A_m are given explicitly using the transverse frame notation of Sudha Rao, Mallesh, and Ramachandran [2] as

$$A_1=-\begin{vmatrix}0 & F\\ C & 0\end{vmatrix},\quad A_0=\begin{vmatrix}B & 0\\ 0 & E\end{vmatrix},\quad A_{-1}=-\begin{vmatrix}0 & D\\ A & 0\end{vmatrix}, \tag{2}$$

while the notation $A^\dagger_m$ is used [20] to denote $(-1)^m$ times the Hermitian conjugate of A_{-m}. Here, the t^k_q are so normalized that $\mathrm{Tr}\,t^0_0$ gives the differential cross section for the inelastic scattering.

The angular distribution of the photons is then of the form

$$I(\theta,\phi)=|Q|^2\sum_{k=0}^{2}C(11k;1-10)[1+(-1)^k](2k+1)^{-1/2}\times[t^kY_k(\theta,\phi)]\ , \tag{3}$$

where (θ,ϕ) denote the angles of photon emission and $|Q|^2$ is essentially proportional to the γ-decay strength parameter. It may be noted that $t^kY_k(\theta,\phi)$ is a scalar under rotations and as such the angles (θ,ϕ) should be measured in the same frame of reference in which the t^k_q are given. In particular, in the transverse frame wherein the amplitude A_m are explicitly given by Eq. (2), the polar angles (θ,ϕ) are defined with respect to a z axis which is along $\mathbf{P}_i\times\mathbf{P}_f$ and x axis chosen along $\mathbf{P}_f$ where $\mathbf{P}_i$, $\mathbf{P}_f$ denote the initial and final center-of-mass proton momenta. This frame is indeed used in experimental measurements [11].

Observing that there is no contribution to (3) from $k=1$, the relevant t^k_q are explicitly given by

$$t^0_0=\tfrac{1}{2}\begin{vmatrix}(|B|^2+|D|^2+|F|^2)+P_z(|B|^2-|F|^2-|D|^2) & P_x(BE^*+FC^*+DA^*)-iP_y(BE^*-FC^*-DA^*)\\ P_x(EB^*+CF^*+AD^*)+iP_y(EB^*-CF^*-AD^*) & (|E|^2+|C|^2+|A|^2)+P_z(|A|^2+|C|^2-|E|^2)\end{vmatrix}, \tag{4}$$

$$t^2_2=\tfrac{1}{2}(3)^{1/2}\begin{vmatrix}(1-P_z)FD^* & (P_x+iP_y)FA^*\\ (P_x-iP_y)CD^* & (1+P_z)CA^*\end{vmatrix}, \tag{5}$$

$$t^2_1=\tfrac{1}{2}(\tfrac{3}{2})^{1/2}\begin{vmatrix}P_x(FB^*-BD^*)+iP_y(FB^*+BD^*) & (FE^*-BA^*)-P_z(FE^*+BA^*)\\ (CB^*-ED^*)+P_z(CB^*+ED^*) & P_x(CE^*-EA^*)-iP_y(CE^*+EA^*)\end{vmatrix}, \tag{6}$$

$$t^2_0=\tfrac{1}{2}(2)^{-1/2}\begin{vmatrix}(|F|^2+|D|^2-2|B|^2)-P_z(|F|^2+|D|^2+2|B|^2) & P_x(FC^*+DA^*-2BE^*)+iP_y(FC^*+DA^*+2BE^*)\\ P_x(CF^*+AD^*-2EB^*)-iP_y(CF^*+AD^*+2EB^*) & (|C|^2+|A|^2-2|E|^2)+P_z(|A|^2+|C|^2+2|E|^2)\end{vmatrix}, \tag{7}$$

and

$$t^k_{-q}=(-1)^q(t^k_q)^\dagger , \quad (8)$$

where $(t^k_q)^\dagger$ denotes the Hermitian conjugate of t^k_q and P_x,P_y,P_z denote the components of the incident proton beam polarization vector $\mathbf{P}$.

All photon-proton correlation observables (except those involving photon polarization measurements) may conveniently be denoted by

$$I_s(\theta,\phi;\mathbf{P})=\mathrm{Tr}[\sigma_s I(\theta,\phi)], \quad s=0,x,y,z , \quad (9)$$

where σ_s denote, respectively, the unit matrix and the three Pauli matrices. It is clear that $I_0(\theta,\phi;\mathbf{P}=0)$ gives the $(p,p'\gamma)$ differential cross section and $I_0(\theta,\phi;\mathbf{P}\neq0)$ the analyzing powers while $I_{s\neq0}(\theta,\phi;\mathbf{P}=0)$ lead to the scattered proton polarization and $I_{s\neq0}(\theta,\phi;\mathbf{P}\neq0)$ denote the proton-proton spin transfers. It may be mentioned that Hicks *et al.* [13] have measured $I_0(\theta,\phi;\mathbf{P}=0)$, the coincidence cross section at $E_{\text{lab}}=400$ MeV while Lyndon *et al.* [6] have reported measurements of analyzing powers $I_0(\theta,\phi;\mathbf{P}\neq0)$ at 318 MeV. The same techniques could possibly be extended to measure $I_{s\neq0}(\theta,\phi;\mathbf{P}=0)$ and $I_{s\neq0}(\theta,\phi;\mathbf{P}\neq0)$. We understand that a proposal to measure the coincidence observables is under way at Indiana [3]. All these photon-proton coincidence observables defined by (9) can readily be expressed in terms of the amplitudes A, B, C, D, E, and F on making use of Eqs. (3)–(8) in (9):

$$\begin{aligned}
I_0(\theta,\phi;\mathbf{P})=(3\pi)^{-1/2}|Q|^2[&\tfrac{1}{2}(\{|A|^2+|B|^2+|C|^2+|D|^2+|E|^2+|F|^2\}+P_z\{|A|^2+|C|^2-|E|^2+|B|^2-|F|^2-|D|^2\})\\
&+\tfrac{1}{8}(3\cos^2\theta-1)(\{|F|^2+|D|^2-2|B|^2+|C|^2+|A|^2-2|E|^2\}\\
&\qquad -P_z\{|F|^2+|D|^2+2|B|^2-|A|^2-|C|^2-2|E|^2\})\\
&+\tfrac{3}{8}(1-\cos2\theta)\mathrm{Re}(e^{-2i\phi}\{(FD^*+CA^*)+P_z(CA^*-FD^*)\})\\
&-[\tfrac{3}{4}(2)^{-1/2}]\sin2\theta\,\mathrm{Re}(e^{-i\phi}\{P_x(FB^*-BD^*+CE^*-EA^*)\\
&\qquad +iP_y(FB^*+BD^*-CE^*-EA^*)\})] ,
\end{aligned} \quad (10)$$

$$\begin{aligned}
I_x(\theta,\phi;\mathbf{P})=(3\pi)^{-1/2}|Q|^2[&(P_x\mathrm{Re}\{BE^*+FC^*+DA^*\}+P_y\mathrm{Im}\{BE^*-FC^*-DA^*\})\\
&+\tfrac{1}{4}(3\cos^2\theta-1)(P_x\mathrm{Re}\{FC^*+DA^*-2BE^*\}-P_y\mathrm{Im}\{FC^*+DA^*+2BE^*\})\\
&+\tfrac{3}{8}(1-\cos2\theta)\mathrm{Re}(e^{-2i\phi}\{P_x(FA^*+CD^*)+iP_y(FA^*-CD^*)\})\\
&-[\tfrac{3}{4}(2)^{-1/2}]\sin2\theta\,\mathrm{Re}(e^{-i\phi}\{(FE^*-BA^*+CB^*-ED^*)\\
&\qquad -P_z(FE^*+BA^*-CB^*-ED^*)\})] ,
\end{aligned} \quad (11)$$

$$\begin{aligned}
I_y(\theta,\phi;\mathbf{P})=(3\pi)^{-1/2}|Q|^2[&(-P_x\mathrm{Im}\{BE^*+FC^*+DA^*\}+P_y\mathrm{Re}\{BE^*-FC^*-DA^*\})\\
&-\tfrac{1}{4}(3\cos^2\theta-1)(P_x\mathrm{Im}\{FC^*+DA^*-2BE^*\}+P_y\mathrm{Re}\{FC^*+DA^*+2BE^*\})\\
&+\tfrac{3}{8}(1-\cos2\theta)\mathrm{Re}(e^{-2i\phi}\{iP_x(FA^*-CD^*)-P_y(FA^*+CD^*)\})\\
&+[\tfrac{3}{4}(2)^{-1/2}]\sin2\theta\,\mathrm{Im}(e^{-i\phi}\{(FE^*-BA^*-CB^*+ED^*)\\
&\qquad -P_z(FE^*+BA^*+CB^*+ED^*)\})] ,
\end{aligned} \quad (12)$$

and

$$\begin{aligned}
I_z(\theta,\phi;\mathbf{P})=\tfrac{1}{2}(3\pi)^{-1/2}|Q|^2[&(\{|B|^2+|D|^2+|F|^2-|E|^2-|C|^2-|A|^2\}+P_z\{|B|^2-|F|^2-|D|^2-|A|^2-|C|^2+|E|^2\})\\
&+\tfrac{1}{4}(3\cos^2\theta-1)(\{|F|^2+|D|^2-2|B|^2-|C|^2-|A|^2+2|E|^2\}\\
&\qquad -P_z\{|F|^2+|D|^2+2|B|^2+|A|^2+|C|^2+2|E|^2\})\\
&+\tfrac{3}{4}(1-\cos2\theta)\mathrm{Re}(e^{-2i\phi}\{(FD^*-CA^*)-P_z(FD^*+CA^*)\})\\
&-[\tfrac{3}{2}(2)^{-1/2}]\sin2\theta\,\mathrm{Re}(e^{-i\phi}\{P_x(FB^*-BD^*-CE^*+EA^*)\\
&\qquad +iP_y(FB^*+BD^*+CE^*+EA^*)\})] .
\end{aligned} \quad (13)$$

Since $\mathrm{Tr}(\sigma_s t^0_0)$, $s=0,x,y,z$ give the differential cross section and all the proton spin observables (denoted by C_{ij} in Ref. [2]) associated with inelastic scattering, it is clear that t^0_0 is completely determined in terms of these measurements. These, however, are insufficient to determine the amplitudes A, B, C, D, E, and F empirically. The recoil nucleus in the excited (1^+) state is polarized and we are essentially using the observables (9) to measure the tensor polarization parameters t^2_q. Such procedures have also been envisaged recently [21] in the context of nucleon transfer reactions. It is worth noting from (4) and (7) that the (p,p') measurements that enable us to know t^0_0 are sufficient to determine completely t^2_0 as well. If t^0_0 and t^2_0 are known, we may readily determine $t^2_{\pm2}$ by setting $\theta=90°$ in (9). With t^0_0, t^2_0, and $t^2_{\pm2}$ known, we can readily determine $t^2_{\pm1}$ from (9) by choosing any angle $\theta\neq n\pi/2$, $n=0,1,2$ or more preferably $\theta=\theta_0$, which serves to eliminate the contribution from the t^2_0 term. It may be noted that there are two choices for θ_0 obtained by setting $Y_{20}(\theta,\phi)=0$.

Several new alternative sets $s_1,s_2,\ldots,s_{32}$ of $(p,p'\gamma)$ observables have thus been identified and are shown in Table I, where the chosen observables in each set are marked by "×". Each set s_i is by itself sufficient to determine the inelastic amplitudes unambiguously. Some of the sets suggested in this Rapid Communication, viz.,

TABLE I. Alternative sets $s_1, s_2, \ldots, s_{32}$ of observables which can be used to determine the amplitudes A, B, C, D, E, and F unambiguously. The observables chosen in each set are marked by ×.

	s_1	s_2	s_3	s_4	s_5	s_6	s_7	s_8	s_9	s_{10}	s_{11}	s_{12}	s_{13}	s_{14}	s_{15}	s_{16}	s_{17}	s_{18}	s_{19}	s_{20}	s_{21}	s_{22}	s_{23}	s_{24}	s_{25}	s_{26}	s_{27}	s_{28}	s_{29}	s_{30}	s_{31}	s_{32}
All C_{ij}'s	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×
$I_0(\theta_0,0;\mathbf{P}=0)$									×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×
$I_0(\theta_0,0;P,0,0)$	×								×	×	×	×	×	×							×		×	×	×			×				
$I_0(\theta_0,90°;P,0,0)$			×						×		×		×		×		×		×				×									×
$I_0(\theta_0,0;0,P,0)$		×									×	×	×	×	×	×						×				×					×	
$I_0(\theta_0,90°;0,P,0)$				×						×		×		×		×		×		×				×	×	×			×			
$I_x(\theta_0,0;\mathbf{P}=0)$			×	×	×	×	×	×									×	×	×	×							×		×			
$I_x(\theta_0,90°;\mathbf{P}=0)$	×	×			×	×	×	×													×	×								×	×	
$I_y(\theta_0,0;\mathbf{P}=0)$			×	×	×	×	×	×									×	×	×	×										×		×
$I_y(\theta_0,90°;\mathbf{P}=0)$	×	×			×	×	×	×													×	×					×	×				
$I_z(\theta_0,0;\mathbf{P}=0)$									×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×
$I_x(90°,0;P,0,0)$																							×	×	×	×	×	×	×	×	×	×
$I_z(\theta_0,0;P,0,0)$		×							×	×	×	×			×	×						×	×	×	×			×				
$I_z(\theta_0,90°;P,0,0)$				×						×		×		×		×		×		×				×								×
$I_x(90°,0;0,P,0)$																							×	×	×	×	×	×	×	×	×	×
$I_z(\theta_0,0;0,P,0)$	×								×	×			×	×	×	×					×					×					×	
$I_z(\theta_0,90°;0,P,0)$			×						×		×		×		×		×		×	×			×		×	×			×			
$I_x(\theta_0,0;0,0,P)$			×	×		×	×	×											×	×							×		×			
$I_x(\theta_0,90°;0,0,P)$	×	×			×	×		×																						×	×	
$I_y(\theta_0,0;0,0,P)$			×	×	×		×	×									×	×												×		×
$I_y(\theta_0,90°;0,0,P)$	×	×			×	×	×														×	×					×	×				

s_1, s_2, s_3, and s_4, involve only 14 observables and the next 18 sets $s_5, s_6, \ldots, s_{22}$ involve only 15 observables, whereas the rest as also each of the sets suggested earlier contain a total of 16 observables.

We illustrate how the amplitudes are determined unambiguously by considering, for example, the first set s_1 containing only 14 observables: we recall that the 8 C_{ij}'s yield the combinations $|B|^2$, $|E|^2$, BE^*, AD^*+CF^*, $|A|^2+|C|^2$, $|D|^2+|F|^2$ and the relative phase of E by choosing B, as before, to be real and positive. The observables $I_x(\theta_0,90°;\mathbf{P}=0)$, $I_x(\theta_0,90°;0,0,P)$, $I_y(\theta_0,90°;\mathbf{P}=0)$, and $I_y(\theta_0,90°;0,0,P)$ obtainable from (11) and (12) determine both the real and imaginary parts of $E(D+F)^* \pm B(A+C)^*$ and hence $(A+C)$ and $(D+F)$ since B and E are known. Consequently, $\mathrm{Re}AC^*$ and $\mathrm{Re}FD^*$ are determined since $|A|^2+|C|^2$ and $|D|^2+|F|^2$ are already known. Knowing $\mathrm{Re}AC^*$ and $\mathrm{Re}FD^*$, we next use

$$I_0(\theta_0,0;P,0,0) = \tfrac{1}{2}|Q|^2(3\pi)^{-1/2}\{C_{00}+\tfrac{3}{4}(1-\cos 2\theta_0)\mathrm{Re}(FD^*+CA^*) + [\tfrac{3}{2}(2)^{-1/2}]\sin 2\theta_0 P\,\mathrm{Re}X\}\ , \tag{14}$$

$$I_z(\theta_0,0;0,P,0) = \tfrac{1}{2}|Q|^2(3\pi)^{-1/2}\{C_{0z}+\tfrac{3}{4}(1-\cos 2\theta_0)\mathrm{Re}(FD^*-CA^*) + [\tfrac{3}{2}(2)^{-1/2}]\sin 2\theta_0 P\,\mathrm{Im}X\}\ , \tag{15}$$

to determine both the real and imaginary parts of X, where

$$X=E(A-C)^*+B(D-F)^*\ . \tag{16}$$

Defining the combination

$$Y=2(AD^*+CF^*)-(A+C)(D+F)^* \tag{17}$$

in terms of the already known entities and observing moreover that $Y=(A-C)(D-F)^*$ leads to the determination of

$$(A-C)=[E|A-C|^2+BY]/X\ , \tag{18}$$

since $|A|^2+|C|^2$, $\mathrm{Re}AC^*$ are known (i.e., $|A-C|^2$ is also known) as also E, B, and Y. Similarly multiplying (16) by $(D-F)$ and expressing the right-hand side as $EY^*+B|D-F|^2$, which is known in terms of the entities already determined, we get

$$(D-F)=[EY^*+B|D-F|^2]/X\ . \tag{19}$$

Since $(A+C)$ and $(D+F)$ are already known, (18) and (19) readily lead to the unambiguous determination of A, C, D, and F. Determination of these together with that of B and E already known from inelastic scattering completes the inversion procedure.

In a similar way each of the other sets listed in Table I could also be seen to lead to the unambiguous determination of the amplitudes.

All the 32 sets of observables suggested here are different from those suggested earlier [2]. Moreover, s_1, s_2, s_3, or s_4 facilitate easier determination of the amplitudes unambiguously since each of these s_i need only 6 p-γ correlation observables to be measured since all the 8 C_{ij}'s have already been measured experimentally from the inelastic scattering at 500 MeV while some of these measurements have been reported earlier at various energies like 150 MeV, 200 MeV [7], and 400 MeV [12,14]. Even the 14 sets $s_9, \ldots, s_{22}$ need only six observables to be measured since the p-γ coincidence cross section $I_0(\theta,\phi;\mathbf{P}=0)$ included in these sets has already been measured [11] at 400 MeV. It must be emphasized that all the observables in a given set have to be measured at the same energy and the same inelastic scattering angle in order to determine the inelastic amplitudes. Even among the six needed observables, experimental measurements of the three analyzing powers have been reported [6], though at different energies. We may, therefore, expect early measurement of the needed six observables at the required energies and angles.

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