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Single-particle spectral function of ^{16}O

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The influence of short-range correlations on the p-wave single-particle spectral function in ^{16}O is studied as a function of energy. This influence, which is represented by the admixture of high-momentum components, is found to be small in the p-shell quasihole wave functions. It is therefore unlikely that studies of quasihole momentum distributions using the $(e,e'p)$ reaction will reveal a significant contribution of high-momentum components. Instead, high-momentum components become increasingly more dominant at higher excitation energy. The above observations are consistent with the energy distribution of high-momentum components in nuclear matter.

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The influence of short-range correlations in finite nuclei has been studied theoretically by calculating the momentum distribution in the ground state of a particular nucleus [1–5]. These results clearly show that for momenta above 400 MeV/c short-range and tensor correlations completely dominate the momentum distribution. The momentum distribution for a given lj combination is given by

$$n_{lj}(k) = \langle \Psi_0 | \, a^{\dagger}_{klj} a_{klj} \, | \Psi_0 \rangle \, . \tag{1}$$

The total momentum distribution is obtained by summing over all lj combinations multiplying each contribution with the relevant degeneracy factor. It is possible to rewrite Eq. (1) by inserting a complete set of $A-1$ particle states with the result

$$n_{lj}(k) = \sum_n \left| \langle \Psi_n | \, a_{klj} \, | \Psi_0 \rangle \right|^2 . \tag{2}$$

In a simple mean-field description this sum is exhausted by the transition from the ground state of the A particle system to the ground state of the $A-1$ particle system when the lj combination corresponds to the last occupied single-particle level. In that case Eq. (2) represents nothing but the square of the corresponding single-particle wave function in momentum space. When correlations beyond the mean field are present, this is no longer true, although the ground state to ground state transition might still dominate at least for small momenta. This has been observed in Ref. [6], where it is demonstrated in the example of ^{3}He, that the ground state to ground state contribution to Eq. (2) contributes only an insignificant fraction of the high-momentum components, which must come from the contribution of the excited states in the $A-1$ system. This has been confirmed in Ref. [7] for the nucleus ^{4}He. In this work the ground state to ground state transition was calculated for both ^{3}He and ^{4}He. In other words, the calculation of high-momentum components in nuclei requires the knowledge of the complete energy dependence of the nucleon hole spectral function

$$S_{lj}(k,E) = \sum_n \left| \langle \Psi_n | \, a_{klj} \, | \Psi_0 \rangle \right|^2 \delta(E - (E_0^A - E_n^{A-1})). \tag{3}$$

By integrating $S_{lj}(k,E)$ from $-\infty$ to $\epsilon_F^- = E_0^A - E_0^{A-1}$, which represents the energy difference between the corresponding ground states, one obtains the contribution from this particular lj combination to the total momentum distribution. Clearly $S_{lj}(k,E)$ contains the information on the location of high-momentum components which can be studied in the $(e,e'p)$ reaction.

A recent proposal to study short-range correlations with the $(e,e'p)$ reaction focuses on the low-lying discrete transitions at high missing momentum [8]. This proposal has been inspired by the work of Ref. [9] for ^{3}He droplets of a finite number of atoms. In this work the corresponding coordinate space contribution to Eq. (2) from the ground state to ground state transition was evaluated. A simple procedure was developed to obtain the amplitude $\langle \Psi_n | \, a_{rlj} \, | \Psi_0 \rangle$, usually referred to as the quasihole wave function, from a corresponding mean-field wave function. In Ref. [10] a phenomenological prescription was developed to study the change from standard Woods-Saxon wave functions to the corresponding quasihole wave function for nuclei. A general discussion of quasihole (quasiparticle) properties and a many-body analysis based on experimental information is available in Ref. [11]. Based on the work in Refs. [10,11] one obtains a suppression of the mean-field wave function in the nuclear interior which results in a corresponding quasihole wave function with high-momentum components, which are sensitive to this suppression. Whether these high-momentum components follow from the inclusion of short-range correlations induced by a realistic nucleon-nucleon interaction is not clear however.

In order to study this question and to elucidate the presence of high-momentum components in nuclei, the nucleon hole spectral function for the p states in ^{16}O has been calculated in a complete energy domain. Short-range and tensor correlations have been evaluated explic-

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itly for the finite system under study. Although the nucleon hole spectral function has been carefully studied in nuclear matter starting with the work of Refs. [12,13] (see also a review in Ref. [14]), no complete microscopic calculations are available for nuclei heavier than ^{3}He [15]. The relevant method for the present study has been developed in Ref. [16]. In Ref. [16] the nucleon self-energy was calculated in ^{16}O using a G-matrix interaction calculated from a realistic nucleon-nucleon interaction. A Hartree-Fock-like contribution was identified which is obtained by using a real G matrix, calculated in nuclear matter at an appropriate starting energy and density, in the corresponding Hartree-Fock diagram for the self-energy in ^{16}O.

The imaginary part of the self-energy is obtained by calculating the relevant second-order diagrams in this G-matrix interaction which contain the two-particle–one-hole and two-hole–one-particle terms appropriate for this nucleus. The former term is responsible for the depletion of strength, which in mean field is located below the Fermi energy, to high energy. The latter term is essential for the accumulation of single-particle strength below the Fermi energy from states (in particular those with high momenta) which are empty in mean field. As a single-particle basis the relevant bound states of the Hartree-Fock term are included and for states at positive energy plane waves are employed with corresponding single-particle energies. These plane waves are properly orthogonalized to the bound states (if present) and enclosed in a box of sufficiently large radius to allow a convenient discretization [16]. The real part of the self-energy is obtained by using dispersion relations relevant for these two self-energy contributions. To avoid double counting for the real part, the corresponding second-order term calculated in nuclear matter at the original starting energy is subtracted.

In the present work the one boson exchange potential C is used as a realistic nucleon-nucleon interaction [17]. Whereas the influence of short-range correlations is carefully considered in this work, no attempt is made to treat the coupling to the very low-lying two-particle–one-hole and two-hole–one-particle states in an adequate way. Attempts at such a treatment can be found in Refs. [18–22] (see also Ref. [14]). To obtain the nucleon hole spectral function one needs to solve the Dyson equation for the single-particle propagator

$$g_{lj}(k_1,k_2;E) = g_{lj}^{(0)}(k_1,k_2;E) + \sum_{k_3,k_4} g_{lj}^{(0)}(k_1,k_3;E)\Sigma_{lj}(k_3,k_4;E)g_{lj}(k_4,k_2;E), \quad (4)$$

where $g^{(0)}$ refers to the Hartree-Fock propagator and Σ_{lj} represents the real and imaginary parts of the irreducible self-energy calculated from the second-order terms discussed above. For the energies E of interest here (below $\epsilon_F^- = E_0^A - E_0^{A-1}$), the solutions of Eq. (4) are insensitive to the discretization of the momentum integrals, if the radius of the box, which determines the grid of momenta k_i, is sufficiently large. The spectral function for hole strength is obtained from the diagonal matrix element of g_{lj} by taking the imaginary part and dividing by π. In the present work the solution of the Dyson equation yields discrete solutions corresponding to the $p\frac{1}{2}$ ground state as well as the first $p\frac{3}{2}$ excited state of the $A = 15$ system. These discrete quasihole solutions are obtained by solving the eigenvalue problem corresponding to Eq. (4). The eigenvector corresponding to these discrete states yields the quasihole wave function in momentum space, which still needs to be normalized by the spectroscopic factor

$$| w_{\alpha_{qh}}^n |^2 = \left(1 - \left.\frac{\partial\Sigma(\alpha_{qh},\alpha_{qh};E)}{\partial E}\right|_{\epsilon_{qh}}\right)^{-1}, \quad (5)$$

where α_{qh} corresponds to the quasihole single-particle quantum numbers and the self-energy, which is real at the quasihole energies ϵ_{qh}, is calculated for these quantum numbers [14]. The quasihole energies obtained in the present work yield -17.9 MeV for the $p\frac{3}{2}$ and -14.1 MeV for the $p\frac{1}{2}$ state, respectively. The results for the strength of the quasihole poles is 0.89 for the $p\frac{1}{2}$ state and 0.91 for the $p\frac{3}{2}$ state, respectively. These numbers can be compared with experimentally determined spectroscopic factors which have recently been determined at the NIKHEF facility [23]. Although the present theoretical result overestimates the experimental result by about 0.2, it is clear that a considerable renormalization of the strength is to be expected due to the coupling of the single-hole states to the low-lying collective excitations, which are not treated in this work. Instead, one should view the quasihole strength that is obtained here, to be

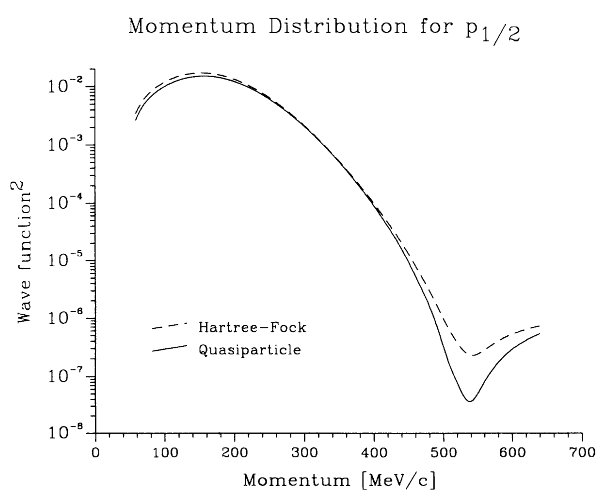

FIG. 1. Square of the quasihole wave function for the $p\frac{1}{2}$ state in ^{16}O (solid curve), normalized to the spectroscopic factor according to Eq. (5), compared to the Hartree-Fock result (dashed curve).

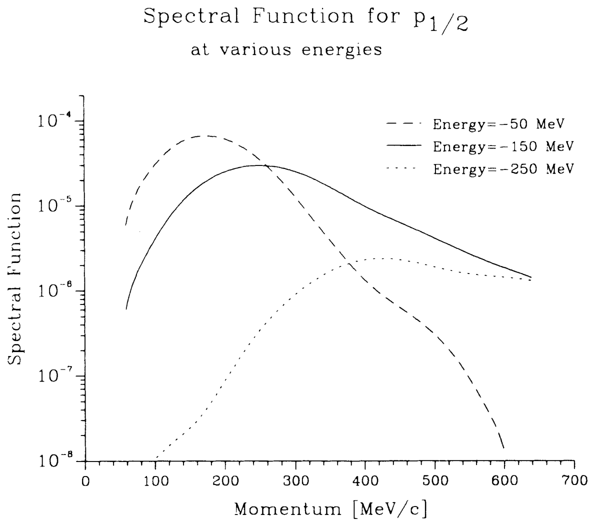

FIG. 2. The $p\frac{1}{2}$ spectral function as a function of momentum at fixed energies corresponding to -50, -100, and -250 MeV. The results demonstrate the increasing importance of high-momentum components with higher excitation energy in $A-1$ system (more negative energy).

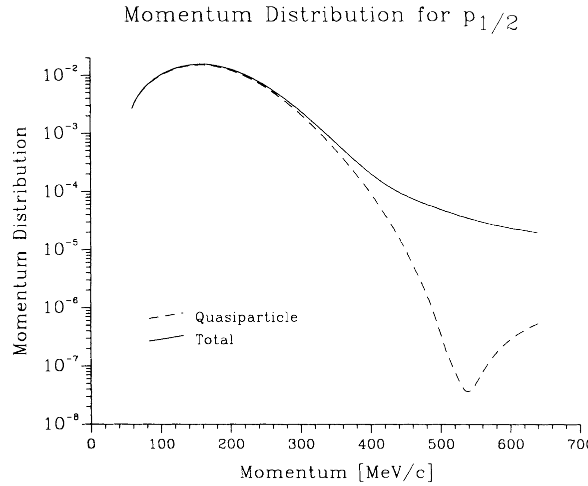

FIG. 3. Total $p\frac{1}{2}$ momentum distribution (solid curve) compared to quasihole contribution (dashed curve).

the result of the influence of short-range correlations [16].

The square of the quasihole wave function for the $p\frac{1}{2}$ state [normalized to the spectroscopic factor, see Eq. (5)] is shown in Fig. 1 as the solid line. For comparison the result for the Hartree-Fock wave function is shown as the dashed line. From the comparison one can infer that at the quasihole energies no substantial change in the wave functions occur and that the Hartree-Fock wave function is a good approximation. It should further be noted that the wave function of a Woods-Saxon potential, which is constructed as the local equivalent of the Hartree-Fock potential [16], is indistinguishable from the Hartree-Fock wave function. This suggests that the explicit inclusion of short-range correlations does not lead to the strong suppression of the wave function in the interior of the nucleus as has been implied by Refs. [10,11]. Again it should be emphasized that the coupling to the low-lying collective excitations may lead to additional changes in the quasihole wave function. It is unlikely, however, that these changes will involve the high-momentum part of the wave function, which is studied in this work. The calculation of the natural orbitals for the p states does not yield much new information. With the emphasis on short-range correlations in the present work, the diagonalization of the one-body density matrix yields for its largest eigenvalue (0.93 for the $p\frac{3}{2}$ orbital) a wave function which is practically identical to the quasihole wave function (which has strength 0.91).

The hole spectral function in the continuum is obtained from the imaginary part of the single-particle propagator by expressing the propagator in terms of the reducible self-energy Σ_{lj}^{R}. The reducible self-energy can be obtained from the irreducible self-energy by a complex matrix inversion, which solves an equation similar to Eq. (4). The results at three different energies are shown in Fig. 2. The long-dashed curve corresponds to -50 MeV, the solid curve to -100 MeV, and finally the short-dashed curve to -250 MeV. From these results it is clear that an important change in the momentum content of the single-particle strength occurs with increasing excitation energy in the $A=15$ system. At higher excitation energy one finds more high-momentum components. Moreover, these high-momentum components are not observed in the quasihole states. This can be concluded from Fig. 3 where the contribution to the momentum distribution of the quasihole state (dashed line) is compared with the total $p\frac{1}{2}$ momentum distribution including the contribution of the continuum (solid line). This requires the integration of the continuum hole strength for each k according to Eq. (2). This result for ^{16}O is very similar to the observation for ^{3}He made in Ref. [7], where the contribution of the ground state to ground state transition exhibits also very few high-momentum components.

To understand this result, it is important to recall that the appearance of high-momentum components at a certain energy in the $A-1$ system is related to the self-energy contribution containing two-hole–one-particle states at this energy. From energy conservation it is then clear that at low energy it is much harder to find such states with a high-momentum particle than at high energy. This same feature is observed in nuclear matter where the peak of the single-particle spectral function for momenta above k_F increases in energy as k^2 [24]. As a result, the hole strength in nuclear matter as a function of momentum shows the same tendency as the result shown in Fig. 2 [25], i.e., higher momenta become more dominant at higher excitation energy.

The additional $p\frac{1}{2}$ strength in the continuum, which is illustrated in Fig. 2, integrated over k yields 4% of single-particle strength. Together with the quasihole strength this leads to an occupation of $p\frac{1}{2}$ quantum numbers which is less than one. This implies that higher waves like d and f will be partially occupied. Results for these higher waves (and the s wave) will be discussed elsewhere together with results for the average separation energy, Koltun sum rule, etc. [26].

In conclusion, it has been shown in this work that the influence of high-momentum components in the quasihole wave function is of minor importance. By calculating the complete energy dependence of the p-wave hole spectral function it has been demonstrated that the presence of high-momentum components in the nuclear ground state will only show up unambiguously at high excitation energy when the $(e, e'p)$ reaction is employed.

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