Optical-model description of time-reversal violation in neutron-nucleus scattering

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A time-reversal-violating spin-correlation coefficient in the total cross section for polarized neutrons incident on a tensor rank-2 polarized target is calculated by assuming a time-reversal-noninvariant, parity-conserving "fivefold" interaction in the neutron-nucleus optical potential. Results are presented for the system $n + {}^{165}$Ho for neutron incident energies covering the range 1–20 MeV. From existing experimental bounds, a strength of 2 ± 10 keV is deduced for the real and imaginary parts of the fivefold term, which implies an upper bound of order 10^{-4} on the relative T-odd strength when compared to the central real optical potential.

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Measurements of the transmission of polarized neutrons through nuclear targets provide sensitive tests of the fundamental symmetries in the nuclear systems [1]. The violation of parity conservation in low energy neutron-nucleus scattering is now well established on the basis of such tests, with measured longitudinal analyzing powers of the order of 10% [2]. Optical-model analyses of the longitudinal analyzing power, utilizing a postulated parity-nonconserving term in the neutron-nucleus interaction, have been reported recently [3,4]. An optical-potential description of nucleon-nucleus scattering observables supplies a useful analytical tool as it relates the observables to the average properties of compound nuclear states, and provides a link to the underlying nucleon-nucleon interaction [5].

A neutron-transmission test of time-reversal invariance in neutron-nucleus scattering has been performed recently, employing polarized 2 MeV neutrons incident on an aligned (tensor rank-2 polarized) ^{165}Ho target [6,7], with a null result at a 10^{-4} level of a time-reversal-violating ("T-odd") spin-correlation coefficient, measured by reversing the direction of neutron transverse polarization. In the present work, we report a coupled-channels calculation in the framework of the optical model of the T-odd spin-correlation coefficient for the system $n + {}^{165}$Ho.

The calculation is based on the presence of a time-reversal-noninvariant, parity-conserving "fivefold" term in the optical potential, expressed in terms of the operator $\mathbf{s}\cdot(\mathbf{I}\times\hat{\mathbf{r}})(\mathbf{I}\cdot\hat{\mathbf{r}})$, where $\mathbf{s}$ and $\mathbf{I}$ are the projectile spin and target spin, respectively. Unlike in the studies of parity

nonconservation, where eV energies of p-wave resonances are relevant and accordingly the compound-elastic cross section dominates the shape-elastic (direct-elastic) cross section, MeV energies with many overlapping, closely-spaced compound-nucleus resonances are under consideration here. Following the general philosophy of the optical potential, the fivefold term is an energy-averaged representation of time-reversal-noninvariant scattering processes, with an imaginary part that accounts for time-reversal-noninvariant contributions to the average compound-elastic and reaction cross sections. As such it gives the most general description of time-reversal violation in the scattering of polarized neutrons from aligned targets, and via folding-model techniques [8] it can in principle be related rigorously to a T-symmetry violation in the effective nucleon-nucleon interaction.

The total cross section σ_t for neutrons (spin $s = 1/2$) incident on a target nucleus with spin I, when the projectile and target are in polarization states that are described by statistical tensors [9] that are "diagonal" in suitably chosen coordinate frames, i.e., $\tilde{t}_{kq}(s) = \tilde{t}_{k0}(s)\delta_{q0}$ and $\tilde{t}_{KQ}(I) = \tilde{t}_{K0}(I)\delta_{Q0}$, respectively, can be written as ·

$$\sigma_t = \sum_{kK} \tilde{t}_{k0}(s)\tilde{t}_{K0}(I)\sigma_{kK}, \qquad (1)$$

where

$$\sigma_{kK} = 4\pi\lambda^2 \frac{\hat{k}\hat{K}}{\hat{s}\hat{I}} \mathrm{Im} \sum_\lambda \hat{\lambda}\, C_{kK\lambda}(\hat{\mathbf{s}}\,\hat{\mathbf{I}}\,\hat{\mathbf{p}}) \sum_{Jlj l'j'} (2J+1)\hat{l}\hat{j}\hat{j}'$$

$$\times \langle l\lambda 00 | l'0 \rangle W(JjIK; Ij') \begin{Bmatrix} l & s & j \\ \lambda & k & K \\ l' & s & j' \end{Bmatrix} T^J_{l'j',lj}, \qquad (2)$$

with

$$C_{kK\lambda}(\hat{\mathbf{s}}\,\hat{\mathbf{I}}\,\hat{\mathbf{p}}) = \frac{(4\pi)^{3/2}}{\hat{k}\hat{K}} \left[[Y_k(\hat{\mathbf{s}}), Y_K(\hat{\mathbf{I}})]_\lambda, Y_\lambda(\hat{\mathbf{p}}) \right]_0. \qquad (3)$$

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Here λ is the reduced wavelength, $\hat{k} = (2k + 1)^{1/2}$, etc., $T^J_{l'j',lj} = (1/2i)(S^J_{l'j',lj} - \delta_{ll'}\delta_{jj'})$, where $S^J_{l'j',lj}$ are elements of the elastic-scattering S-matrix in the spin-orbit coupling representation, $[\ ,\ ]_k$ denotes a spherical-tensor product of rank k, and $\hat{s}$, $\hat{I}$, and $\hat{p}$ are unit vectors along the z axes of the frames in which the projectile and target statistical tensors are diagonal, and along the beam direction, respectively; the angular brackets, W, and braces denote the Clebsch-Gordan, Racah, and 9-j coefficients, respectively. The scalar quantities $C_{kK\lambda}(\hat{s}\,\hat{I}\,\hat{p})$ are the so-called correlation terms in the forward elastic-scattering amplitude, which are real (pure imaginary) for $k + K + \lambda$ even (odd). Expressions of differing generality for the total cross section with the projectile and target in polarization states described by statistical tensors have been given also elsewhere [10–13].

A correlation term $C_{kK\lambda}(\hat{s}\,\hat{I}\,\hat{p})$ indicates the presence of a term in the projectile-target interaction that has the same spherical-tensor structure. For example, the $k = K = 1$ cross section σ_{11} has spin-spin correlation terms with orbital angular momentum transfers $\lambda = 0$ and 2:

$$C_{11\lambda}(\hat{s}\,\hat{I}\,\hat{p}) = \begin{cases} -\sqrt{\tfrac{1}{3}}\,\hat{s}\cdot\hat{I}, & \lambda = 0 \\[2mm] \sqrt{\tfrac{3}{2}}\left[(\hat{s}\cdot\hat{p})(\hat{I}\cdot\hat{p}) - \tfrac{1}{3}\hat{s}\cdot\hat{I}\right], & \lambda = 2 \end{cases}$$

(4)

and these reflect the presence of spherical and tensor spin-spin terms, $\mathbf{s}\cdot\mathbf{I}$ and $(\mathbf{s}\cdot\hat{\mathbf{r}})(\mathbf{I}\cdot\hat{\mathbf{r}}) - \tfrac{1}{3}\mathbf{s}\cdot\mathbf{I}$, respectively, in the projectile-target interaction (here $\hat{\mathbf{r}}$ is a unit vector along the direction from the target to the projectile; an operator quadratic in $\hat{\mathbf{r}}$ transfers two units of orbital angular momentum, $\lambda = 2$). Another example is the case of $k = 0$ and $K = 2$, the deformation cross section σ_{02} for an unpolarized projectile incident on an aligned target. The deformation cross section σ_{02} has a correlation term with $\lambda = 2$:

$$C_{022}(\hat{s}\,\hat{I}\,\hat{p}) = \tfrac{3}{2}\left[(\hat{I}\cdot\hat{p})^2 - \tfrac{1}{3}\right] = P_2(\hat{I}\cdot\hat{p}),$$

(5)

which corresponds to the tensor potential $(\mathbf{I}\cdot\hat{\mathbf{r}})^2 - \tfrac{1}{3}I(I+1)$ of a nucleus with spin $I > 1/2$, or the quadrupole reorientation interaction of a rotational, statically deformed nucleus, which has the same tensor form.

The $k = 1$, $K = 2$ cross section σ_{12} is of our interest, as it has a parity-even, time-reversal-odd correlation term with $\lambda = 2$, which has the following "fivefold" form in terms of the Cartesian vectors $\hat{s}$, $\hat{I}$, and $\hat{p}$:

$$C_{122}(\hat{s}\,\hat{I}\,\hat{p}) = i\sqrt{\tfrac{3}{2}}\,\hat{s}\cdot(\hat{I}\times\hat{p})(\hat{I}\cdot\hat{p}).$$

(6)

This correlation term is imaginary as here $k + K + \lambda$ is odd. A projectile-target interaction that has the same spherical-tensor structure is

$$T_5 = \tfrac{1}{2}[\mathbf{s}\cdot(\mathbf{I}\times\hat{\mathbf{r}})(\mathbf{I}\cdot\hat{\mathbf{r}}) + (\mathbf{I}\cdot\hat{\mathbf{r}})(\mathbf{I}\times\hat{\mathbf{r}})\cdot\mathbf{s}].$$

(7)

As $\mathbf{I}\times\hat{\mathbf{r}}$ and $\mathbf{I}\cdot\hat{\mathbf{r}}$ do not commute ($\mathbf{s}$ and $\mathbf{I}$ are *operators* of the projectile and target spins, respectively, while

the quantities $\hat{s}$, $\hat{I}$, and $\hat{p}$ in the correlation terms are "c-numbers"), the fivefold operator T_5 is symmetrized as above. It is Hermitian and conserves parity, but it anticommutes with the operator of time reversal. The operator T_5 generates an antisymmetric elastic-scattering S matrix, $S^J_{l'j',lj} = -S^J_{lj,l'j'}$, as it is odd on time reversal and Hermitian; an operator iT_5, which is time-reversal-even and anti-Hermitian, leads similarly to an antisymmetric S matrix. This parallels the behavior of the familiar central terms in the optical potential: the central real part $V(r)$, which is time-reversal-even and Hermitian, and the central imaginary part $iW(r)$, which is time-reversal-odd and anti-Hermitian, both generate a symmetric S matrix [9]. Table I summarizes the symmetry properties of these terms in the optical potential. The presence of an interaction with the operator T_5 or iT_5 in the neutron-nucleus optical potential leads to a nonzero cross section σ_{12}, or a T-odd spin-correlation coefficient [14] A_5, defined as

$$A_5 = \frac{\sigma_{12}^{\max}}{\sigma_{00}} = \frac{1}{2\,\hat{s}\cdot(\hat{I}\times\hat{p})(\hat{I}\cdot\hat{p})}\frac{\sigma_{12}}{\sigma_{00}},$$

(8)

where $\sigma_{12}^{\max}$ corresponds to the maximum value of the fivefold correlation term and σ_{00} is the total cross section for an unpolarized beam and target (note that A_5 of this definition is by a factor of $(15/32)^{1/2}$ smaller than the "T-odd analyzing power" used in [6,7]). Experimentally, the T-odd spin-correlation coefficient is determined from the ratio $(\sigma_\uparrow - \sigma_\downarrow)/2\sigma_{00}$, where $\sigma_\uparrow$ ($\sigma_\downarrow$) is the total cross section for neutrons incident on an aligned target and polarized up (down) with respect to a direction parallel to $\hat{I}\times\hat{p}$ [6,7]. It should be mentioned that an interaction of the same form as in Eq. (7), but with the position operator $\hat{\mathbf{r}}$ replaced by the momentum operator $\mathbf{p}$ has the same spherical-tensor structure and symmetry properties as the operator T_5. However, such an interaction has a second-order velocity dependence, as opposed to the static character of T_5, and such interactions are generally considered to be of much less importance when a static interaction is available (cf. the case of the tensor spin-spin interaction).

Using the techniques of spherical-tensor algebra [15], the fivefold operator T_5 can be expressed as a scalar product of rank-2 spherical tensors

$$T_5 = -i\sqrt{4\pi}\{[Y_2(\hat{\mathbf{r}}),\mathbf{s}]_2,[\mathbf{I},\mathbf{I}]_2\}_0,$$

(9)

TABLE I. The symmetry S of the elastic-scattering S matrix and the properties under Hermitian conjugation H and time reversal T for the central and fivefold interactions. The positive (negative) sign denotes that an interaction is even (odd) under a transformation and that the corresponding S matrix is symmetric (antisymmetric).

Interaction	H	T	S
$V(r)$	+	+	+
$iW(r)$	−	−	+
T_5	+	−	−
iT_5	−	+	−

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and thus it is seen to be responsible for transfers $\lambda = 2$, $j_s = 1$, and $j_I = 2$ of the orbital, projectile-spin, and target-spin angular momenta, respectively, in the spin-orbit coupling representation $\boldsymbol{\lambda} + \mathbf{j}_s = \mathbf{j}_I$. The reduced matrix element [15] of T_5 is then proportional to

$$-i\sqrt{4\pi}\langle s\|\mathbf{s}\|s\rangle\langle I\|[\mathbf{I},\mathbf{I}]_2\|I\rangle$$

$$= -i\sqrt{\tfrac{2\pi}{3}}s(s+1)I(I+1)(2I+3)(2I-1), \quad (10)$$

and a calculation of elastic-scattering S-matrix elements with the interaction T_5 included in the optical potential can be performed using a standard coupled-channels code. The reduced matrix element of T_5 is imaginary, in accordance with the operator being odd on time reversal and Hermitian.

Using the coupled-channels code CHUCK [16], calculations of the T-odd spin-correlation coefficient A_5 were performed for the system $n + {}^{165}\text{Ho}$ (spin $I = 7/2$) for neutron incident energies covering the range 1–20 MeV. A deformed optical potential of the standard Woods-Saxon parametrization was employed, with the real part of strength $49.8 - 16(N - Z)/A - 0.325E$ MeV, surface imaginary part of strength $5 - 8(N - Z)/A + 0.51E$ ($E \leq 6.5$ MeV) and $8.3 - 8(N - Z)/A - 0.09(E - 6.5)$ ($E > 6.5$ MeV), volume imaginary part of strength $-1.8 + 0.2E$ ($E > 6.5$ MeV), and a spin-orbit strength of 6 MeV; the reduced radius and diffuseness parameters of all the terms of the potential were 1.26 and 0.63 fm, respectively, with the exception of 0.48 fm for the diffuseness of the surface imaginary part; the central part of the potential had a quadrupole deformation parameter $\beta_2 = 0.29$ [17]. A fivefold interaction term

$$[V_5 f_V(r) + iW_5 f_W(r)]T_5, \quad (11)$$

with volume Woods-Saxon form factors $f_V(r)$ and $f_W(r)$ of the same geometries as the corresponding terms in the central potential was added to the optical potential. Apart from the coupling due to the fivefold term, the calculations included the reorientation coupling of the
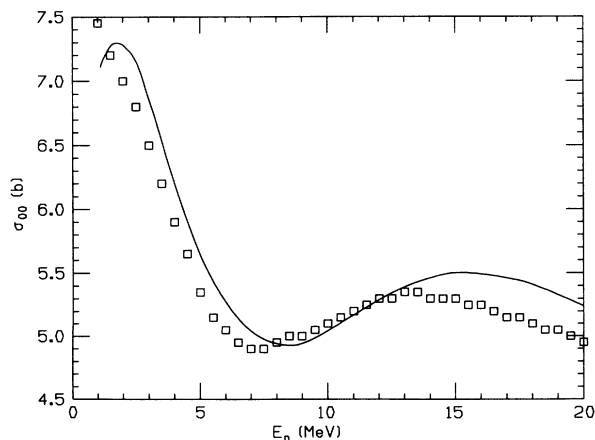

FIG. 2. The deformation cross section σ_{def} for $n + {}^{165}\text{Ho}$ as a function of the neutron incident energy E_n. The experimental data are from [22] (diamonds), [23] (squares), and [24] (circles). Typical experimental errors are $\pm(50$–$80)$ mb.

ground state of ${}^{165}\text{Ho}$, with angular momentum transfers $\lambda = 2$, 4, and 6, assuming the rotational model and a ground-state bandhead $K = I = 7/2$. Performing coupled-channels calculations, instead of a distorted-wave Born approximation treatment of the small five-fold term, thus had the advantage of being able to account easily for the large static deformation of ${}^{165}\text{Ho}$. The optical potential used is an adequate representation of the average n-${}^{165}\text{Ho}$ interaction in the energy range considered; this can be seen in Figs. 1 and 2, where the experimental total cross sections σ_{00} and deformation cross sections $\sigma_{\text{def}} = \sigma_{02}/P_2(\hat{\mathbf{I}} \cdot \hat{\mathbf{p}})$ are compared with the predictions calculated with the potential. Figure 3 presents the results for the T-odd spin-correlation coefficient A_5, separately for a pure real fivefold term of strength $V_5 = +0.1$ MeV and a pure imaginary fivefold term of strength $W_5 = +0.1$ MeV. As a function of the incident neutron energy, the spin-correlation coefficients are seen to oscillate in a typical Ramsauer fashion [18], reflecting the small changes in the overall strength of the

FIG. 1. The total cross section σ_{00} for an unpolarized beam and target for $n + {}^{165}\text{Ho}$ as a function of the neutron incident energy E_n. The experimental data are from [21].
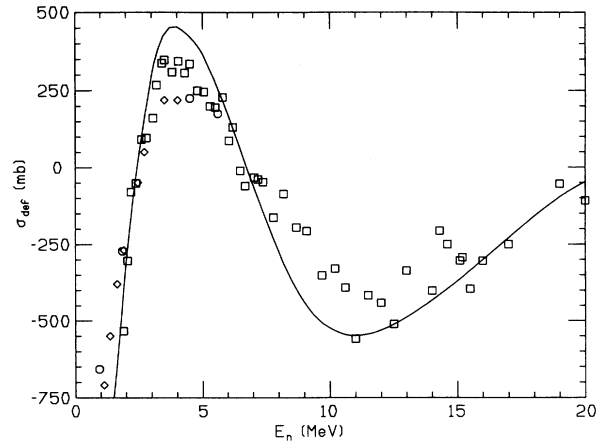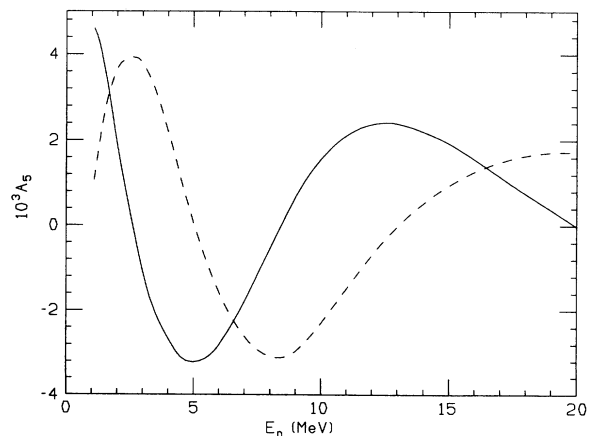

FIG. 3. T-odd spin-correlation coefficient A_5 for $n + {}^{165}\text{Ho}$ as a function of the neutron incident energy E_n. The solid and dashed lines are for a real strength $V_5 = +0.1$ MeV and an imaginary strength $W_5 = +0.1$ MeV, respectively, of the fivefold interaction.

nucleon-nucleus interaction due to the fivefold term. The amplitude of the oscillation is about 4×10^{-3} at the low-energy end of the 1–20 MeV range. The calculated values of A_5 are proportional to the small strengths V_5 and W_5.

Using these results and the experimental value of $A_5 = (0.7 \pm 4.1) \times 10^{-4}$ for the T-odd spin-correlation coefficient in the system $n + {}^{165}$Ho at 2 MeV [6], both the real and imaginary strengths of the T-odd fivefold term in the optical potential are estimated as 2 ± 10 keV. In order to be able to relate quantitatively this estimate to the strength of the T-odd, parity-even part of an effective nucleon-nucleon interaction, the fivefold term in the optical potential would have to be calculated from the underlying nucleon-nucleon force, which is still an outstanding task. An order-of-magnitude estimate can be made, however, of the bound on the ratio α_T of the strengths of the T-odd, parity-even and T-even, parity-even parts of the effective nucleon-nucleon interaction simply by taking the ratio of the strengths of the fivefold and central real parts in the optical potential: $\alpha_T < (10\,\text{keV})/(50\,\text{MeV}) = 2 \times 10^{-4}$. This is of the same order as the best sensitivity in α_T obtained from analyses of detailed-balance experiments [19] and from a recent analysis of energy shifts in neutron p-wave resonances [20].

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