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Intrinsic charge radius of the neutron: Discrepancy between the Garching and Dubna results

Yu. A. Alexandrov
Frank Laboratory of Neutron Physics, Joint Institute for Nuclear Research, 141980 Dubna, Moscow Region, Russia

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This Rapid Communication discusses possible reasons for the discrepancy between the results of the Garching and Dubna determinations of the mean square intrinsic charge radius of the neutron related to the inner structure of the neutron. It is shown that the most probable reason for the discrepancy between the values for bismuth is the differences in accounting for the influence of negative energy resonances on the (ne)-scattering length value measured in the experiments. The Dubna result seems to be more reliable for the present.

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Widely discussed recently [1–7] is the question of what the mean square intrinsic radius of the neutron, related to the electric charge distribution $\rho(\mathbf{r})$ inside the neutron [8],

$$\langle r_{\rm in}^2\rangle_N = \int \rho(\mathbf{r}) r^2 d^3\mathbf{r} = 6(dF_1/dq^2)_{q^2=0} = \frac{3\hbar^2}{Me^2}(a_{ne}-a_F) \tag{1}$$

is actually equal to. In Eq. (1) a_{ne} is the measurable scattering length of a slow neutron on an electron (ne-interaction), $a_F = \mu_n \frac{e^2}{2Mc^2} = -1.468 \times 10^{-3}$ fm is the Foldy scattering length related to a free neutron satisfying the Dirac equation and exhibiting an anomalous moment μ_n, and F_1 is the Dirac form factor.

From the table of experimental data given in [1] it follows that the most accurate experiments can be divided into two groups: the measurements of [9,10] resulting in $\langle a_{ne}\rangle = (-1.309 \pm 0.024) \times 10^{-3}$ fm, which leads to $\langle r_{\rm in}^2\rangle_N > 0$, in contradiction with modern theoretical representations of the neutron, and the measurements of [11–13] giving $\langle a_{ne}\rangle = (-1.577 \pm 0.034) \times 10^{-3}$ fm leading to $\langle r_{\rm in}^2\rangle_N < 0$, in confirmation of modern ideas of the neutron.

Earlier in [5,6] the possibility of the errors now present in [9] was noted.

A more promising direction in (ne)-interaction studies is the method applied in Dubna of thermal neutron diffraction on tungsten-186 crystals [12]. In this case the sought-for effect reaches a value of 20% (in [9,10] the effect is no higher than 0.5–1.2%), and as a result one obtains $a_{ne} = (-1.60 \pm 0.05) \times 10^{-3}$ fm. Note that this value is in agreement with the result reported in an earlier work [11] and has not been an object of criticism as yet.

The main discussion centers around the results of the Garching experiment [10] and the Dubna one [13]. The data obtained in each for the energy dependence (from 1 to 100 eV) of the total cross section, $\sigma_{\rm tot}$, of bismuth almost coincide. However, different data treatments gave a difference in values for $a_{ne} = (-1.32 \pm 0.04) \times 10^{-3}$ fm [10] and $a_{ne} = (-1.55 \pm 0.11) \times 10^{-3}$ fm [13].

First note that some of the discrepancy between the results of [10] and [13] comes from the different methods of data treatment. With Eqs. (1), (2), and (5) in [10] one can obtain, for the s-wave nuclear interaction [at $e(k) = 1$, $\Delta E \gg \Gamma/2$ and $R = (\sin\delta_0)/k$],

$$\frac{\sigma_{\rm tot}}{4\pi} = \frac{\sigma_{\rm coh}+\sigma_i+\sigma_{n\gamma}}{4\pi} = \frac{\sin^2\delta_0}{k^2} - \frac{\sin\delta_0}{k^2}\left[\sum_+ \frac{g_+\Gamma_n\Delta E}{\Delta E^2+\Gamma^2/4} + \sum_- \frac{g_-\Gamma_n\Delta E}{\Delta E^2+\Gamma^2/4}\right] + \frac{1}{4k^2}\left[\sum_+ \frac{g_+\Gamma_n\Delta E}{\Delta E^2+\Gamma^2/4} + \sum_- \frac{g_-\Gamma_n\Delta E}{\Delta E^2+\Gamma^2/4}\right]^2 + \frac{\sigma_i}{4\pi} + \frac{\sigma_{n\gamma}}{4\pi}, \tag{2}$$

where σ_i is the nuclear incoherent cross section, $g_{+-} = \frac{2J+1}{2(2I+1)}$ (in [10] they do not write g_{+-} in their formulas), $J = I \pm 1/2$, $I = 9/2$ (for Bi), and $\Delta E = E - E_{0j}$.

From [13] it follows that

$$\frac{\sigma_{\rm tot}}{4\pi} = \frac{\sin^2\delta_0}{k^2} - \frac{\sin\delta_0}{k^2}\left[\sum_+ \frac{g_+\Gamma_n\Delta E}{\Delta E^2+\Gamma^2/4} + \sum_- \frac{g_-\Gamma_n\Delta E}{\Delta E^2+\Gamma^2/4}\right] + \frac{1}{4k^2}\left[\sum_+ \frac{g_+\Gamma_n^2}{\Delta E^2+\Gamma^2/4} + \sum_- \frac{g_-\Gamma_n^2}{\Delta E^2+\Gamma^2/4}\right] + \frac{\sigma_{n\gamma}}{4\pi}. \tag{3}$$

The first two and the last terms in Eqs. (2) and (3) coincide, while the others are different. The first reason for this difference is the fact that Eq. (3) was derived on the basis of a generally accepted S matrix of neutron scattering

$$S_{nn}=\left(1-i\sum\frac{\Gamma_n}{\Delta E+i\Gamma/2}\right)\exp(2i\delta_{\rm pot}) \qquad (4)$$

which does not take into account small interresonance interference. However, as it will be shown below, taking this phenomenon into account cannot influence the result of the a_{ne} determination in [13]. An attempt to take the interresonance interference into account was undertaken in [2]. As shown in [3] it cannot be considered correct. Meanwhile, there are well known S matrices that do account for this phenomenon of interresonance interference (e.g., see Refs. [14–16]). Calculations of $\frac{\sigma_{\rm tot}}{4\pi}$ based on them were performed in [5]. The calculations have shown that with the known resonances $0 < E_{0j} < 265$ keV [17] being taken into account, the additional interresonance interference term in Eq. (3) for bismuth at an energy of about 10 eV makes $\frac{\sigma_{\rm int}}{4\pi} = 0.0086\times10^{-24}$ cm^2/sr (the total cross section of bismuth at this energy is $\frac{\sigma_{\rm tot}}{4\pi}=0.74\times10^{-24}$ cm^2/sr, i.e., nearly 90 times larger). At energies below 50 eV the value of $\frac{\sigma_{\rm int}}{4\pi}$ does not depend on energy, and neither, for example, does the interresonance interference term calculated with S matrix [15] (see also [5])

$$\frac{\sigma_{\rm int}}{4\pi}=\frac{g_+}{4k^2}\left\{\sum_i\Gamma_{ni}\Delta E_i\frac{\sum_{j\neq i}\Gamma_j/\Delta E_j}{\Delta E_i^2+\frac14(\Gamma_i+\Delta E_i\sum_{j\neq i}\Gamma_j/\Delta E_j)^2}\right\}+\text{(a similar term for the other spin)}. \qquad (5)$$

Far from the resonance energy, because $\Gamma=\Gamma_n+\Gamma_\gamma$, the term containing $\Gamma_{ni}\Gamma_{nj}$ in Eq. (5) does not change with energy, and the second term containing $\Gamma_{ni}\Gamma_{\gamma j}$ is much less than the first one (for bismuth, by 40 times at an energy of 10 eV).

In the Dubna work [13] to find the value of a_{ne} for the case of bismuth we analyzed the value of

$$Y=\frac{\sigma_{\rm tot}(E')}{4\pi}-b^2_{\rm coh}\frac{A^2}{(A+1)^2}=a^2_{ne}(Z^2-2ZF')$$
$$-2a_{ne}b_{\rm coh}\frac{A}{A+1}(Z-F')-p_1\left[b_{\rm coh}\frac{A}{A+1}-a_{ne}(Z-F')\right]+p_2+\frac{\sigma_{\rm abs}(E')}{4\pi} \qquad (6)$$

(the electric polarizability of the neutron is accepted to be equal to zero). Here $b_{\rm coh}$ is the coherent neutron scattering length, $F=\frac12\int_0^\pi f(\frac{\sin\theta}{\lambda})\sin\theta d\theta$, $f(\frac{\sin\theta}{\lambda})$ is the atomic form factor describing the electron charge distribution of an atom, $p_1=\sum_1-\sum'$, $p_2=\frac14(\sum_1)^2-\frac12\sum_1\sum'+\frac14\sum'_2$,

$$\sum\nolimits_1=\sum_i\frac{g_i\Gamma_{ni}\Delta E_i}{k(\Delta E_i^2+\Gamma_i^2/4)},$$

$$\sum\nolimits'=\sum_i\frac{g_i\Gamma'_{ni}\Delta E'_i}{k'(\Delta E_i'^2+\Gamma_i'^2/4)},$$

$$\sum\nolimits'_2=\sum_i\frac{g_i\Gamma'^2_{ni}}{k'^2(\Delta E_i'^2+\Gamma_i'^2/4)},$$

$\sigma_{\rm abs}(E')$ is the absorption cross section, $\Delta E_i=E-E_i$, $\Gamma_i=\Gamma_{ni}+\Gamma_{\gamma i}$, E_i, Γ_{ni}, $\Gamma_{\gamma i}$ are the energy, the neutron, and γ widths of the ith resonance, and E and E' are the neutron energies at which $b_{\rm coh}$ and $\sigma_{\rm tot}$ were measured. In the region far from the resonance energies (for bismuth $E < 30$ eV) the equation for Y [$Y\simeq(0.015-0.020)\times10^{-24}$ cm^2/sr] has an energy-independent term, p_2, that can be varied to achieve the best experimental description. Since p_2 does not depend on energy, by introducing a constant term $\frac{\sigma_{\rm int}}{4\pi}$ [Eq. (5)] one cannot affect the result of the a_{ne} determination in [13]. However, this will change the analytical expression somewhat, for p_2 will be $p_2+\sigma_{\rm int}/4\pi$ instead, and, moreover, it can be shown [18] that

$$p_2+\sigma_{\rm int}/4\pi=\frac{g_+g_-}{4}\left[\sum_+\frac{\Gamma_{ni}}{k_iE_i}-\sum_-\frac{\Gamma_{ni}}{k_iE_i}\right]^2$$
$$+\frac{g_+}{4k^2}\sum_i\frac{\Gamma_{ni}}{\Delta E_i}\sum_{j\neq i}\frac{\Gamma_{\gamma j}}{\Delta E_j}$$
$$+\text{(a similar term for the other spin)}. \qquad (7)$$

The second and third terms in Eq. (7) may be negative. Their signs depend on the influence on them of the neighboring levels with $E_i < 0$. Thus there exists no direct argument in favor of excluding the possibility of the negative sign for $p_2+\frac{\sigma_{\rm int}}{4\pi}$. For an even-even nucleus ($g_+=1$, $g_-=0$)

$$p_2+\sigma_{\rm int}/4\pi=\frac{1}{4k^2}\sum_i\frac{\Gamma_{ni}}{\Delta E_i}\sum_{j\neq i}\frac{\Gamma_{\gamma j}}{\Delta E_j}.$$

For ^{208}Pb [17,19] with the energy of 1 eV, $p_2+\sigma_{\rm int}/4\pi\simeq(6.4\times10^{-7})\times10^{-24}$ cm^2/sr and $p_1b_{\rm coh}\simeq(1.3\times10^{-7})\times10^{-24}$ cm^2/sr, the neutron resonance scattering can be neglected.

For bismuth the situation is much more complicated: $p_2+\sigma_{\rm int}/4\pi$ can be smaller than zero (as it follows from [13]). However, one has to be very careful when speaking about $p_2+\sigma_{\rm int}/4\pi$ being independent of energy, because the second and the third terms in Eq. (7) depend on energy as $1/E^{1/2}$.

So, from [11–13] it follows that $\langle r_{\rm in}^2\rangle_N < 0$. What kind of error comes into [10]?

Let us compare the formulas (2) and (3) for bismuth for the energy 10 eV taking into account resonances with the energy $E_{0j} > 0$ and the additional interresonance interference term Eq. (5):

$$\frac{1}{4k^2}\left[\sum_{+}\frac{g_+\Gamma_n\Delta E}{\Delta E^2+\Gamma^2/4}+\sum_{-}\frac{g_-\Gamma_n\Delta E}{\Delta E^2+\Gamma^2/4}\right]^2+\frac{\sigma_i}{4\pi}=(0.0113+0.0006)\times10^{-24}\ \mathrm{cm^2/sr}=0.0119\times10^{-24}\ \mathrm{cm^2/sr}. \tag{8}$$

$$\frac{1}{4k^2}\left[\sum_{+}\frac{g_+\Gamma_n^2}{\Delta E^2+\Gamma^2/4}+\sum_{-}\frac{g_-\Gamma_n^2}{\Delta E^2+\Gamma^2/4}\right]+\frac{\sigma_{\rm int}}{4\pi}=(0.0029+0.0086)\times10^{-24}\ \mathrm{cm^2/sr}=0.0115\times10^{-24}\ \mathrm{cm^2/sr}. \tag{9}$$

Thus, if the contribution of the $\frac{\sigma_{\rm int}}{4\pi}$ term is taken into account [Eq. (5)], expressions (2) and (3) give practically the same results (at $E_{0j} > 0$).

However, there is some difference between works [10] and [13] in their approach to calculating the contribution of negative energy resonances ($E_{0j} < 0$) and of unknown resonances to the total cross section. In [10] this contribution of one bound and of the unknown levels has been calculated using the average parameters of s-wave scattering: the strength function, $S_0 = 0.65 \pm 0.15$, and the mean level distance $\langle D_0\rangle = 4.5 \pm 0.6$ keV [17]. In this situation I think an error may easily creep in, since a resonance at $E_{01} < 0$, for example, may be at a distance $|E_{01}| < \langle D_0\rangle$ from the point $E = 0$ and it will hardly be possible to estimate its influence on the term b_R with any accuracy, because the uncertainty in the determination of S_0 is large (on the order of ±23%).

In [13] we have used a more realistic method consisting of the variation of the parameter p_2. This is the main reason for the discrepancy between the results of Garching and Dubna obtained for bismuth. A treatment of the experimental data of [10], taking into account the parameter $p_2 = -0.0023 \times 10^{-24}$ cm^2/sr found in [13] by the least square method, will lead to a 1.2 times increase in the absolute value of a_{ne}, i.e., to $a_{ne} = -1.58 \times 10^{-3}$ fm.

Thus, to my thinking, the values of a_{ne} obtained in [9,10] are not grounded enough and, consequently, the actual $\langle r_{\rm in}^2\rangle < 0$ [if Eq. (1) is correct]. This conclusion is in agreement with the measurements in [11–13] and with modern ideas of the inner structure of the neutron [6,20–23], but it disagrees with the result of the analysis of the available data made in [7] that favors a value of a_{ne} which is less negative than the Foldy scattering length.

Sometimes the question arises as to with what quantities the theoretically calculated charge radius (for instance, obtained in the Cloudy Bag Model [21]) should be compared: with $\langle r_{\rm in}^2\rangle_N$ or with $\langle r_{\rm in}^2\rangle_N + \frac{3\hbar^2}{Me^2}a_F$? Since all calculations of the nucleon radius are performed in the approximation of a motionless (recoilless) heavy nucleon ($M \to \infty$), it only seems correct to compare the results of the calculations with $\langle r_{\rm in}^2\rangle_N$, i.e., subtracting the Foldy scattering length [in accordance with Eq. (1)] from the measured value of a_{ne}.

Though the value of $\langle r_{\rm in}^2\rangle_N > 0$ does not agree with modern representations of the neutron, it is the viewpoint of an experimentalist that the question of the sign of $\langle r_{\rm in}^2\rangle_N$ is nevertheless to be solved by an experiment. A rather easy and reliable way to investigate this problem consists in a comparison of $\sigma_{\rm tot}$ measured at different energies with $b_{\rm coh}$ measured at very small energies [13]. It is important that the measurements are carried out with one and the same sample to exclude the influence of different admixtures. This kind of measurement is now being carried out by the Dubna-Germany-Czech Republic collaboration.

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