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Resonant P_{33} $S_{1+}^{(3/2)}(q^2)$ electroproduction multipole amplitude and the $\Delta N\gamma$ scalar form factor $G_C^*(q^2)$

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The resonant P_{33} single-pion electroproduction multipole amplitude $S_{1+}^{(3/2)}(q^2)$ and the $\Delta N\gamma$ scalar form factor $G_C^*(q^2)$ are calculated nonperturbatively in terms of the well-known nucleon form factors $g_A(q^2)$ and $G_E^V(q^2)$. Results are in good agreement with experiment. Our treatment is completely relativistic with current conservation guaranteed. Assuming the usual dipole behavior for $g_A(q^2)$ and $G_E^V(q^2)$, we confirm that $G_C^*(q^2)$ falls off asymptotically at the rate q^{-6} as expected in perturbative QCD.

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The $\Delta N\gamma$ transition form factor [1,2] $G_M^*(q^2)$, $G_E^*(q^2)$, and $G_C^*(q^2)$ in elementary particle physics are very important in that they provide a basis for testing theories of effective quark forces or production models [3,4]. They are especially important in the analysis of perturbative QCD (PQCD) models involving gluon exchange mechanisms, tensor interactions, or possible hybrid baryonic states; in enhanced quark models in which the transition form factors may be calculated as a function of q^2; in electroproduction and photoproduction processes; in symmetry schemes such as SU(6) and U(6,6), and Melosh transformations; in bag models; in dispersion relation and Bethe-Salpeter approaches; in current algebra baryon sum rules; and in nonperturbative methods such as lattice QCD, QCD sum rules, and algebraic formulations [5–8]. The fundamental reason that the transition form factors are such good QCD probes lies in the fact in many quark, symmetry, or potential models, $G_E^*(q^2)$ and/or $G_C^*(q^2)$ are identically zero, thus giving rise to pure magnetic dipole M_{1+} transitions. However, we wish to point out that while much theoretical and experimental attention has been focused in the past few years on the ratio $(E_{1+}/M_{1+})_{q^2=0} = -(G_E^*/G_M^*)_{q^2=0}$, one should not forget the equally important scalar quadrupole amplitude $S_{1+}(q^2) \propto G_C^*(q^2)$ which arises in the electroproduction of the Δ in the reaction $\gamma_v + N \to \Delta \to \pi + N$. This is particularly true when one considers that available data indicate that $|S_{1+}(q^2)|$ is greater than $|E_{1+}(q^2)|$ at low momentum transfer.

In this paper, we utilize the nonperturbative methods of asymptotic flavor $SU_F(2)$ symmetry and asymptotic level realization [8,9] to derive an *explicit* form for the Coulomb form factor G_C^* as a function of q^2 and the well-known nucleon weak axial-vector form factor $g_A(q''^2)$ [parametrized by $g_A(q''^2)/g_A(0) \cong (1 - q''^2/m_A^2)^{-2}$], where

$$\langle p,p_2|A^\mu_{\pi^+}(0)|n,p_1\rangle \approx (2\pi)^{-3}\sqrt{(mm_n)/(E_{p_1}E_{p_2})} \times \bar{u}_p(p_2)[g_A(q''^2)\gamma^\mu\gamma_5]u_n(p_1)\ ,$$

m_n = neutron mass, and $q''^2 = (p_2 - p_1)^2$ and the nucleon isovector Sachs form factor $G_E^V(k^2)$ [parametrized by $G_E^V(k^2) \approx \frac{1}{2}(1 - k^2/0.71)^{-2}$] [10]. From this relation, we then determine $S_{1+}^{(3/2)}(q^2)$ and compare our results with available data and a dispersion relation numerical calculation.

One may write [11] for the $\Delta^+ p\gamma$ transition helicity amplitude (including an explicit isospin factor of $\sqrt{2/3}$) the expression

$$\langle p(\mathbf{p},\lambda_p)|j_\mu(0)|\Delta^+(\mathbf{p}^*,\lambda_\Delta)\rangle = \frac{1}{(2\pi)^3}\sqrt{\tfrac{2}{3}}\sqrt{mm^*/E_pE_\Delta}\,\bar{u}_p(\mathbf{p},\lambda_p)[\Gamma_{\mu\beta}]u_\Delta^\beta(\mathbf{p}^*,\lambda_\Delta) \tag{1}$$

where

$$\Gamma_{\mu\beta} = i\frac{3(m^*+m)}{2m}(G_M^* - 3G_E^*)\Theta^{-1}m^*q_\beta\epsilon_\mu(qp\gamma) - \frac{3(m^*+m)}{2m}(G_M^* + G_E^*)\Theta^{-1}[2\epsilon_{\beta\sigma}(p^*p)\epsilon_\mu^\sigma(p^*p)\gamma_5 - im^*q_\beta\epsilon_\mu(qp\gamma)] + \frac{3(m^*+m)}{m}G_C^*\Theta^{-1}q_\beta[pqq_\mu - q^2p_\mu]\gamma_5. \tag{2}$$

In Eqs. (1) and (2), the electromagnetic current is denoted by j_μ, $q \equiv p^* - p$, p^* and p are the four momenta of the Δ^+ and nucleon, respectively. $\Theta^{-1} \equiv \{[(m^*+m)^2 - q^2][(m^*-m)^2 - q^2]\}^{-1}$ is a kinematic factor which depends on q^2, m^* (the Δ^+ mass), and m (the proton mass); λ_P and λ_Δ are the helicities of the proton and Δ^+, respectively. We note that the first, second, and third terms in Eq. (2) induce transverse $\frac{1}{2}$, transverse $\frac{3}{2}$, and longitudinal helicity transitions, respectively, in the rest frame of the Δ^+ isobar [2].

The magnetic, electric, and Coulombic multipole (CM) transitions given by $M_{1+}(q^2)$, $E_{1+}(q^2)$, and $S_{1+}(q^2)$ can be written in terms of $G_M^*(q^2)$, $G_E^*(q^2)$, and $G_C^*(q^2)$. Specifically, the isospin $\frac{3}{2}$ scalar quadrupole multipole amplitude $S_{1+}^{(3/2)}(q^2)$ is given in terms of $G_C^*(q^2)$ by the expression [12]

$$S_{1^+}^{(3/2)}(q^2) = -\frac{1}{A}\frac{3\sqrt{2}}{16}\frac{(m^*+m)}{m^{*2}m}\sqrt{Q^+}Q^-G_C^*(q^2) \quad (3)$$

where $Q^\pm(q^2) \equiv (m^* \pm m)^2 - q^2$, $A \equiv 2\sqrt{3}m^*[(p_\pi^*\Gamma)/\alpha]^{1/2}$, $p_\pi^*\equiv$magnitude of the pion CM three-momentum for the reaction $\gamma+p \to \Delta^+ \to \pi^0+p$, $\Gamma \equiv$ total Δ width ≈ 0.12 GeV, and $\alpha\equiv$ fine-structure constant $\approx \frac{1}{137}$.

For the virtual process $p \to p+\gamma$, we have similarly

$$\langle p(\mathbf{s},\lambda)|j_\mu(0)|p(\mathbf{t},\lambda^*)\rangle = \frac{1}{(2\pi)^3}\sqrt{m^2/(E_{\mathbf{p}}E_{\mathbf{p}^*})}\bar{u}_p(\mathbf{s},\lambda)[\Gamma_\mu]u_p(\mathbf{t},\lambda^*) \quad (4)$$

where

$$\Gamma_\mu = [1-\tilde{q}^2/(4m^2)]^{-1}[(i/(4m^2))G_M(\tilde{q}^2)\epsilon_\mu(\tilde{P}\tilde{q}\gamma)\gamma_5 + (1/(2m))G_E(\tilde{q}^2)\tilde{P}_\mu],$$

$\tilde{P}_\mu \equiv \tilde{p}+\tilde{p}^*$, $\tilde{q}=\tilde{p}^*-\tilde{p}$ with $\tilde{p}^*=(\tilde{p}^{*0},\mathbf{t})$, and $\tilde{p}=(\tilde{p}^0,\mathbf{s})$. $G_M(\tilde{q}^2)$ and $G_E(\tilde{q}^2)$ are the familiar nucleon Sachs form factors.

In order to proceed with the calculation of $G_C^*(q^2)$, we consider only helicity states with $\lambda = +\frac{1}{2}$ (i.e., spin nonflip sum rules) and the nonstrange ($S=0$) $L=0$ ground-state baryons ($J^{PC} = \frac{1}{2}^+, \frac{3}{2}^+$). It is well known [8,13] that if one defines the axial-vector matrix elements $\langle p,1/2|A_{\pi^+}|n,1/2\rangle \equiv f = g_A(0)$, $\langle\Delta^{++},1/2|A_{\pi^+}|\Delta^+,1/2\rangle \equiv -\sqrt{(3/2)}g$, and $\langle\Delta^{++},1/2|A_{\pi^+}|p,1/2\rangle \equiv -\sqrt{6}h$, and applies asymptotic level realization to the chiral SU(2)$\otimes$SU(2) charge algebra $[A_{\pi^+},A_{\pi^-}] = 2V_3$, then $h^2 = \frac{4}{25}f^2$ [the sign of $h = +\frac{2}{5}f$, can be fixed by requiring that $G_M^*(0)>0$ [9]] and $g=(-\sqrt{2}/5)f$. If one further defines (suppressing the index μ) $\langle\Delta^+,\frac{1}{2},\mathbf{s}|j_3|\Delta^+,\frac{1}{2},\mathbf{t}\rangle \equiv a$, $\langle p,\frac{1}{2},\mathbf{s}|j_3|p,\frac{1}{2},\mathbf{t}\rangle \equiv b$, $\langle n,\frac{1}{2},\mathbf{s}|j_3|\Delta^0,\frac{1}{2},\mathbf{t}\rangle \equiv c$, $\langle\Delta^0,\frac{1}{2},\mathbf{s}|j_3|n,\frac{1}{2},\mathbf{t}\rangle \equiv d$ (note that other required matrix elements of j_3 can then be obtained easily from the double commutator $[[j_3^\mu(0),V_{\pi^+}],V_{\pi^-}] = 2j_3^\mu(0)$) and inserts the algebra $[j_3^\mu(0),A_{\pi^+}] = A_{\pi^+}^\mu(0)$ ($j^\mu \equiv j_3^\mu + j_S^\mu$, where $j_3^\mu \equiv$ isovector part of j^μ and j_S^μ is isoscalar) between the ground states $\langle B(\alpha,\lambda=\frac{1}{2},\mathbf{s})|$ and $|B'(\alpha,\lambda=\frac{1}{2},\mathbf{t})\rangle$ with $|\mathbf{s}|\to\infty$, $|\mathbf{t}|\to\infty$, where $\langle B(\alpha)|$ and $|B'(\beta)\rangle$ are the following SU$_f$(2) related combinations: $\langle p,n\rangle$, $\langle p,\Delta^0\rangle$, $\langle\Delta^{++},p\rangle$, $\langle n,\Delta^-\rangle$, $\langle\Delta^{++},\Delta^+\rangle$, $\langle\Delta^+,\Delta^0\rangle$, $\langle\Delta^0,\Delta^-\rangle$, and $\langle\Delta^+,n\rangle$, then one obtains (we use $\langle N|j_S^\mu|\Delta\rangle = 0$) the constraint equations (not all independent):

$$2fb-\sqrt{2}h(c+d) = f^{L=0}(\lambda=\tfrac{1}{2})\langle p|A_{\pi^+}^\mu|n\rangle, \quad (5)$$

$$\sqrt{2}h(a+b)+(-\sqrt{2}g-f)c = f^{L=0}(\lambda=\tfrac{1}{2})\langle p|A_{\pi^+}^\mu|\Delta^0\rangle, \quad (6)$$

$$\sqrt{6}h(-3a+b)+\sqrt{3/2}gd = f^{L=0}(\lambda=\tfrac{1}{2})\langle\Delta^{++}|A_{\pi^+}^\mu|p\rangle, \quad (7)$$

$$\sqrt{6}h(3a-b)-\sqrt{3/2}gc = f^{L=0}(\lambda=\tfrac{1}{2})\langle n|A_{\pi^+}^\mu|\Delta^-\rangle, \quad (8)$$

$$-\sqrt{6}ga+\sqrt{6}hc = f^{L=0}(\lambda=\tfrac{1}{2})\langle\Delta^{++}|A_{\pi^+}^\mu|\Delta^+\rangle, \quad (9)$$

$$-2\sqrt{2}ga+\sqrt{2}h(c+d) = f^{L=0}(\lambda=\tfrac{1}{2})\langle\Delta^+|A_{\pi^+}^\mu|\Delta^0\rangle, \quad (10)$$

$$-\sqrt{6}ga+\sqrt{6}hd = f^{L=0}(\lambda=\tfrac{1}{2})\langle\Delta^0|A_{\pi^+}^\mu|\Delta^-\rangle, \quad (11)$$

$$-\sqrt{2}h(a+b)+(f+\sqrt{2}g)d = f^{L=0}(\lambda=\tfrac{1}{2})\langle\Delta^+|A_{\pi^+}^\mu|n\rangle. \quad (12)$$

Applying asymptotic level symmetry, Eqs. (5)–(12) immediately imply that $d=c$ and $a=b+[-\frac{1}{4}(g/h)-(1/2\sqrt{2})(f/h)]c$. One can calculate $f^{L=0}(\lambda=\frac{1}{2})$ easily by setting $\mu=0$, restoring the x dependence to the matrix elements and integrating over $d\mathbf{x}$. We find that $f^{L=0}(\lambda=\frac{1}{2})=1$. Finally, we obtain [14]

$$\langle p,\tfrac{1}{2},\mathbf{s}|j^\mu(0)|\Delta^+,\tfrac{1}{2},\mathbf{t}\rangle = \frac{5\sqrt{2}}{4}\left\{-\frac{\langle p,\frac{1}{2},\mathbf{s}|A_{\pi^+}^\mu(0)|n,\frac{1}{2},\mathbf{t}\rangle}{2f}+\langle p,\tfrac{1}{2},\mathbf{s}|j_3^\mu(0)|p,\tfrac{1}{2},\mathbf{t}\rangle\right\}. \quad (13)$$

Now take the limit $|\mathbf{t}|\to\infty$ where $|\mathbf{s}| = r|\mathbf{t}|$ ($\mathbf{s}$ and $\mathbf{t}$ are taken along the z axis, $0<r\le m^2/m^{*2}$) and evaluate directly each of the matrix elements in Eq. (13). We find that

$$G_C^*(q^2) = -\frac{5rmm^*}{(1-r)(rm^*-m)(m+m^*)}\left\{-\frac{1}{2}\left[1-\frac{q''^2}{m_A^2}\right]^{-2}+\frac{(1+r)^2}{4r}\left[1-\frac{\tilde{q}^2}{4m^2}\right]^{-1}G_E^V(\tilde{q}^2)\right\},$$

$$q^2 = \frac{(1-r)}{r}(m^{*2}r-m^2),\quad q''^2 = \frac{(1-r)}{r}(m_n^2r-m^2),\quad \tilde{q}^2 = -\frac{(1-r)^2}{r}m^2,\quad G_E^V(0)=\tfrac{1}{2}. \quad (14)$$

Equation (14) is the main result of this work and effectively gives G_C^* as a function of q^2. One finds analytically that $|q^6|G_C^*(q^2)\to\{(5m^*m^6)/[2(m+m^*)]\}\{(m_A/m_n)^4-(\sqrt{0.71}/m)^4\}$ as $-q^2\to\infty$. In Fig. 1, we see immediately that $G_C^*(q^2)$ falls off asymptotically at the rate $1/q^6$ as expected in PQCD [4] but that $|q^6|G_C^*(q^2)$ does not show significant signs of approaching its *asymptotic* value of $\approx 0.4(1.3)$ GeV$^6/c^6$ for $m_A = 0.95(1.1)$ GeV/c until $-q^2$

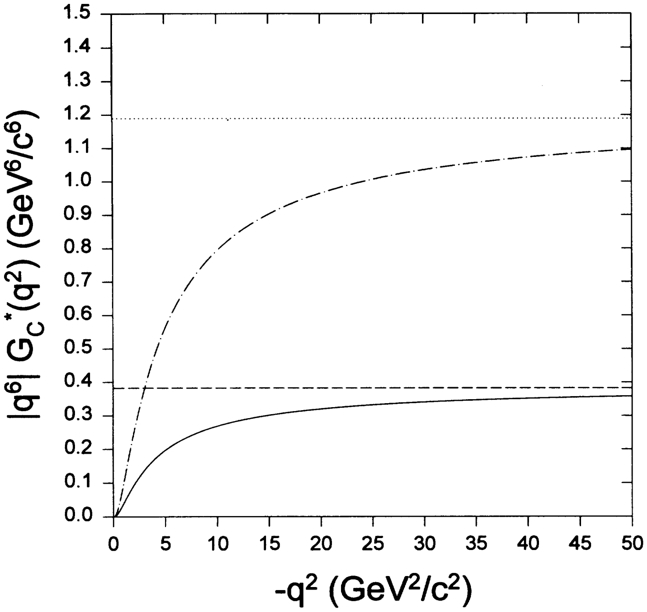

FIG. 1. The quantity $|q^6|G_C^*(q^2)$ for $m^* = 1.232$ GeV/c. The solid line has $m_A = 0.95$ GeV/c. As $-q^2 \to \infty$, the dashed line represents its asymptotic value of 0.382 GeV$^6/c^6$. The dot-dashed line has $m_A = 1.10$ GeV/c. As $-q^2 \to \infty$, the dotted line represents its asymptotic value of 1.189 GeV$^6/c^6$.

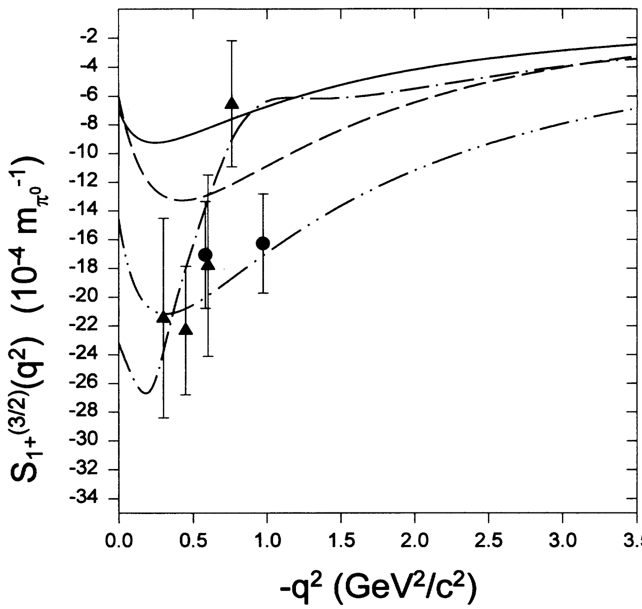

FIG. 2. The $\Delta N\gamma$ scalar quadrupole amplitude $S_{1+}^{(3/2)}(q^2)$ for $m^* = 1.232$ GeV/c. The solid line represents our theoretical calculation for $m_A = 0.95$ GeV/c. The dash-dot-dotted line represents our theoretical calculation for $m_A = 1.10$ GeV/c. The dash-dotted line is the dispersion relation result of Ref. [20]. The dashed line is the result of a fit to data from Refs. [2,16,19]. The triangular data points are from Ref. [18] and the circular data points are from Ref. [17].

is in the range of 10–20 GeV$^2/c^2$. Since one would expect that the q^2 region corresponding to the onset of PQCD scaling for $G_C^*(q^2)$ is the same as that for $G_M^*(q^2)$ and $G_E^*(q^2)$, it may be very difficult to observe this "leveling off" behavior even at the accelerator energies that Continuous Electron Beam Accelerator (CEBAF) can provide. On the other hand, for low momentum transfer, we expect that our results will be tested in the very near future by new electroproduction experiments designed to produce data near the photon point and also designed to yield new more precise measurements of m_A or equivalently the axial root-mean-square radius of the nucleon [15]. Equations (3) and (14) allow one to obtain $S_{1+}^{(3/2)}$ explicitly as a function of q^2. In Fig. 2, we plot $S_{1+}^{(3/2)}(q^2)$ ($m^* = 1.232$ GeV/c) as given by our theoretical calculation for $m_A = 0.95$ GeV/c and $m_A = 1.10$ GeV/c, by experiment [2,16–19], and as given by a numerical calculation using projected fixed t dispersion relations [20]. One observes that (especially for the value $m_A = 1.10$ GeV/c) our calculation is in very good numerical agreement with the data and also predicts the *explicit analytic* q^2 behavior of $S_{1+}^{(3/2)}(q^2)$ consistent with the *fitted values and form factor model assumptions* of Refs. [2,19] and the *implicit* q^2 behavior of the results of Ref. [20].

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