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Coulomb effects in deuteron breakup by proton impact

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We present the first results of a calculation of kinematically complete differential cross sections for the proton-induced deuteron breakup reaction, obtained by using a three-body formalism based on momentum space integral equations which correctly takes into account the Coulomb repulsion between the two protons. Comparison with experimental data is made.

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The nucleon-induced breakup of deuterons is a major field of applications of three-body theories. In recent years, based upon momentum space integral equations of the Faddeev type, great progress has been made in the calculation of kinematically complete differential cross sections for rather realistic forms of the nucleon-nucleon (NN) interaction *provided the projectile is neutral* (see, e.g., [1,2] and references therein). The motivation for the enormous effort required is that the richness of possible, and the freedom in the choice of specific, kinematic conditions for the three-body final states may reveal more, and in particular also clearer, signatures of interesting physical phenomena such as three-nucleon forces, or off-shell behavior of the nuclear interaction used, than the investigation of properties of the three-nucleon bound or of elastic nucleon-deuteron scattering states. Furthermore, tests of the limits of validity of the charge independence hypothesis, or even the extraction of NN on-shell information, which is difficult to deduce from NN scattering experiments, might be possible. Such expectations are, however, tempered by the experimental situation which for the $^2\mathrm{H}(n,nn)^1\mathrm{H}$ reaction is rather unsatisfactory: Despite heroic efforts, the available data are sparse and of an accuracy which does not yet allow one to meaningfully differentiate between the various theoretical assumptions entering the calculations.

For the proton-induced reaction $^2\mathrm{H}(p,pp)n$, on the other hand, a body of accurate experimental data is available. There, however, the theoretical treatments up to now have been rather unsatisfactory: The Coulomb potential acting between the two protons has in general been either completely neglected or only partially taken into account in a rather *ad hoc* manner. But from the one published case where $^2\mathrm{H}(n,nn)^1\mathrm{H}$ [3] and $^2\mathrm{H}(p,pp)n$ [4] experiments have been performed under identical kinematical conditions, it has become clear that Coulomb repulsion influences the spectrum of the emitted particles in a way which, at least in certain kinematical configurations, might completely veil the expected (smaller) effects of the above-mentioned interesting topics.

For the inclusion of long-ranged forces in composite-particle scattering theories, essentially only two proposals exist in the literature yielding equations that are *practical* and usable also *beyond the bound state dissociation threshold.* One is based on integro-differential equations in coordinate space [5], which up to now has only been applied to pd elastic scattering and breakup calculations in a very approximate manner [6]. The other one, formulated in terms of momentum space integral equations, makes use of the screening and renormalization method [7]. Among other things, it has been employed successfully for the calculation of elastic pd scattering phase parameters and cross sections (see [8,9] and references therein). It has now also been applied to the breakup of deuterons by proton impact.

We only sketch here the basic idea of this approach; for details see [7,10]. We restrict ourselves to the case in which the (nuclear parts of the) T matrices describing the scattering in the various NN subsystems are separable, where the formalism becomes particularly simple and transparent. Of course, according to the two-potential picture, the total T matrix for the pp subsystem contains, in addition, the pure (screened) Coulomb amplitude T^R that is, and should be kept, nonseparable (R denotes the screening radius). Note that by switching off the Coulomb interaction the formalism yields the amplitudes for the neutron-induced reaction.

We denote the identical particles [the two protons (neutrons) in the proton- (neutron-) induced reaction] by 1 and 2, and the odd particle by 3. Thus the α-subsystem T matrix is assumed to be given as $T_\alpha^{(R)} = \sum_m |\tilde{\chi}_{\alpha m}\rangle \Delta_{\alpha m} \langle \tilde{\chi}_{\alpha m}| + \delta_{\alpha 3} T^R$, $\alpha = 1,2,3$, with $|\tilde{\chi}_{\alpha m}\rangle = (1+\delta_{\alpha 3} T^R G_0)|\chi_{\alpha m}\rangle$ and $G_0(z) = (z - H_0)^{-1}$. Furthermore, $\Delta_{\alpha m}^{-1} = \lambda_{\alpha m}^{-1} - \langle \chi_{\alpha m}|G_0|\tilde{\chi}_{\alpha m}\rangle$. Here and in the following the energy dependence of the various operators is suppressed. Usually, such a representation for $T_\alpha^{(R)}$ is derived from the assumption that the nuclear parts of the underlying potentials are separable, i.e., $V_\alpha^{(R)} = \sum_m |\chi_{\alpha m}\rangle \lambda_{\alpha m} \langle \chi_{\alpha m}| + \delta_{\alpha 3} V^R$. The form factors $|\chi_{\alpha m}\rangle$ and the coupling parameters $\lambda_{\alpha m}$ are then chosen so as to reproduce the low energy scattering observables in subsystem α, in the states with quantum numbers "m." For the Coulomb potential, exponential screening is chosen for convenience, i.e., $V^R(r) = (e^2/r)\exp(-r/R)$. Hence, for finite R all potentials are of short range so that conventional scattering theory is applicable.

As is well known (see, e.g., [11]) the three-body breakup amplitudes can be calculated either by solving integral equations or by quadrature from the arrangement amplitudes. We have chosen the latter procedure. Correspondingly, the (screened) effective two-body operator $\mathcal{T}_{0,1d}^{(R)}$ describing the breakup of a deuteron, composed of proton 2 and neutron 3, by the impinging proton

1, leading to a three-free particle final state, is obtained from the screened two-fragment amplitudes $\mathcal{T}^{(R)}_{\gamma r,1d}$ via

$$\mathcal{T}^{(R)}_{0,1d} = \mathcal{V}^{(R)}_{0,1d} + \sum_{\gamma=1}^{3}\sum_{r} \mathcal{V}^{(R)}_{0,\gamma r}\, \mathcal{G}^{(R)}_{0;\gamma r}\, \mathcal{T}^{(R)}_{\gamma r,1d}. \tag{1}$$

We denote by $\mathbf{q}_1$ the center-of-mass momentum in the initial state, and by $\mathbf{p}'_3$ and $\mathbf{q}'_3$ the Jacobi relative momenta within the (pp) pair 3 and between the center of mass of the latter and the neutron in the final state. Then on the energy shell, i.e., when $E = 3q_1^2/4M+\hat{E}_d = 3q_3'^2/4M+p_3'^2/M$ (M denotes the nucleon mass and $\hat{E}_d$ the deuteron binding energy), the quantity $\mathcal{T}^{(R)}_{0,1d}(\mathbf{p}'_3,\mathbf{q}'_3,\mathbf{q}_1;E+i0) = \langle \mathbf{p}'_3|\langle \mathbf{q}'_3|\mathcal{T}^{(R)}_{0,1d}(E+i0)|\mathbf{q}_1\rangle$ is the physical (screened) breakup amplitude. The effective breakup potential $\mathcal{V}^{(R)}_{0,\alpha m}$ is given by

$$\begin{aligned}\mathcal{V}^{(R)}_{0,\alpha m}&(\mathbf{p}'_3,\mathbf{q}'_3,\mathbf{q}_\alpha;E+i0)\\ &= \langle \mathbf{p}'_3|\langle \mathbf{q}'_3|[1+T^R(E+i0)G_0(E+i0)]|\chi_{\alpha m}\rangle|\mathbf{q}_\alpha\rangle\\ &=: \mathcal{V}^{(0)}_{0,\alpha m}(\mathbf{p}'_3,\mathbf{q}'_3,\mathbf{q}_\alpha;E+i0)\\ &\quad+\tilde{\mathcal{V}}^{(R)}_{0,\alpha m}(\mathbf{p}'_3,\mathbf{q}'_3,\mathbf{q}_\alpha;E+i0),\end{aligned} \tag{2}$$

with the first part $\mathcal{V}^{(0)}_{0,\alpha m}$ being the well-known driving term for neutron-induced breakup. We stress that for separable nuclear potentials, and with only two of the three particles being charged, expression (2) is *exact.* Moreover, $\langle \mathbf{q}'_\gamma|\mathcal{G}^{(R)}_{0;\gamma r}(E+i0)|\mathbf{q}_\gamma\rangle = \delta(\mathbf{q}'_\gamma-\mathbf{q}_\gamma)\,\Delta_{\gamma r}(E+i0-3q_\gamma^2/4M)$. The two-fragment amplitudes $\mathcal{T}^{(R)}_{\beta n,1d}$ are solutions of a set of coupled Lippmann-Schwinger (LS)-type equations with effective arrangement potentials $\mathcal{V}^{(R)}_{\beta n,\gamma m}$ [11]. (Again, for separable nuclear potentials and two charged and one neutral particles, $\mathcal{V}^{(R)}_{\beta n,\gamma m}$ assumes a simple *exact* and *closed* form.) As discussed in [7], for $\alpha \neq 3$ and with the α-subsystem being in the deuteron state, the elastic potential matrix element $\mathcal{V}^{(R)}_{\alpha d,\alpha d}(\mathbf{q}'_\alpha,\mathbf{q}_\alpha;E+i0)$ contains, as its longest-ranged part the so-called center-of-mass Coulomb potential, which, for exponential screening, looks like $v^R_\alpha(\mathbf{q}'_\alpha,\mathbf{q}_\alpha) = e^2/2\pi^2[(\mathbf{q}'_\alpha-\mathbf{q}_\alpha)^2+R^{-2}]$. It describes the Coulomb scattering of proton α (= 1 or 2) off the total charge of the deuteron concentrated in its center of mass. Let t^R_α be the amplitude, and $\psi^{(+)}_{R,\mathbf{q}_\alpha}$ the scattering wave function (for an asymptotic momentum $\mathbf{q}_\alpha$ and energy $3q_\alpha^2/4M$), to be calculated from a two-body LS equation with potential v^R_α. Then the Coulomb-modified short-range (CMSR) arrangement amplitudes are obtained via

$$\begin{aligned}\mathcal{T}^{SR}_{\beta n,1d}(\mathbf{q}'_\beta,\mathbf{q}_1;E+i0) &:= \mathcal{T}^{(R)}_{\beta n,1d}(\mathbf{q}'_\beta,\mathbf{q}_1;E+i0)\\ &\quad-\delta_{\beta 1}\delta_{nd}t^R_1\left(\mathbf{q}'_1,\mathbf{q}_1;\frac{3q_1^2}{4M}+i0\right).\end{aligned} \tag{3}$$

Insertion of (3) into (1) leads to an analogous decomposition of the breakup amplitudes

$$\begin{aligned}\mathcal{T}^{(R)}_{0,1d}(\mathbf{p}'_3,\mathbf{q}'_3,\mathbf{q}_1;E+i0) &= \mathcal{B}^R_{0,1d}(\mathbf{p}'_3,\mathbf{q}'_3,\mathbf{q}_1;E+i0)\\ &\quad+\mathcal{T}^{SR}_{0,1d}(\mathbf{p}'_3,\mathbf{q}'_3,\mathbf{q}_1;E+i0).\end{aligned} \tag{4}$$

The driving term $\mathcal{B}^R_{0,1d}$, which describes the pure Coulomb breakup, is given on the energy shell as

$$\begin{aligned}\mathcal{B}^R_{0,1d}&(\mathbf{p}'_3,\mathbf{q}'_3,\mathbf{q}_1)\\ &= \int d^3\mathbf{q}''_1\,\mathcal{V}^{(R)}_{0,1d}(\mathbf{p}'_3,\mathbf{q}'_3,\mathbf{q}''_1)\psi^{(+)}_{R,\mathbf{q}_1}(\mathbf{q}''_1).\end{aligned} \tag{5}$$

The CMSR breakup part $\mathcal{T}^{SR}_{0,1d}$ follows from the corresponding arrangement amplitudes by quadrature

$$\begin{aligned}\mathcal{T}^{SR}_{0,1d}(\mathbf{p}'_3,\mathbf{q}'_3,\mathbf{q}_1) &= \sum_{\gamma r}\int d^3\mathbf{q}''_\gamma\,\mathcal{V}^{(R)}_{0,\gamma r}(\mathbf{p}'_3,\mathbf{q}'_3,\mathbf{q}''_\gamma)\\ &\quad\times\mathcal{G}^{(R)}_{0;\gamma r}(\mathbf{q}''_\gamma)\mathcal{T}^{SR}_{\gamma r,1d}(\mathbf{q}''_\gamma,\mathbf{q}_1).\end{aligned} \tag{6}$$

Then, as shown in [7], after renormalization by explicitly known factors $Z_3^{(R)\,-1/2}(p'_3)$ and $Z_1^{(R)\,-1/2}(q_1)$, the zero-screening limit exists for both

$$\begin{aligned}Z_3^{(R)\,-1/2}(p'_3)\,\mathcal{B}^R_{0,1d}(\mathbf{p}'_3,\mathbf{q}'_3,\mathbf{q}_1;E+i0)\,Z_1^{(R)\,-1/2}(q_1) &\xrightarrow{R\to\infty} \mathcal{B}^C_{0,1d}(\mathbf{p}'_3,\mathbf{q}'_3,\mathbf{q}_1)\\ &= \int d^3\mathbf{q}''_1\,\psi^{(-)*}_{C,\mathbf{p}'_3}(\mathbf{k}'[\mathbf{q}'_3,\mathbf{q}''_1])\,\chi_{1d}(\mathbf{k}[\mathbf{q}'_3,\mathbf{q}''_1])\,\psi^{(+)}_{C,\mathbf{q}_1}(\mathbf{q}''_1),\end{aligned} \tag{7}$$

and

$$\begin{aligned}Z_3^{(R)\,-1/2}(p'_3)\,\mathcal{T}^{SR}_{0,1d}&(\mathbf{p}'_3,\mathbf{q}'_3,\mathbf{q}_1;E+i0)\,Z_1^{(R)\,-1/2}(q_1)\\ &\xrightarrow{R\to\infty} \mathcal{T}^{SC}_{0,1d}(\mathbf{p}'_3,\mathbf{q}'_3,\mathbf{q}_1).\end{aligned} \tag{8}$$

In (7), $\mathbf{k}$ and $\mathbf{k}'$ have to be expressed as the standard linear combinations of the indicated momenta. Moreover, $\psi^{(-)}_{C,\mathbf{p}'_3}$ denotes the unscreened Coulomb wave function for the two outgoing protons, and $\psi^{(+)}_{C,\mathbf{q}_1}$ the unscreened center-of-mass Coulomb wave function describing the initial-state Coulomb distortion. In this way we obtain the full breakup amplitude for *unscreened* Coulomb potentials as

$$\begin{aligned}\mathcal{B}^C_{0,1d}(\mathbf{p}'_3,\mathbf{q}'_3,\mathbf{q}_1) &+ \mathcal{T}^{SC}_{0,1d}(\mathbf{p}'_3,\mathbf{q}'_3,\mathbf{q}_1)\\ &= \mathcal{T}_{0,1d}(\mathbf{p}'_3,\mathbf{q}'_3,\mathbf{q}_1).\end{aligned} \tag{9}$$

Finally we stress that one of the main advantages of the present approach is that the (screened or unscreened)

Coulomb amplitude is taken into account in full three-dimensional form. Thus no problems arise here from possible lack of convergence of the Coulomb partial wave series.

In the actual calculation we proceed as follows. We solve the appropriately antisymmetrized equations for the arrangement amplitudes and the two-body LS equation for t_1^R to get the CMSR arrangement amplitude (3). The latter is then introduced in (6) to find the CMSR breakup amplitude. The whole procedure is repeated for increasing values of R until the left-hand side of (8) becomes independent of the screening radius R. In this way we numerically perform the unscreening limit. To this we add (7) which has been evaluated analytically in the coordinate space representation, thereby arriving at the desired physical pd breakup amplitude in (9). We point out that while the integral equations for $\mathcal{T}^{(R)}_{\beta n,1d}$ were solved with the single approximation of replacing the pp Coulomb T matrix T^R by V^R, the breakup potentials $\mathcal{V}^{(R)}_{0,\gamma r}$ have been calculated exactly.

We have used charge- and spin-dependent separable NN potentials of rank one acting in S waves only, with the parameters given in [8]. By proceeding as described above we have calculated the fivefold differential cross sections $d^5\sigma/d\Omega_1^L d\Omega_2^L dE_S$, where Ω_1^L and Ω_2^L are the laboratory angles of the two measured protons (or neutrons for the neutron reaction) and E_S is the arclength along the kinematically allowed curve. As a first example we present in Fig. 1 proton and neutron cross sections at 14.1 MeV for a np final-state interaction (FSI) configuration. In the peak region the Coulomb corrections are definitively non-negligible and bring the theoretical calculations into rather good agreement with the experimental data [12]. However, it is to be pointed out that the neutron-induced FSI cross sections, in general, appear to depend appreciably on the nuclear potential model. This is exemplified by showing a few selected values of a $^1S_0$, $^3S_1$-$^3D_1$ calculation for the Paris potential [2]. Moreover, we include the cross section obtained by retaining the Coulomb effects in the arrangement amplitudes, but omitting those in the breakup potential, i.e., when $\mathcal{V}^{(R)}_{0,\gamma r}$ is replaced by $\mathcal{V}^{(0)}_{0,\gamma r}$ in (6). Comparison with the full result shows that the Coulomb effects due to $\tilde{\mathcal{V}}^{(R)}_{0,\gamma r}$ practically do not contribute in the FSI peaks. In Fig. 2 the theoretical cross sections at 14.1 MeV for a collinear configuration are compared with data [12]. Inspection shows that Coulomb effects are significant over the whole kinematic curve, and again arise essentially from the arrangement and not from the breakup part in (6). The sensitivity to the choice of the nuclear interaction can be estimated from a comparison of our neutron reaction cross section with the one obtained for the Paris potential [2]. Cross sections for a quasifree scattering (QFS) region at the same energy are shown in Fig. 3. In the neighborhood of the QFS point Coulomb effects are seen to reduce the peak cross section significantly. Consequently, the expectation [13], based on discrepancies between neutron calculations and proton data, that signatures of three-body forces might be visible in such kinematical regions cannot be corroborated. Moreover, the situation differs from the previous cases in that Coulomb effects from the arrangement amplitudes play a negligible role here; that is, practically all the Coulomb corrections arise from the breakup potential. Finally, in Fig. 4 a space star (SST) configuration is presented, but now at 13 MeV where experimental neutron [3] and proton [4] data are available. The Coulomb effects produce an approximately 10% reduction of the corresponding neutron cross section over a wide range of arclengths, which is in qualitative agreement with the data although there the quantitative difference appears to be distinctly larger. It

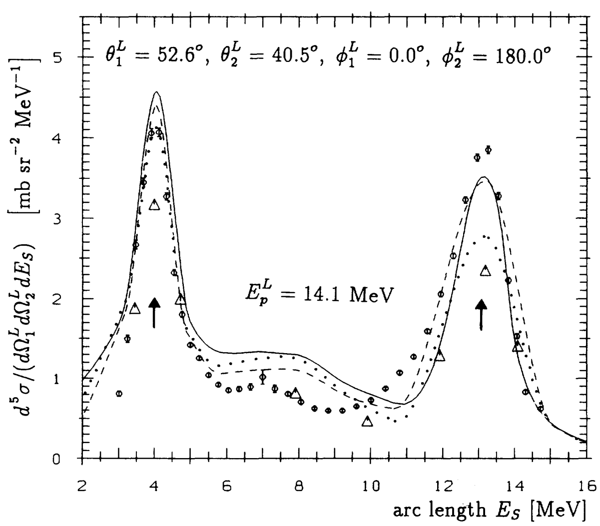

FIG. 1. ^{2}H$(p,pp)n$ cross section at 14.1 MeV proton laboratory energy, for a np FSI configuration, against the arclength. Arrows indicate the FSI conditions. Experimental data are from [12]. (—), Full calculation, $(\cdots)$ ^{2}H$(n,nn)^1$H cross section; (- - -), proton cross section but with Coulomb effects taken into account in the arrangement amplitude only. For comparison a few Paris potential results ($\triangle$) for ^{2}H$(n,nn)^1$H are shown [2].

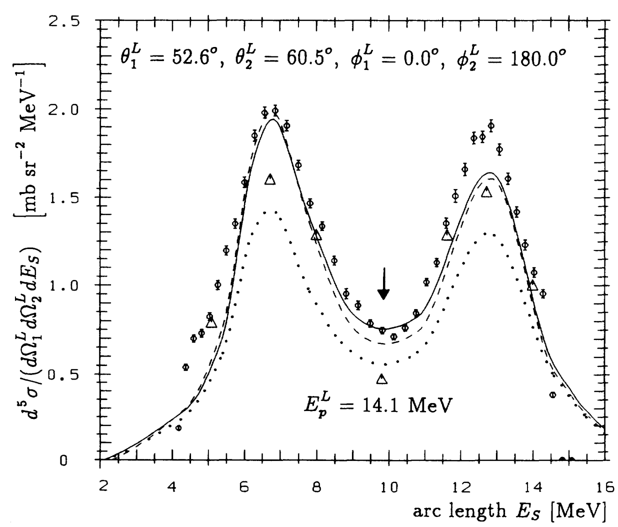

FIG. 2. As in Fig. 1, but for a pp (nn) collinear configuration. The collinearity point is marked by an arrow.

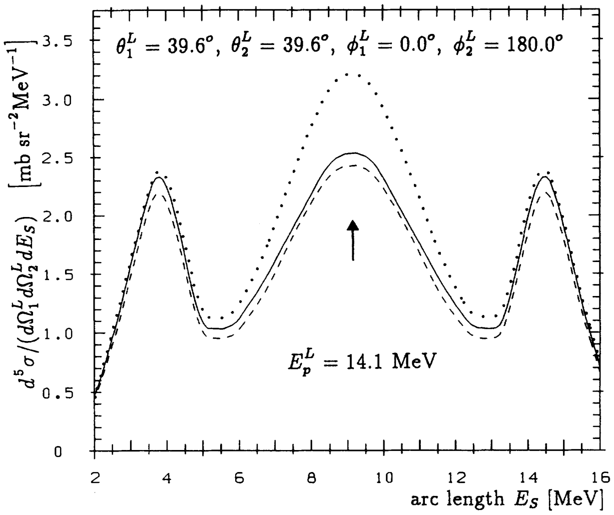

FIG. 3. Cross section for pp (nn) QFS. Curves as described in Fig. 1. The arrow shows the QFS point.

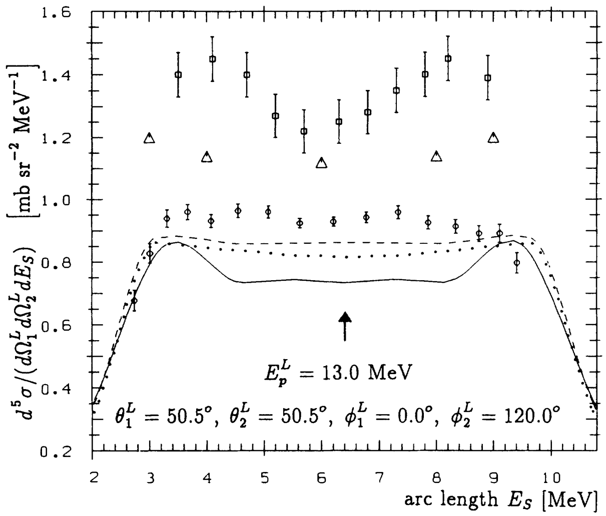

FIG. 4. A SST configuration at 13 MeV. Curves as described in Fig. 1. The SST point is marked by an arrow. (□), ^{2}H$(n,nn)^1$H data from [3]; (○) ^{2}H$(p,pp)n$ data from [4]; (△), Paris potential results for ^{2}H$(n,nn)^1$H [13].

should be mentioned that a more realistic calculation for the neutron reaction, which includes higher NN partial waves, gives a cross section that lies higher than ours, but still falls significantly below the experimental one [13]. We point out that the Coulomb effects arise here in roughly equal parts in the breakup potential and the arrangement amplitudes.

Hence the following general conclusions can be drawn. (i) In some parts of three-body configuration space Coulomb effects are important, while in others they play a minor role. Hence, only in the latter situations can a comparison of sophisticated calculations of the neutron-induced reaction with experimental proton-induced reaction data be expected to provide new physical information. (ii) In FSI and collinear configurations the main effects stem from the Coulomb corrections in the CMSR arrangement amplitudes $\mathcal{T}^{SR}_{\beta r,1d}$. However, in other kinematical regions, the Coulomb contributions from the breakup potentials are also important. (iii) As is apparent, in most of the configurations investigated the experimental cross section data can be described with reasonable accuracy by the inclusion of Coulomb effects, despite the simplicity of the nuclear model employed. Hence little room is left for "nonstandard" effects. (iv) In order to further test the importance of Coulomb effects in the deuteron breakup reaction, more proton and neutron cross sections at *identical kinematical configurations* are needed.

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