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Angular momentum dependence of the parity splitting in nuclei with octupole correlations

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Experimental data on the angular momentum dependence of parity splitting of yrast bands in different nuclei are analyzed using a one-dimensional model of octupole motion with axial symmetry. A two parameter formula, based on a solution of the Schrödinger equation with a double-minimum potential, predicts that the parity splitting exponentially decreases with $I(I+1)$ and gives a good fit to these data. Moreover, it relates the observed parity splitting directly to the internal barrier height of the potential.

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Octupole correlations have attracted widespread interest [1–5] as a characteristic collective mode that has been observed in many nuclei. These correlations have been treated in geometrical, mean field, and algebraic approaches [6–8]. Experimentally the most characteristic feature of the presence of strong octupole correlations is the appearance of alternating parity bands. If the octupole motion can be characterized as vibrational then the negative parity states appear at rather high excitation energies and are well separated from the positive parity states. If significant octupole correlations or deformation are present, the negative parity states lie much lower in excitation energy and form an alternating parity band of molecular type with the low-lying positive parity states. Thus, the energy splitting of the negative and positive parity states in a band can be used as a measure of the strength of octupole correlations (or of the potential energy as a function of the octupole deformation parameter). Moreover, and this is the central focus of the present work, the parity splitting of the octupole correlations can depend on the angular momentum.

The purpose of this Rapid Communication is to investigate the relation of the octupole potential energy surface to the parity splitting in the alternating parity bands. In particular, we derive a formula for the angular momentum dependence of the parity splitting in a band and show how this can be connected to the angular momentum dependence of the effective octupole potential energy, which includes a rotational energy term. Furthermore, we compare this formula to the data for a variety of alternating parity bands.

We investigate the dependence of the parity splitting on the potential well depth of the octupole deformed minimum for a one-dimensional Schrödinger equation with double-minimum potential reflecting the symmetry with respect to change of sign of the octupole deformation (Fig. 1) and with constant mass parameter. Probably, other effects such as Coriolis coupling to higher $K \neq 0$ octupole states [1,3,9,10] or two quasiparticle admixtures are also involved, but we consider them less important or assume that they can be included effectively through a renormalization of the potential. For instance the effect of the Coriolis coupling to higher $K \neq 0$ states can be expressed as a renormalization of the moment of inertia [1], whose influence on the angular momentum dependence of the potential is discussed below. Thus we rather focus on the octupole potential. The potentials of the type shown in Fig. 1 with different values of softness can be used to investigate the dependence of the parity splitting on the barrier height ΔV between the deformed minima. For physically reasonable cases the parity splitting $(E_- - E_+)$ (i.e., the energy difference between two lowest eigenvalues for negative and positive parities, respectively) decreases as ΔV increases [11]. With a reasonable accuracy the quantity $-\ln(E_- - E_+)$ linearly increases with ΔV with the same slope for all potentials considered, which are smooth functions of the octupole variable β_3.

This result can be exploited to parametrize the angular momentum dependence of the parity splitting in a simple fashion. With increasing angular momentum, centrifugal forces modify the effective potential. This potential thus depends on the angular momentum via a rotational energy term $A(\beta_3)I(I+1)$. Since the rotational parameter $A(\beta_3)$ depends on the octupole deformation β_3 and increases with β_3 [12–15], the depth of the minimum (lo-

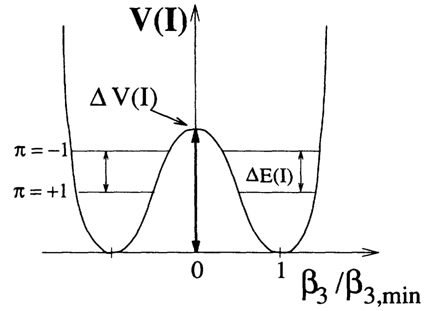

FIG. 1. The potential $V(\beta_3, I)$ for constant I as a function of β_3 showing the two symmetric minima. The vertical scale depends on the mass parameter.

cated at $\beta_{3,\min}$) or the barrier height between the reflection asymmetric shape and its mirror image will increase proportionally to $I(I+1)$. Allowing for a possible increase of the moment of inertia with spin I (described by a variable moment of inertia or the Lipas-Holmberg expression) we take the following expression for the potential well depth

$$V(\beta_3, I) = V(\beta_3, 0) + A(\beta_3)\frac{I(I+1)}{1+aI(I+1)}, \tag{1}$$

where a parametrizes changes in the moment of inertia with I. Hence $\Delta V(I)$ is

$$\Delta V(I) = V(0,I) - V(\beta_{3,\min}, I) = \text{const} + [A(\beta_3 = 0) - A(\beta_{3,\min})]\frac{I(I+1)}{1+aI(I+1)}, \tag{2}$$

Since $-\ln(E_- - E_+)$ linearly increases with $\Delta V(I)$, the parity splitting $\Delta E(I)$ is

$$\Delta E(I) \equiv E_-(I) - E_+(I) = c\,\exp\left(-\frac{I(I+1)/J_0(J_0+1)}{1+aI(I+1)}\right), \tag{3}$$

where J_0 is a parameter. It can be shown from [11] and from a numerical solution of one-dimensional Schrödinger equation with the potential shown in Fig. 1 that the value of J_0 is

$$[J_0(J_0+1)]^{-1} = 0.2\frac{B_3\beta^2_{3,\min}}{\hbar^2}[A(\beta_3 = 0) - A(\beta_{3,\min})]. \tag{4}$$

In Eq. (3) and Eq. (4) c is a constant and B_3 is the octupole mass parameter. The factor 0.2 comes from the numerical solution of the Schrödinger equation. We note that we use the same picture of the dependence of the relative energy of the positive and negative parity states on the barrier height between the reflection asymmetric shape and its mirror image as discussed in [2] but we consider in addition the angular momentum dependence of the barrier height, which is of crucial importance for describing the data. We will see that this gives another sensitive measure of octupole correlation effects.

If the potential has a minimum at $\beta_3 = 0$, the parity splitting is equal to a frequency of vibration. As I increases the potential becomes softer due to centrifugal forces. Then the vibrational frequency decreases and the parity splitting decreases, showing the same tendency as discussed above. Thus Eq. (2) may be useful in this case as well.

Let us consider the data for the angular momentum dependence of the parity splitting. If we approximate a smooth part of an energy $E(I)$ of an alternating parity band by the rotor expression, we obtain

$$E(I) = AI(I+1) - \frac{1}{2}(-1)^I\Delta E(I), \tag{5}$$

where positive $\Delta E(I)$ is the parity splitting [see Eq. (3)]. From Eq. (5) we derive

$$E(I) - 2E(I-1) + E(I-2) = 2A - 2(-1)^I\frac{1}{4}[\Delta E(I) + 2\Delta E(I-1) + \Delta E(I-2)]. \tag{6}$$

The left-hand side of (6) is the staggering index introduced in [16] times $E(2^+_g)$. On the right is a parity splitting term averaged over three neighboring values of I

$$\overline{\Delta E(I)} = \frac{1}{4}[\Delta E(I) + 2\Delta E(I-1) + \Delta E(I-2)] \tag{7}$$

and a term proportional to the rotational parameter A, which is small compared to the second term in Eq. (6) at low and middle spins I and becomes important only for spins at which parity splitting decreases significantly. For instance, in ^{226}Th at $I = 9, 10$ the first term is ten times smaller than the second one. Nevertheless, it is convenient to subtract this term in order to get an expression for an averaged parity splitting. This can be done approximately by subtracting from both sides of Eq. (7) the expression

$$\frac{1}{4}[E(I+2) - 2E(I) + E(I-2)] \tag{8}$$

which contains the energies of states of the same parity and for this reason the parity splitting contributions approximately cancel each other in Eq. (8). As a result we get

$$\overline{\Delta E(I)} = \frac{1}{2}\left|E(I) - 2E(I-1) + E(I-2) - \frac{1}{4}[E(I+2) - 2E(I) + E(I-2)]\right|. \tag{9}$$

It is convenient to use instead the normalized parity splitting $\Delta\epsilon(I)$

$$\Delta\epsilon(I) \equiv \overline{\Delta E(I)}/\overline{\Delta E(2)}. \tag{10}$$

Moreover, we want to compare experimental normalized parity splitting data [17–31] with calculated values based on Eq. (3), which gives for $\Delta\epsilon(I)$

$$\Delta\epsilon(I) = \exp\left[-\frac{I(I+1)/J_0(J_0+1)}{1+aI(I+1)} + \frac{6/J_0(J_0+1)}{1+6a}\right]. \tag{11}$$

Note the extremely simple result that $\ln\Delta\epsilon(I)$ is exactly linear in $I(I+1)$ if $a = 0$. Hence the dependence on $I(I+1)$ can be conveniently studied if we plot the data for $-\ln\Delta\epsilon(I)$.

This is shown in Fig. 2. Considered as a function of $I(I+1)$ the data fall into three groups. The first group, to which Th belongs, shows a linear dependence of $-\ln\Delta\epsilon(I)$ on $I(I+1)$. The second group corresponding to positive a, is exemplified by the U, Kr, Se isotopes and various rare earth nuclei. This corresponds to a decrease of the rotational parameter $A(\beta_3)$ with I. The third group, with negative a, comprises the Ra isotopes and also ^{146}Ce and ^{144,146}Ba. (Note that the negative a cannot be justified as an angular momentum dependence of the moment of inertia.)

Examples of the fit of experimental data for $\Delta\epsilon(I)$ with Eq. (11) are presented in Fig. 3. The overall agreement is very satisfactory. Although a is very small, its inclusion is often important to get a good fit at large I. The values of the parameters used in the fit are given in Table I. We give also the quantities $\Delta = [\frac{1}{I_{max}-2}\sum_{I=3}^{I_{max}}(\Delta\epsilon_{\rm exp}(I) - \Delta\epsilon_{\rm theor}(I))^2]^{1/2}$ and $E(1^-)/E(2_g^+)$, which allow the reader to judge the very good quality of the fit.

Relation (4) connects the phenomenological parameter J_0, which can be extracted from the experimental data, to quantities which can be calculated microscopically and therefore should give an important test of the model. Taking J_0 from the Table I we extract the ratio $A(\beta_3 = 0)/A(\beta_{3,\rm min})$ from Eq. (4). Of course the results depend on the octupole mass parameter B_3, which can depend on an isotope [32] and on β_3. If we take $B_3/\hbar^2$=200 MeV^{-1} as in [2], we get for $A(\beta_3 = 0)/A(\beta_{3,\rm min})$ 1.5 in ^{232}Th, 2.9 in ^{228}Th, 4.6 in ^{226}Th, and 2.7 in ^{146}Ce, which seem reasonable. An increase of B_3 by a factor of 2 decreases the value of $A(\beta_3 = 0)/A(\beta_{3,min}) - 1$ by the same factor.

In conclusion, we have suggested a formula for the angular momentum dependence of the parity splitting in alternating parity bands from a solution of the one-dimensional Schrödinger equation with a double-

TABLE I. Values of the parameters J_0, a, the maximum value of the angular momentum $I_{\rm max}$ used in the fit of the experimental data [17–31] for $\Delta\epsilon(I)$ [Eq.(10)], and the ratio $E(1^-)/E(2_g^+)$. The quantity Δ, which measures the deviation of the theoretical values of $\Delta\epsilon(I)$ from the experimental ones, averages 0.012 and is always less than 0.026.

Nucleus	J_0	a	$I_{\rm max}$	$E(1^-)/E(2_g^+)$
^{224}Th	4.9	0	9	2.53
^{226}Th	7.9	0	17	3.19
^{228}Th	12.1	0	12	5.68
^{230}Th	15.9	0	20	9.55
^{232}Th	22	0.0002	26	14.47
^{220}Ra	4.3	0.004	9	2.32
^{224}Ra	5.8	0	9	2.56
^{226}Ra	7.9	0	13	3.75
^{144}Ba	6.15	0	10	3.81
^{146}Ba	6.2	0	8	4.08
^{146}Ce	5	0.0008	9	3.58
^{232}U	24	0	10	11.84
^{234}U	37	0	10	18.08
^{236}U	21	0.0016	18	15.2
^{238}U	17.8	0.0011	22	15.14

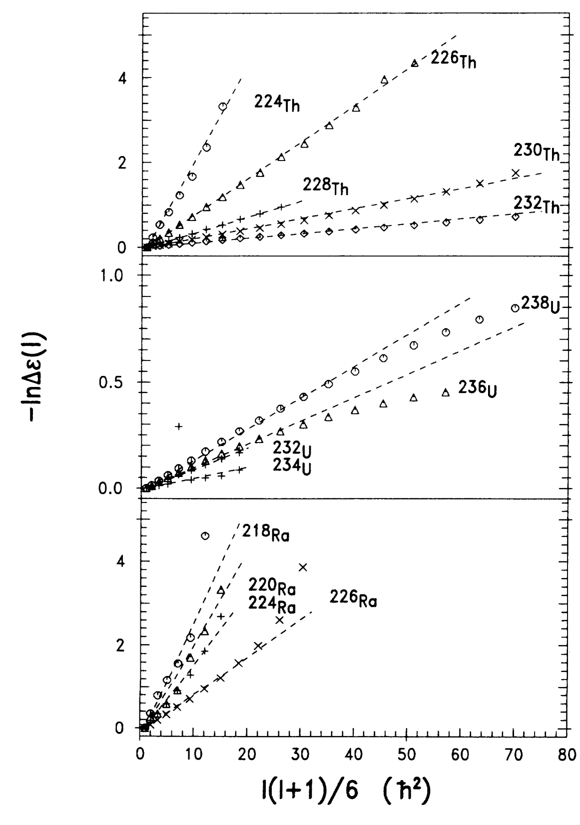

FIG. 2. Experimental data for $-\ln\Delta\epsilon(I)$ versus $I(I+1)/6$ for Th, U, and Ra isotopes [see Eq. (10)]. The straight lines show the quality of the linear approximation.

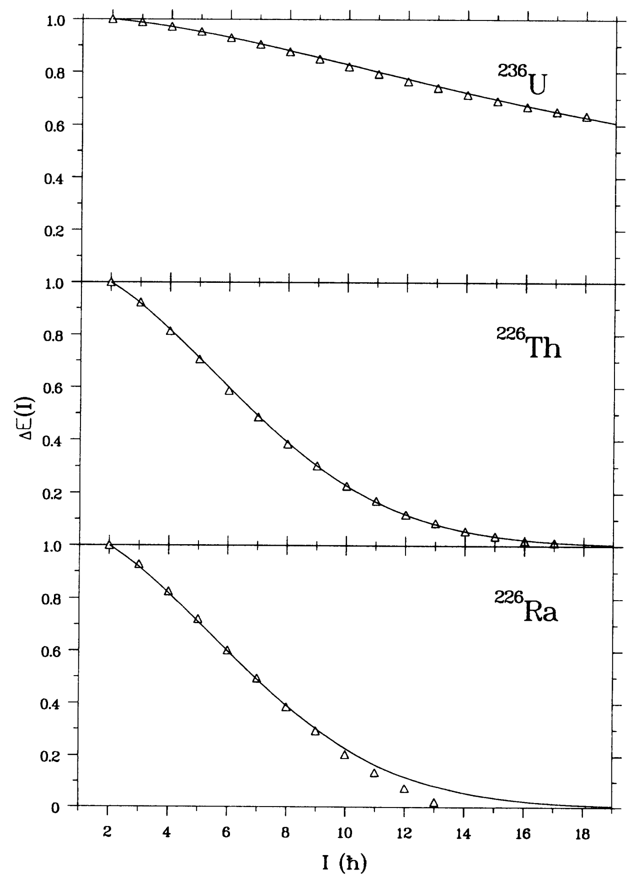

FIG. 3. Angular momentum dependence of the relative parity splitting $\Delta\epsilon(I) = \overline{\Delta E(I)}/\overline{\Delta E(2)}$ for three nuclei. Solid lines give the theoretical results; triangles, the data.

minimum potential. A good overall fit to the experimental data is obtained. This supports the basic validity of this model of the octupole correlations and allows the observed parity splitting to be directly related to the barrier height. An additional consequence is that nuclei with dynamical octupole deformations at low spins will develop static octupole deformations at sufficiently high collective spins.

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