

***s*-wave pion-nucleus interaction and weak coupling constants**

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The chiral expansion of the off-shell πN amplitude suggests that the *s*-wave interaction with the nucleus of spacelike pions is appreciably repulsive. This has important consequences for the weak hadronic nuclear current, which are discussed here.

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The importance in a number of nuclear phenomena of the *s*-wave part of the interaction of off-shell pions with the nucleus has gradually emerged. One of the places where this interaction is manifest is the quark condensate which signals the spontaneous violation of chiral symmetry. It is modified in the nucleus, with a decrease of its magnitude, so that there is in the nuclear medium a partial restoration of chiral symmetry [1–3]. The change is governed by the sigma commutator matrix element, which is nothing else than the scattering amplitude for soft pions. Another case where this interaction might play an important role is in the experiments aiming at the detection of a pion excess in the nucleus, such as the (p, n) reaction [4]. We have investigated the role of the *s*-wave interaction with the nucleus of the pion exchanged between the projectile and the target, which is appreciably off-shell with a spacelike character [5]. We have suggested that, in this case, the interaction is repulsive and that it could be responsible for the long standing discrepancy between the data and the calculations based on a random phase approximation (RPA) where only the *p*-wave interaction is incorporated. Since the weak interactions are linked through partially conserved axial current (PCAC) to pionic phenomena it is natural to investigate also in this case the role of this *s*-wave component. The influence of the *p*-wave interaction on the weak coupling constants was discussed long ago [6]. More recently the in-medium pion decay constant has been discussed in connection with the Gell-Mann–Oakes–Renner relation in which appears the time part of the axial current (in the medium the space and time components renormalize differently) [3,7]. It was shown to be linked to the properties of the *s*-wave pion self-energy [7].

The aim of the present work is to study the weak hadronic current in the nucleus, both space and time components, with a unique formalism which takes into account both the *p*- and *s*-wave parts of the pion self-energy. We derive the renormalization of the different coupling constants entering the weak axial current. The basis of our approach is PCAC in its standard form: $\partial^\mu A_\mu = f_\pi m_\pi^2 \Phi$ where Φ is the pion field and $f_\pi = 94$ MeV the pion decay constant. It is important to remark that the proportionality constant on the right-hand side

should not change in the medium since it is fixed by a given underlying Lagrangian. This does not prevent the two quantities f_π and m_π^2 from being renormalized when they appear in the expressions of A_μ and Φ . We give a general derivation and illustrate our results by a method based on meson exchange diagrams.

We consider the case of a homogeneous isospin symmetric infinite nuclear matter. We introduce the pion self-energy Π , denoting Π_s and Π_p its *s*- and *p*-wave parts, respectively. To leading order in the density this quantity is related to the isospin symmetric πN forward amplitude $T^{(+)}$ by $\Pi = -\rho T^{(+)}$. For the *p*-wave self-energy we keep the notations of our previous work [6] with the parametrization $\Pi_p = \alpha q^2$. The quantity $\alpha = \partial \Pi_p / \partial q^2$ which arises from the Δ -hole excitations is weakly energy dependent at low energies and we take it to be constant. Notice that we have discarded the important part of the self-energy which comes from the nucleon poles. As is common in studies of renormalization effects we consider that these *N*-hole excitations are separately accounted for by a convenient RPA treatment.

The attention will be focused here on the *s*-wave self-energy which is energy and momentum dependent. We introduce the following linear expansion in q^2 and q_0^2 around the soft pion point ($q_\mu = q'_\mu = 0$):

$$\Pi_s = \Pi_s(0) + q_0^2 \partial \Pi_s / \partial q_0^2 + q^2 \partial \Pi_s / \partial q^2 + \dots \quad (1)$$

with self-explaining notations. This expansion parallels that of the *s*-wave part of the πN amplitude, which in the static limit (i.e., infinite nucleon mass) is written:

$$T_s^{(+)} = \frac{-\Sigma_N}{f_\pi} \left(1 - \frac{q^2 + q'^2}{m_\pi^2} \right) + \beta q_0^2 + \dots \quad (2)$$

Here q (q') is the four-momentum of the incident (outgoing) pion (with the condition $q_0 = q'_0$ since we work in the static limit); Σ_N is the sigma commutator term which has a value of about 45 MeV. The coefficient β is linked to the effective range of the *s*-wave amplitude; its empirical value is such that the scattering length $a^{(+)} = (\Sigma_N / f_\pi^2 + \beta m_\pi^2) / 4\pi$ nearly vanishes, so that $\beta \approx -\Sigma_N / f_\pi^2 m_\pi^2$. Notice that the expansion (2) satisfies the Adler consistency condition of the vanishing of the amplitude for one pion soft and the other on-shell. The coefficient of the Σ_N term, which will play an important role in our discussion, is fixed by this constraint.

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Applying the expansion (2) to the forward amplitude, i.e., $q_\mu = q'_\mu$, we obtain the corresponding expansion of the self-energy:

$$\begin{aligned}\Pi_s &= \frac{\Sigma_{N\rho}}{f_\pi^2} \left(1 + \frac{2\mathbf{q}^2}{m_\pi^2} \right) - q_0^2 \rho \left(\beta + \frac{2\Sigma_{N\rho}}{f_\pi^2 m_\pi^2} \right) \\ &\approx \frac{\Sigma_{N\rho}}{f_\pi^2} \left(1 + \frac{2\mathbf{q}^2}{m_\pi^2} - \frac{q_0^2}{m_\pi^2} \right).\end{aligned}\quad (3)$$

At the soft point the interaction is repulsive: $\Pi_s(0) = \Sigma_{N\rho}/f_\pi^2$. We have pointed out in Ref. [5] that the expansion (3) implies that the repulsion becomes more important for spacelike pions of small energy. This is for instance the situation in muon capture where the energy transfer corresponds to typical nuclear excitations energies, in particular giant Gamow-Teller resonances, while the momentum transfer is of the order of the muon mass. The pion propagator in the nuclear medium is

$$D(\mathbf{q}, q_0) = [m_\pi^2 + \Pi_s(0) + \mathbf{q}^2(1 + \partial\Pi_s/\partial\mathbf{q}^2 + \partial\Pi_p/\partial\mathbf{q}^2) - q_0^2(1 - \partial\Pi_s/\partial q_0^2)]^{-1}. \quad (4)$$

Let us now turn to the axial nucleonic current. In free space a nonrelativistic reduction leads to the following expressions of its space and time components:

$$\begin{aligned}\mathbf{A} &= g_A \boldsymbol{\sigma} - \frac{f_\pi g}{M} \frac{\mathbf{q}(\boldsymbol{\sigma} \cdot \mathbf{q} - q_0 \boldsymbol{\sigma} \cdot \mathbf{p}/M)}{m_\pi^2 + \mathbf{q}^2 - q_0^2}, \\ A_0 &= g_A \frac{\boldsymbol{\sigma} \cdot \mathbf{p}}{M} - \frac{f_\pi g}{M} \frac{q_0(\boldsymbol{\sigma} \cdot \mathbf{q} - q_0 \boldsymbol{\sigma} \cdot \mathbf{p}/M)}{m_\pi^2 + \mathbf{q}^2 - q_0^2}.\end{aligned}\quad (5)$$

In the nuclear medium all quantities entering these expressions can be modified. However, the axial coupling constant as well as the πN coupling constants belong to vertex parts which are irreducible with respect to pion lines and remain unchanged since we study for the moment the case of a homogeneous medium where correlations are ignored. The in-medium expression of the current is

$$\begin{aligned}\mathbf{A}^* &= g_A \boldsymbol{\sigma} - \frac{(C f_\pi) g}{M^*} \frac{\mathbf{q}(\boldsymbol{\sigma} \cdot \mathbf{q} - q_0 \boldsymbol{\sigma} \cdot \mathbf{p}/M^*)}{m_\pi^2 + \Pi(0) + \mathbf{q}^2(1 + \partial\Pi_s/\partial\mathbf{q}^2 + \partial\Pi_p/\partial\mathbf{q}^2) - q_0^2(1 - \partial\Pi_s/\partial q_0^2)}, \\ A_0^* &= g_A \frac{\boldsymbol{\sigma} \cdot \mathbf{p}}{M^*} - \frac{(C_0 f_\pi) g}{M^*} \frac{q_0(\boldsymbol{\sigma} \cdot \mathbf{q} - q_0 \boldsymbol{\sigma} \cdot \mathbf{p}/M^*)}{m_\pi^2 + \Pi(0) + \mathbf{q}^2(1 + \partial\Pi_s/\partial\mathbf{q}^2 + \partial\Pi_p/\partial\mathbf{q}^2) - q_0^2(1 - \partial\Pi_s/\partial q_0^2)}.\end{aligned}\quad (6)$$

Here C and C_0 are the renormalization factors of the space and time parts of the pion decay vertex, respectively, and M^* is the effective nucleon mass. They are determined by the PCAC relation $\partial^\mu A_\mu = f_\pi m_\pi^2 \Phi$. In momentum space the divergence of the axial current reads

$$q_0 A_0^* - \mathbf{q} \cdot \mathbf{A}^* = -g_A [\boldsymbol{\sigma} \cdot \mathbf{q} - q_0 \boldsymbol{\sigma} \cdot \mathbf{p}/M^*] \left[1 + \frac{M}{M^*} (C_0 q_0^2 - C \mathbf{q}^2) D(\mathbf{q}, q_0) \right] \quad (7)$$

where we have made use of the Goldberger-Treiman relation $g_A = f_\pi g/M$. On the other hand, the operator $f_\pi m_\pi^2 \Phi$ has the matrix element

$$f_\pi m_\pi^2 \Phi^* = -\frac{f_\pi g}{M^*} m_\pi^2 [\boldsymbol{\sigma} \cdot \mathbf{q} - q_0 \boldsymbol{\sigma} \cdot \mathbf{p}/M^*] D(\mathbf{q}, q_0). \quad (8)$$

One can see in Eqs. (6)–(8) that we have used the πN pseudoscalar coupling. With the pseudovector coupling one should work with a modified coupling constant $f^* = g m_\pi/2M^*$ in order to keep untouched the PCAC relation. Identifying Eqs. (7) and (8) for all values of the momentum and the energy provides the renormalization factors (we would get the same answer by identifying the residues at the modified pion pole):

$$\frac{M^*}{M} = \left(1 + \frac{\Pi(0)}{m_\pi^2} \right)^{-1}, \quad C = (1 + \partial\Pi_s/\partial\mathbf{q}^2 + \partial\Pi_p/\partial\mathbf{q}^2) \frac{M^*}{M}, \quad C_0 = (1 - \partial\Pi_s/\partial q_0^2) \frac{M^*}{M} \quad (9)$$

or

$$\frac{M^*}{M} \approx \left(1 + \frac{\Sigma_{N\rho}}{f_\pi^2 m_\pi^2} \right)^{-1}, \quad C \approx \left(1 + \alpha + \frac{2\Sigma_{N\rho}}{f_\pi^2 m_\pi^2} \right) \left(1 + \frac{\Sigma_{N\rho}}{f_\pi^2 m_\pi^2} \right)^{-1}, \quad C_0 \approx 1. \quad (10)$$

The expressions (9) are general while the expressions (10) are those obtained in the approximation (3) for the self-energy. The factors C and C_0 represent the renormalization of the pion decay vertex, for the space and time components, respectively. Let us first comment on the time component C_0 , which we find here to be practically 1, while in Ref. [7] we found that it decreases as the square root of the condensate. Formally in the expression of C_0 the factor $(1 - \partial\Pi_s/\partial q_0^2)$ appeared with a square root. The reason for this difference lies in different definitions of the decay constant, since in the nuclear medium there is a latitude, as explained in Ref. [7]. Our present definition goes along with a full pion propagator and hence is different from our previous one which was chosen because of its compatibility with the Gell-Mann–Oakes–Renner relation. As for the factor C which enters in the space component, it differs in two points from that of our previous work [6] where the s -wave influence was ignored. The first one is the appearance here of a denominator which comes from the effective mass term. The second and most important one is the appearance of the momentum derivative of the s -wave self-energy. Indeed the parameter α in C arising from the p -wave interaction has an effective value of ≈ -0.63 at normal density. If it were alone, it would imply a strong suppression of the pion decay constant and hence of the pion pole term, as was stressed in Ref. [6]. However, when the s -wave term is incorporated with $2\Sigma_N\rho/f_\pi^2 m_\pi^2 = 0.7\rho/\rho_0$, the influence of the p -wave factor is nearly totally counterbalanced. The suppression effect is no longer present. The introduction of the s -wave self-energy thus introduces a major change in the perspective developed in previous works where this influence was ignored.

We introduce now the effect of the correlations which renormalize the axial coupling constants differently for the space and time components. For the space component the role of short-range correlations associated with p -wave axial production amounts to a quenching of g_A by the Lorentz-Lorenz factor $(1 - g'_{N\Delta}\alpha)^{-1}$ where $g'_{N\Delta}$ is the Landau-Migdal parameter which enters in the mixing force between N -hole and Δ -hole states. Additional quenching is provided by second order core polarization [8,9]. As for the time component, g_{A_0} , it is well known to be enhanced by soft pion exchange associated to Pauli correlations [10–13]. To be fully consistent we would have to introduce the effect of short range correlations also for s -wave pions. This question has been discussed in the case of soft pions [14]. It requires a delicate study of the comparative range of the scattering amplitude and the correlations that we do not attempt here. We define πN coupling constants g^* and g_0^* corresponding to the p - and s -wave parts of the vertex, respectively. At $q_\mu = 0$ PCAC implies that they are renormalized in the same way as the axial ones according to

$$g^*/g = g_A^*/g_A \text{ and } g_0^*/g_0 = g_{A_0}^*/g_{A_0}, \quad (11)$$

respectively. The axial current now becomes

$$\begin{aligned} \mathbf{A} &= g_A^* \boldsymbol{\sigma} + f_\pi C \mathbf{q} j_\pi^*(\mathbf{q}, q_0) D(\mathbf{q}, q_0), \\ A_0 &= g_{A_0}^* \boldsymbol{\sigma} \cdot \mathbf{p} + f_\pi C_0 q_0 j_\pi^*(\mathbf{q}, q_0) D(\mathbf{q}, q_0) \end{aligned} \quad (12)$$

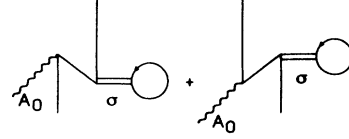


FIG. 1. Mechanism for enhancement by M^* denominators in the σ model: excitation of $N\bar{N}$ pairs (Hartree approximation).

where the pion source function is

$$j_\pi^*(\mathbf{q}, q_0) = -(g^* \boldsymbol{\sigma} \cdot \mathbf{q} - g_0^* q_0 \boldsymbol{\sigma} \cdot \mathbf{p}/M^*)/M^*. \quad (13)$$

This expression allows the evaluation of the nuclear axial current for any momentum and energy transfer, provided they are small enough ($\leq m_\pi$) for the linear expansion to hold. The situation of muon capture provides a good application case.

Before we turn to this question, it is illustrative to see how the renormalizations that we have derived are realized in a specific Lagrangian model. A good example is the linear σ model which satisfies chiral symmetry. The modification of the nucleon mass is governed by the exchange of the σ meson according to $M^*/M = 1 - g^2\rho/Mm_\sigma^2 = 1 - \Sigma_N\rho/f_\pi^2 m_\pi^2$ where we have expressed m_σ in terms of $\Sigma_N \approx gf_\pi m_\pi^2/m_\sigma^2$ and used the Goldberger-Treiman relation of the model $M = gf_\pi$. To lowest order in the density this expression is nothing else than the first of relations (10) [we have actually shown that relation (9) for the effective mass works up to second order in the density]. It is then natural that the matrix elements of Dirac odd operators, i.e., axial charge $\boldsymbol{\sigma} \cdot \mathbf{p}/M^*$, pseudoscalar vertex $\boldsymbol{\sigma} \cdot \mathbf{q}/2M^*$, etc., should be enhanced by the presence of a M^* denominator. Indeed in the language of exchange currents [15], there is an enhancement of the axial charge matrix elements by a nucleon effective mass effect corresponding to the excitation of nucleon-antinucleon pair via σ exchange (see Fig. 1). Such an estimate was, however, model dependent due to the fictitious character of the scalar meson with unknown mass and coupling. The relation that we make here with Σ_N which is a measured quantity seems quite safe. As for the renormalization of the pion axial decay vertex it corresponds to the $\sigma \rightarrow \pi$ current (Fig. 2). One gets in the Hartree limit

$$f_\pi q_\lambda \rightarrow f_\pi q_\lambda - \frac{g\rho}{m_\sigma^2} q_\lambda = f_\pi q_\lambda \left(1 - \frac{\Sigma_N\rho}{f_\pi^2 m_\pi^2}\right), \quad (14)$$

a result already known from Ref. [16]. Since time and space components of the vertex are renormalized in the

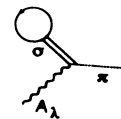


FIG. 2. Renormalization of f_π in the σ model: σ - π current.

same way, it is legitimate to consider this modification as a renormalization of f_π in the medium which turns out to follow that of the nucleon mass: $f_\pi^*/f_\pi = M^*/M$. One recovers here a feature common to chiral models. The above expression is again in agreement with the M^*/M factors in C and C_0 of Eqs. (9). Except for the absence of energy-momentum dependence in the self-energy, the σ model presents the same features as our general derivation.

We come now to a numerical application to muon capture and more specifically the effective pseudoscalar coupling constant. In free space it is defined as $g_P = 2m_\mu f_\pi g(m_\pi^2 - q^2)^{-1}$. In ordinary muon capture the energy transfer can be taken to be zero while the momentum transfer $|\mathbf{q}| \approx m_\mu$. In the medium the relevant coupling constant is $g_P^* = 2m_\mu C f_\pi g^* D(|\mathbf{q}| = m_\mu, 0)$. It is obvious in Eqs. (12) and (13) that it always appears in the current under the combination g_P^*/M^* . The effective mass factor present in C is thus canceled. As the remaining factor in C is practically unity, owing to strong cancellation between the s - and p -wave terms, we are left with two effects for the renormalization of g_P . First is the quenching of the πN vertex g^*/g for which we take the empirical value 0.70 at $\rho = \rho_0$ according to the observed quenching of Gamow-Teller strength [17]. Second is the renormalization of the pion propagator. At $|\mathbf{q}| = m_\mu$ and $q_0 = 0$ its expression is $D(m_\mu, 0) \approx [m_\pi^2 + \Pi_s(0) + m_\mu^2]^{-1}$ which is quenched by the repulsive s -wave interaction $\Pi_s(0)$. In our previous work where s -wave interaction was neglected one had instead $D(m_\mu, 0) = [m_\pi^2 + (1 + \alpha)m_\mu^2]^{-1}$ which corresponds to enhancement. Altogether our present renormalization of g_P amounts to a factor 0.60 to be compared to the stronger quenching 0.33 in the absence of s -wave interaction. Furthermore, experiments measuring correlations [18–20] are actually sensitive to the ratio g_P^*/g_A^* where the quenching factors of g^* and g_A^* factors cancel. One is left there with a quenching of 0.85 only while the p -wave alone would give 0.47. For the lower densities, relevant for light nuclei such as ^{12}C , this ratio would be even closer to 1. This is a result worth emphasizing since it is in much more satisfactory agreement with experiment [18–21] than the strong quenching we obtained previously. Another important test case is provided by the celebrated $^{16}\text{O}(0^+) \leftrightarrow ^{16}\text{N}(0^-)$ transition. It has attracted interest for a long time because it gives access to the soft pion exchange enhancement of the axial charge $g_{A_0}^*/g_{A_0}$, expected to be close to 50% [10–13] (see Ref. [22] for a recent analysis). In these nuclei, both the inverse processes of μ capture and β decay are measured so that the disposal of two related observables reduces the uncertainties of the interpretation. It is clear that they

should be reanalyzed in the light of our present work with due account of s -wave renormalization.

The process of radiative muon capture with photons in the upper part of the spectrum offers a kinematical situation where the energy carried by the virtual pion can be large. It is, however, much more complicated to interpret since both the weak and electromagnetic currents are involved giving rise to many Feynman graphs. It is thus not possible to define in a simple way effective pseudoscalar coupling constants. Furthermore the medium effects should be introduced in agreement with gauge invariance. We will be content here to calculate the renormalization of the pion propagator entering in the graph where the nucleon radiates. This is the pion pole graph to which the capture rate is the most sensitive. Indeed at maximum photon energy $E_\gamma \approx m_\mu$ the free space propagator is $(m_\pi^2 - m_\mu^2)^{-1}$, rather close to the pion pole. For a more realistic kinematical situation we select $E_\gamma = 70$ MeV, hence the pion carries momentum-energy $\mathbf{q} = 30$ MeV, $q_0 = 70$ MeV. We find a quenching factor 0.74 instead of 0.79 for spacelike pions. With p -wave interaction only we would have found a small enhancement of 1.04 (1.30 in ordinary muon capture).

To conclude, we have studied the renormalizations of the weak coupling constants in nuclei introducing the s -wave interaction of the pions with the nuclear medium, besides the p -wave one. The indications are that the s -wave interaction is appreciably repulsive leading to quenching of the pion propagator at momentum-energy transfer relevant to ordinary muon capture. The PCAC constraint fixes then the pion decay vertex. Its previously known quenching arising from the p -wave pion interaction is practically canceled by the s -wave one. In addition there appears in its expression a nucleon effective mass factor which is common to chiral models. Another requirement of PCAC is a relation between the nucleon effective mass in the medium and the soft pion self-energy $\Pi_s(0)$ which in turn can be expressed in terms of a measurable quantity, the Σ commutator. A consequence for the induced pseudoscalar coupling constant is a much more moderate quenching than once believed. We stress that the present treatment applies only to infinite nuclear matter. It should be extended to specific nuclear transitions for a meaningful comparison. Surface effects which can play an important role have to be taken into account [23]. Moreover, the role played by the s wave in presence of short range correlations remains to be investigated.

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- [1] E. G. Drukarev and E. M. Levin, Nucl. Phys. **A511**, 679 (1990); Prog. Part. Nucl. Phys. **27**, 77 (1991).
- [2] T. D. Cohen, R. J. Furnstahl, and D. K. Griegel, Phys. Rev. C **45**, 1881 (1992).
- [3] M. Lust, S. Klimt, and W. Weise, Nucl. Phys. **A542**, 521 (1992).

- [4] X. Y. Chen *et al.*, Phys. Rev. C **47**, 2159 (1993).
- [5] M. Ericson, IPN Lyon Report No. LYCEN 9359, 1993 (unpublished).
- [6] M. Ericson, A. Figureau, and C. Th  venet, Phys. Lett. **45B**, 19 (1973).
- [7] G. Chanfray, M. Ericson, and M. Kirchbach, Mod. Phys.

- Lett. (to be published).
- [8] K. Shimizu, M. Ichimura, and A. Arima, Nucl. Phys. **A226**, 282 (1974).
 - [9] I. S. Towner and F. C. Khanna, Phys. Rev. Lett. **42**, 51 (1979).
 - [10] K. Kubodera, J. Delorme, and M. Rho, Phys. Rev. Lett. **40**, 755 (1978).
 - [11] P. Guichon, M. Giffon, and C. Samour, Phys. Lett. **74B**, 15 (1978).
 - [12] I. S. Towner and F. C. Khanna, Nucl. Phys. **A372**, 331 (1981).
 - [13] E. K. Warburton and I. S. Towner, Phys. Rep. (to be published).
 - [14] T. E. O. Ericson, Phys. Lett. (to be published).
 - [15] J. Delorme and I. S. Towner, Nucl. Phys. **A475**, 720 (1987).
 - [16] E. K. Akhmedov, Nucl. Phys. **A500**, 596 (1989).
 - [17] C. Gaarde, J. Rapaport, T. N. Taddeucci, C. D. Goodman, C. C. Foster, D. E. Bainum, C. A. Goulding, M. B. Greenfield, D. J. Horen, and E. Sugarbaker, Nucl. Phys. **A369**, 258 (1981).
 - [18] A. Possoz, D. Favart, L. Grenacs, J. Lehmann, P. Macq, D. Meda, L. Palffy, J. Julien, and C. Samour, Phys. Lett. **50B**, 438 (1974).
 - [19] L. Ph. Roesch, V. L. Telegdi, P. Truttmann, A. Zehnder, L. Grenacs, and L. Palffy, Phys. Rev. Lett. **46**, 1507 (1981).
 - [20] J. Deutsch and L. Grenacs, private communication.
 - [21] P. A. M. Guichon and C. Samour, Nucl. Phys. **A382**, 461 (1982).
 - [22] E. K. Warburton, I. S. Towner, and B. A. Brown, Phys. Rev. C **49**, 824 (1994).
 - [23] J. Delorme, M. Ericson, A. Figureau, and C. Thévenet, Ann. Phys. (N.Y.) **102**, 273 (1976).